Computation of Yield-optimized Pareto Fronts for Analog Integrated Circuit Specifications

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Abstract—For any analog integrated circuit, a simultaneous analysis of the performance trade-offs and impact of variability can be conducted by computing the Pareto front of the realizable specifications. The resulting Specification Pareto front shows the most ambitious specification combinations for a given minimum parametric yield. Recent Pareto optimization approaches compute a so-called yield-aware specification Pareto front by applying a two-step approach. First, the Pareto front is calculated for nominal conditions. Then, a subsequent analysis of the impact of variability is conducted. In the first part of this work, it is shown that such a two-step approach fails to generate the most ambitious realizable specification bounds for mismatch-sensitive performances. In the second part of this work, a novel single-step approach to compute yield-optimized specification Pareto fronts is presented. Its optimization objectives are the realizable specification bounds themselves. Experimental results show that for mismatch-sensitive performances the resulting yield-optimized specification Pareto front is superior to the yield-aware specification Pareto front.

I. INTRODUCTION

During the sizing step of an analog integrated circuit, performances such as gain, bandwidth or power show distinct trade-offs. Pareto optimization offers a method to analyze the performance trade-offs by calculating the optimal compromises between the competing performances. The result is a discrete approximation of the Pareto front of the performances. Many methods have been proposed to calculate the Pareto front under nominal operating and process conditions, either by applying deterministic optimization algorithms, e.g., [1], [2], or stochastic ones, e.g., [3], [4], [5], [6], [7]. Pareto fronts are applied to optimize large-scale analog circuits in a hierarchical fashion, e.g., [3], [8], [9].

It must be assured that variability due to process variations and changing operating conditions as well as resulting device mismatch does not lead to an unacceptable degradation of the performances. The impact of variability on the performance trade-offs can be shown by computing a discrete approximation of the Specification Pareto front (SPF). The SPF is composed of the most ambitious combinations of specification bounds that can be realized such that a given minimum yield requirement is met. Approaches [10], [11], [12], [13] have been proposed to compute the so-called yield-aware SPF in a two-step process. First, the Pareto front for nominal conditions is calculated. In a second step, the impact of variability is either analyzed [11], [12], [13] or the solutions, which are Pareto-optimal under nominal conditions, are subject to local yield optimization [10].

As first contribution of this work, we take a look at the two-step computation of the SPF. It is shown that such a two-step approach is successful and efficient for those performances, for which optimal specification bounds result from optimal nominal performance values. Then, it is shown that such a two-step approach fails to generate the most ambitious realizable specification bounds for mismatch-sensitive performances. This is due to the fact that these performances only take meaningful values if variability is considered. For example, the power source rejection ratio (PSRR) of an operational amplifier will obtain very high values under nominal conditions due to perfectly matching devices. If variability is considered, high PSRR specification bounds can only be realized by finding sizings that are very insensitive to mismatch.

As second contribution, a novel approach is presented that computes a yield-optimized SPF that is capable of handling mismatch-sensitive performances. During computation of the yield-optimized SPF, the specification bounds are the objectives of the optimization. The specification bounds are found by evaluating the performances at worst-case conditions for the operating parameters and process parameters. These worst-case conditions must be recomputed for any new circuit sizing that is encountered during the optimization process. Since this can be computationally very expensive, an efficient method is presented that updates the worst-case conditions efficiently for new sizings. Experiments on two test circuits show the superior results for the proposed computation of the yield-optimized SPF compared to the two-step computation of the yield-aware SPF in the case of mismatch-sensitive performances.

The work is structured as follows. First, basic concepts are reviewed in section II. The yield-aware SPF and yield-optimized SPF are discussed in section III. The efficient computation of the yield-optimized SPF is presented in section IV. Experimental results for a two-stage operational amplifier and a charge-pump circuit are shown in section V. Section VI concludes.

II. BASIC CONCEPTS

A. Circuit parameters

The design parameter vector \( \mathbf{d} \) includes all selectable parameters such as transistor sizes and resistor values. The region of valid parameter vectors, \( \mathcal{D} \), is determined by upper and lower limits and by nonlinear sizing constraints, which assure, e.g., that certain transistors operate in the saturation region:

\[
\mathcal{D} = \{ \mathbf{d} \mid d_L \leq \mathbf{d} \leq d_U \land c(\mathbf{d}) \geq 0 \} \quad (1)
\]

The operating parameter vector \( \mathbf{\theta} \) models environmental conditions such as temperature or supply voltage. The circuit
is required to work correctly for all operating conditions inside some limits defining a box-shaped tolerance region \( T_\theta \):

\[ T_\theta = \{ \theta \mid \theta_L \leq \theta \leq \theta_U \} \] (2)

Process variations that occur during production are modeled by distributions of transistor model parameters such as oxide thickness or threshold voltage. These variations can be described by a multi-dimensional Gaussian probability density function (pdf) of a process parameter vector \( s \) with \( n_s \) components, mean value vector \( s_0 \) and covariance matrix \( C \).

\[ pdf_s(s) = \frac{1}{\sqrt{2\pi} \sqrt{\text{det}(C)}} e^{-\frac{s^2}{2 \text{det}(C)}} \] (3)

\[ \beta^2(s) = (s - s_0)^T C^{-1} (s - s_0) \] (4)

B. Performances, specification and yield

The circuit performances \( f \) include, e.g., gain, bandwidth and power consumption. They are evaluated by analog simulation:

\[ d, \theta, s \xrightarrow{\text{Simulation}} f(d, \theta, s), \quad \mathbb{R}^{n_i}, \mathbb{R}^{n_s}, \mathbb{R}^{n_f} \rightarrow \mathbb{R}^{n_f} \] (5)

Without loss of generality, we assume that performance values become optimal for minimal values. Thus, the specifications \( f_{\text{spec}} \) are given as upper bound limits:

\[ f(d, \theta, s) \leq f_{\text{spec}} \] (6)

The acceptance region \( A_s \) defines the set of process parameters, for which the circuit satisfies the specification for all operating conditions. The parametric yield \( Y_p \) is given as:

\[ Y_p = \int_{A_s} \int_\theta \frac{1}{\sqrt{2\pi} \sqrt{\text{det}(C)}} e^{-\frac{(s - s_0)^T C^{-1} (s - s_0)}{2}} ds \] (7)

III. YIELD-AWARE AND YIELD-OPTIMIZED SPECIFICATION PARETO FRONT (SPF)

A. Yield-aware SPF

The yield-aware SPF is generated in two steps: In the first step, the Pareto front for nominal conditions is generated by solving a multi-objective optimization problem:

\[ \min_{d \in D} f(d, \theta_0, s_0) \rightarrow d_k^+, \quad k = 1, \ldots, K. \] (8)

The result is a set of \( K \) design parameter vectors \( d_k^+ \), which are Pareto-optimal under nominal conditions \( \theta_0 \) and \( s_0 \). In the second step, the impact of variability due to process variations or changing operating conditions is investigated. In [10], a combined Genetic Algorithm and Simulating Annealing method is used to generate the nominal Pareto front. By an efficient Monte-Carlo analysis and local yield optimization, the yield-aware Pareto front is generated in a subsequent step. In [11], another two-step method to generate the yield-aware Pareto front is presented. It uses a numerical performance model called Kriging model to speed up the investigation of the impact of variability on the performances. In [13], a Monte-Carlo Analysis is conducted at each sizing with nominally Pareto-optimal performance vector. In [12], worst-case analysis is applied in order to compute the specification Pareto front for a given minimum yield requirement \( Y_{min} \). The minimum yield value \( Y_{min} \) determines a maximum deviation \( \beta_w \) of the process parameter vector \( s \) from its mean value vector \( s_0 [14] \). It defines an ellipsoid-shaped tolerance region \( T_s \) for the process parameters \( s \):

\[ T_s = \{ s \mid \beta(s) \leq \beta_w \} \] (9)

For minimum yield values \( Y_{min} \), the corresponding \( \beta_w \) is determined by the inverse of the cumulative distribution function of the standard Gaussian distribution (cdf): \( \beta_w = \text{cdf}^{-1}(Y_{min}), \text{e.g., } \beta_w = \text{cdf}^{-1}(0.841) = 1. \)

The worst-case analysis (WCA) is done for each performance \( f \) individually by solving the following worst-case problem at the nominally Pareto-optimal design parameter vectors \( d_k^+ \):

\[ \max_{\theta \in T_\theta, s \in T_s} f_i(d_k^+, \theta, s) \rightarrow \min f_{Y,i}(d_k^+, \theta, s, Y_{i}) \] (10)

It is shown, that the performance value \( f_{Y,i}(d_k^+) \) at the worst-case conditions \( \theta_{Y,i} \) and \( s_{Y,i} \) can be interpreted as realizable specification value as follows: For this design parameter set \( d_k^+ \), a parametric yield of \( Y_{min} \) is obtained considering the single specification bound of \( f_{Y,i}(d_k^+) \) on the performance \( f_i \). In [12], (10) is solved once for each of the \( n_f \) performances at each of the \( K \) nominally Pareto-optimal design parameter vectors \( d_k^+ \). The specification Pareto front is approximated by the set of \( K \) computed specification vectors \( f_{Y}(d_k^+) \).

B. Yield-optimized SPF

The impact of variability can be considered by optimizing the performance values at their respective worst-case conditions:

\[ \min_{d \in D} f_Y(d) \triangleq \begin{bmatrix} f_1(d, \theta_{Y,1}, s_{Y,1}) \\ \vdots \\ f_{n_f}(d, \theta_{Y,n_f}, s_{Y,n_f}) \end{bmatrix} \] (11)

In order to find the worst-case conditions for each performance, the worst-case analysis (WCA) problem of (10) must be solved. Plugging (10) in (11) leads to a multi-objective optimization problem with two inter-dependent optimization tasks [12]:

\[ \min_{d \in D} \begin{bmatrix} f_1(d, \theta_{max}, s) \\ \vdots \\ f_{n_f}(d, \theta_{max}, s) \end{bmatrix} \rightarrow d_k^+, \quad k = 1, \ldots, K. \] (12)

The final performance values at worst-case conditions, \( f_Y(d_k^+) \), are the specification vectors that compose the yield-optimized SPF. Since the specification bounds \( f_{Y,i}(d) \) are the objective of the optimization, the resulting design parameter vectors, \( d_k^+ \), are not only optimized in respect to the nominal performance values, but also in respect to performance degradation due to variability as is discussed in section III-C.

Solving (12) straight forward is usually not possible in reasonable time because the inter-dependent optimization tasks lead to several nested iteration loops, which are illustrated in figure 1. We apply a Sequential Quadratic Programming (SQP)
algorithm to solve the Pareto optimization problem [15]. This SQP algorithm minimizes one scalar optimization problem for each point on the SPF [16]. If the WCA is also solved by gradient-based methods, the gradient for process and operating parameters must be computed in each WCA iteration by sensitivity analysis. Since the sensitivity analysis is nested into the Pareto optimization and WCA loops, it must be conducted prohibitively often. In section IV, it is shown that optimization problem (12) can be solved much more efficiently by updating the worst-case conditions adaptively without solving the complete WCA problem. But first, we illustrate the need for computing the yield-optimized SPF according to problem formulation (12) for mismatch-sensitive performances.

| For each point on the SPF 
| --- |
| **For each iteration of Pareto optimization** 
| --- |
| For each new design parameter vector 
| --- |
| For each performance 
| --- |
| **For each iteration of WCA** 
<table>
<thead>
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<tbody>
<tr>
<td>Sensitivity analysis for θ and s</td>
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</table>

Fig. 1. Nested optimization loops

C. Need for Yield-optimized SPF

The presented alternative formulations (8) or (12) are multi-objective optimization problems for several design parameters. For simplification, the case of an optimization problem with one performance \( f \) and one design parameter \( d \) is discussed in the following. Such one-dimensional examinations are familiar from yield optimization and keep their significance in Pareto optimization considering variability. Figure 2 shows two illustrative examples for the dependency of the nominal performance value \( f(d) = f(d, θ₀, s₀) \) and the specification bound \( f_Y(d) = f(d, θ_Y, s_Y) \) for some yield requirement \( Y_p ≥ Y_{min} \).

In the upper-left part of figure 2, the optimal (minimal) nominal performance value \( f^*_Y \) is obtained at the design parameter \( d^* \). The corresponding specification value \( f_Y^* \) is inferior due to the impact of variability. This is illustrated in the upper-right part. It shows the pdf of the performance values, which results from variability. In order to meet the yield requirement, the specification bound \( f_Y^* \) must be placed such that \( Y_p \), which is equal to the area under the pdf up to \( f_Y^* \), is at least equal to \( Y_{min} \). The resulting performance degradation due to variability \( Δ^* = f_Y^* - f^*_Y \) depends on the shape of the pdf.

If we take a closer look at the upper part of figure 2, it can be seen that the specification bound \( f_Y^* \) at \( d^* \) is the most ambitious (minimal) specification bound realizable for \( Y_p ≥ Y_{min} \). Thus, in this case, the two-step approach calculates \( d^* \) by minimizing \( f(d) \) (nominal optimization) and, in the subsequent step, obtains \( f_Y^* \) with a single WCA. The new proposed approach would minimize \( f_Y(d) \) and obtain \( d^* = d^* \). The resulting most ambitious specification bounds are equal \( f_Y^* = f_Y^* \) and optimal. In this case, the two-step approach would be more efficient because it is based on nominal optimization.

But this is not always the case. The lower part of figure 2 shows a different functional relationship: The nominal optimum \( f^* \) is located at a different design parameter vector than the desired optimum for the specification bound \( f_Y^* \). In this case, the two-step approach does not yield the most ambitious specification bound but instead \( f_Y^* \). Comparing the pdfs of the performance at \( d^* \) and \( d^+ \), it is obvious that the specification value \( f_Y^* \) is inferior to \( f_Y^* \) due to a large impact of the variability at \( d^* \), leading to a large \( Δ^* \). The minimization of \( f_Y(d) \), as is proposed for the computation of the yield optimized SPF in eq. (12), results in \( d^+ \). At \( d^+ \), even though the nominal performance value \( f^* \) is inferior to the nominally optimized nominal performance value \( f^* \), we obtain the most ambitious specification bound \( f_Y^* \) due to smaller performance degradation \( Δ^* \), which results from the more narrow pdf. This narrow pdf indicates smaller sensitivities towards variability at \( d^+ \).

The two-step approach will compute a good approximation of the SPF for any performance set, for which nominally optimal performance vectors and optimal specification vectors are encountered at the same design parameter vectors (or near to each other, if local yield optimization is applied). Usually, such a behavior may not be expected for mismatch-sensitive performances. Figure 3 illustrates the yield-aware and yield-optimized SPFs for the mismatch-sensitive case. The yield-optimized SPF features superior specification vectors compared to the two-step approach, showing the true realizable specification capabilities of the circuit block. This can be seen for the two-stage opamp in section V. Another effect was encountered for the charge-pump. For this circuit, only the yield-optimized SPF shows the trade-off in circuit performances.
In order to solve (12) in reasonable time, linear models for the dependency of each performance \( f_i \) on the operating parameter vector \( \theta \) and process parameter vector \( s \) are set up and updated repeatedly during the optimization. This is similar to the so-called realistic worst-case analysis presented in [14]. The optimization flow of the proposed method is illustrated in table I. For easier understanding, the steps are simplified and only statistical parameters \( s \) are considered. In the following, each step is discussed shortly:

**Start**: The optimization starts with an initial design parameter vector, \( d^{(0)} \).

**WCA 1**: A first estimation of the worst-case parameter vectors, \( s_Y^{(1)} \), is found by setting up the linear model at \( d^{(0)} = d_{\text{start}} \) and nominal conditions \( s_0 \).

**SIM 1**: The realizable specification vector at the initial design is equal to the performance values at their respective worst-case conditions \( s_{Y,i}^{(1)} \).

**PO 1**: The SQP method calculates a new design parameter vector, \( d^{(1)} \), (or often more than one).

**WCA 2**: The worst-case parameter vectors, \( s_{Y,i}^{(2)} \), are updated for the parameter vector \( d^{(1)} \). For each performance, the linear model is set up by selecting the worst-case parameter vector of the last iteration step, \( s_{Y,i}^{(1)} \), as linearisation point. Because the true solutions to the WCAs (true worst-case process parameter vectors) depend usually only weakly on \( d \), \( s_{Y,i}^{(1)} \) found at \( d^{(0)} \) is a good guess for \( s_{Y,i}^{(2)} \) computed for \( d^{(1)} \). During the Pareto optimization, the worst-case conditions of the last iteration step are passed on and used as linearisation points repeatedly. The worst-case process parameter vectors are updated based on the information collected in several iteration steps. This increases the accuracy of the estimated worst-case conditions as the Pareto optimization proceeds.

**SIM 2**: The realizable specification vector at the new design parameter vector \( d^{(1)} \) can be simulated. It is checked, if a better objective function value according to [16] is obtained. In this case, the new design parameter vector \( d^{(1)} \) is accepted and the optimization is continued. If a step leads to no further improvement in the scalar objective function, the optimization terminates.

This adaptive update removes the iterative loop to solve the WCA optimization problem. This is illustrated in figure 4.

<table>
<thead>
<tr>
<th>Step</th>
<th>Linear model</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start</td>
<td>( d^{(0)} = d_{\text{start}} ): ( s_0 )</td>
<td>( \nabla_s f_i</td>
</tr>
<tr>
<td>WCA 1</td>
<td>( f_i(d^{(0)}, s_{Y,i}^{(1)}): i = 1, \ldots, n_f )</td>
<td>( d^{(1)} )</td>
</tr>
<tr>
<td>SIM 1</td>
<td>( f_i(d^{(1)}, s_{Y,i}^{(2)}): i = 1, \ldots, n_f )</td>
<td>( d^{(2)} )</td>
</tr>
<tr>
<td>PO 2</td>
<td>( f_i(d^{(2)}, s_{Y,i}^{(2)}): i = 1, \ldots, n_f )</td>
<td>( d^{(3)} )</td>
</tr>
</tbody>
</table>

**TABLE I**

**OPTIMIZATION FLOW**

For each point on the SPF

<table>
<thead>
<tr>
<th>For each iteration of Pareto optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>For each new design parameter vector</td>
</tr>
<tr>
<td>For each performance</td>
</tr>
<tr>
<td>Sensitivity analysis for ( \theta ) and ( s )</td>
</tr>
</tbody>
</table>

**V. EXPERIMENTAL RESULTS**

**A. Two-stage operational amplifier**

The two-stage opamp is realized in an industrial 180 nm technology with 1.8 V supply voltage. Its schematic is shown in figure 5. The optimization includes 11 design parameters. Two operating parameters (vdd and temperature) are considered. The process variations are modeled in this technology with 13 global and 16 local (2 per transistor) statistical parameters. In addition to the saturation constraints, a DC Gain of 70 dB and a Phase Margin of 60 degree is demanded.

Figure 6 shows the Pareto fronts for the performances Power Supply Rejection Ratio (PSRR) and transit frequency. The yield requirement on both performances is set to 99.86%, a so-called three-sigma design (\( \beta = 3 \)). The two-step approach computes the nominal Pareto front first [12]. Due to perfectly matching transistors, very high nominal PSRR values up to 138 dB are obtained on the nominal Pareto front. The subsequent analysis of the impact of variability leads to the yield-aware SPF (two-step). Since variability was ignored during the nominal Pareto optimization, large degradation of the PSRR values result for this yield-aware SPF. No specification bound for PSRR higher than 72 dB seems to be realizable. In contrast, the proposed novel method targets at the specification bounds from the beginning. In figure 6, the yield-optimized SPF (two-step)
SPF is superior to the yield-aware SPF(two-step). For the given yield requirement, PSRR specification values up to 98 dB can be realized showing the true specification capabilities of the opamp. In order to investigate the results further, a Monte-Carlo analysis (MCA) is conducted at each Pareto-optimal design parameter vector found by both approaches with 5000 sample elements each. The variability clouds in the performance space, shown in figure 7, illustrate that the spread for PSRR values is larger for the yield-aware SPF (two-step) compared to the yield-optimized SPF. The yield-optimized SPF finds variability-insensitive sizings, reducing the impact of variability on PSRR performance. Table II shows the computational time. The computation of the yield-optimized SPF takes about 2.5x longer compared to the two-step approach.

**B. Charge-Pump**

The structure of the charge-pump is shown in figure 8. It is realized in a 180 nm demonstrator technology. Design parameters are the current mirror sizes \( w_n, l_n, w_p \) and \( l_p \). Process variations can be considered for five globally varying process parameters (\( \delta t_{ox}, \delta \mu_n, \delta \mu_p, \delta v_{th,n} \) and \( \delta v_{th,p} \)). The operating temperature is also taken into account. This circuit has three operating modes. If up and down are low, switches S1b and S2b are closed and the output current \( I_{out} \) is close to zero. If up is high, \( I_{out} = I_{up} \approx I_b \). The bias current \( I_b \) is mirrored via current mirrors N1/N2 and P1/P2 and switch S1b is open. If down is high, \( I_{out} = I_{down} \approx -I_b \). The bias current is mirrored via current mirror N1/N3 and switch S2b is open. An important performance of this circuit is the up-down current matching: \( I_{up} = -I_{down} \). Perfect matching is difficult to achieve because p-type and n-type current mirrors are used. Process variations will cause mismatch of up and down current. The relative impact of variability can be reduced by increasing the size of the transistors at the cost of a larger total transistor area.

Figure 9 shows the Pareto fronts for the investigation of the trade-off between up-down current difference and transistor area. Under nominal condition, no trade-off is visible and the computed design parameter vectors can realize almost perfect current matching at minimal transistor area. Applying the two-step approach, the subsequent analysis of the impact of variability shows that one may only specify a current difference as high as 2.4 \( \mu A \) for a yield requirement of 99.86%. No sizings with less current difference are computed by the two-step approach.

In contrast, the yield-optimized SPF is able to show the trade-off between total transistor area and up-down current difference. Several optimal compromises between the two performances are found. An up-down current difference of about 1 \( \mu A \) is obtained for an area of 3.4 \( \mu m^2 \). The tightening of the current matching specification comes at the cost of a larger total transistor area, as was expected. Figure 10 shows the variability clouds for the yield-optimized SPF, found by Monte-Carlo analysis at each point of the SPF. Each MC sample had a size of 1000. The area is not affected by the variations such that the MCA results in lines parallel to the current difference axis. As can be seen, larger transistor area

<table>
<thead>
<tr>
<th>Circuit</th>
<th>yield-aware</th>
<th>yield-optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-stage OpAmp</td>
<td>2h 17min</td>
<td>5h 41min</td>
</tr>
<tr>
<td>Charge pump</td>
<td>30 min</td>
<td>8h 11 min</td>
</tr>
</tbody>
</table>

**TABLE II**

**Computational time on Dual quad-core Xeon PC with 1.86 GHz and 4 GB RAM.**
can be used to reduce the impact of variability on current mismatch, visible in a decreasing spread for the samples at larger transistor area.

Table II shows the computational time. The yield-optimized SPF takes about 16x longer than the two-step approach. The long optimization times result from the simulation setup. A transient simulation of the complete switching behavior is performed. This results in a relative long simulation time for the rather small circuit size of about 12 seconds.

VI. CONCLUSIONS

The presented yield-optimized SPF computes superior specification vectors for mismatch-sensitive performances compared to the state of the art. It can be computed efficiently by an adaptive update of the worst-case conditions.

REFERENCES


