A Coupled Battery State-of-Charge and Voltage Model for Optimal Control Applications

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Abstract—Optimal control of electric vehicle (EV) batteries for maximal energy efficiency, safety and lifespan requires that the Battery Management System (BMS) has accurate real-time information on both the battery State-of-Charge (SoC) *and its dynamics*, i.e. long-term and short-term energy supply capacity, at all times. However, these quantities cannot be measured directly from the battery, and, in practice, only SoC estimation is typically carried out. In this article, we propose a novel parametric algebraic voltage model coupled to the well-known Manwell-McGowan dynamic Kinetic Battery Model (KiBaM), which is able to predict both battery SoC dynamics and its electrical response. Numerical simulations, based on laboratory measurements, are presented for prismatic Lithium-Titanate Oxide (LTO) battery cells. Such cells are prime candidates for modern heavy offroad EV applications.

I. INTRODUCTION

A key challenge in optimal control of battery systems is that the energy content (*State-of-Charge*, SoC) is not measurable outside the battery. In practice, mathematical modeling of SoC is thus often based on voltage (V) and current (I), which can be measured. One seeks a SoC dependent dynamical Equivalent Circuit Model (ECM), that can predict the electrical response of the battery (see e.g. [1], [2]), and that model is then used for SoC estimation via simple Coulomb counting. However, it is important to emphasize that the obtained SoC estimate is *static* and *memoryless*, i.e., it does not address the effect of the usage history of the battery on the energy (or power).

To address battery energy dynamics, in this article, we take an opposite battery modeling approach: We propose a simple algebraic electrical subsystem model coupled to a more complex dynamical model of the energy subsystem. The energy model considered herein is the Kinetic Battery Model (KiBaM) specification introduced by Manwell and McGowan in 1993 [3], see Fig. 1. It is well known to be able to represent the recovery and rate-capacity effects seen in real batteries [4], among others. The challenge with the KiBaM model is relating it to the battery voltage and current, which is necessary for parameter identification and practical use. Manwell et al. [5] proposed the simple algebraic specification $V = V_{oc} - R_s I$, where V_{oc} denotes the open-circuit voltage and R_s the internal resistance of the battery, but without relation to SoC. Further, Bako et al. [6] and Manwell et al. [5] proposed a rational voltage models with SoC dependence. On the other hand, Fenner et al. [7] proposed a parametric rational-exponential voltage law targeted at replicating the response seen in constant current discharge tests. However, the nonlinearities in these electrical subsystem models potentially



Fig. 1. Overall view of the proposed battery model and parameter identification methodology

make parameter identification complex, and also impose a heavier computational burden on the battery SoC estimation during runtime. It is, therefore, of considerable theoretical and practical interest to establish a simple and computationally lightweight but accurate electrical subsystem model to be augmented with the KiBaM. Such a model is presented herein, along with its offline parameter identification method, which utilizes standard dynamic discharge profile laboratory test data [8].

II. BATTERY MODEL

We couple the well-known Kinetic Battery Model (KiBaM) to our proposed linear voltage model. The KiBaM models the electro-chemical dynamics of battery's internal states and represent the energy/charge balance, while the voltage model relates KiBaM to the battery's terminal voltage. Then, given the training data, we jointly optimize the parameter set of the coupled model $p = (c, k, E_0, E_1, E_2)$ using interior-point optimization method [9].

Fig. 1 illustrates the overall view of the proposed method. The KiBaM stores the electric charge in two tanks, *bound charge* tank and *available charge* tank which are connected through a limited-rate valve. Variables y_1 and y_2 represent the amount of charge in each tank, k represents the flow rate through the valve and $c \in (0, 1)$ represents the relative capacity of the tanks. The dynamics of charge in each tank is modeled by the following differential equations:

$$\frac{dy_1}{dt} = \frac{k}{1-c}y_2(t) - \frac{k}{c}y_1(t) - I(t)$$
(1a)

$$\frac{dy_2}{dt} = -\frac{k}{1-c}y_2(t) + \frac{k}{c}y_1(t)$$
(1b)

The proposed voltage model is a linearly parameterized equation describing voltage response of the battery based on the Rint equivalent circuit model [10] and internal energy levels of the two tanks as:



Fig. 2. KiBaM parameters (k, c) and voltage model parameters $(E_0, E_1, E_2, V_{oc}, R_s)$ over SoC level. Red points indicate the optimized parameters of each subrange and blue lines indicate their linear interpolation over the whole range of SoC.

$$V(t) = V_{OC} - R_s \cdot I(t) + E_0 + E_1 \cdot \frac{y_1(t)}{Q_0} + E_2 \cdot \frac{y_2(t)}{Q_0}$$
(2)

where R_s is the battery internal resistance, V_{OC} is the battery open-circuit voltage, Q_0 is the nominal capacity of the battery and E_0 , E_1 and E_2 are the parameters. Finally, we defined the SoC at each time by:

$$SoC(t) = \frac{y_1(t) + y_2(t)}{Q_0}$$
(3)

It is essential to mention that, in our experiments, parameters of Rint, KiBaM and voltage model are all dependent on current SoC level. Therefore, all should be subscripted with SoC(t) in equations 1a, 1b and 2, but for notational convenience, this dependence is not explicitly denoted.

We fit the proposed model to the measurement data by utilizing interior-point optimization. It is well-known that non-linear programming methods such as interior-point are sensitive to the chosen initial point. Therefore, to mitigate this effect, we propose a four-stage parameter identification method that finds a reasonably good initial point by incrementally increasing the complexity of the optimization (details are omitted due to space constraints).

III. EXPERIMENTS

The battery cell considered herein is a 23Ah Toshiba SCiBTM LTO battery cell. To train the model and validate the results, we used data from Discharge Pulse Power Characterization (DPPC) tests [8]. The test was performed in room temperature at two different charge rates.

We obtained 0.065% mean percentage error and 1.6 mV mean absolute error for voltage prediction on out-of-sample test data that outperforms previous methods and is marginal for various demands in battery voltage estimation. The optimized parameters are presented in Fig. 2 as functions of SoC. The change in SoC-dependent parameters shows that the battery behaves significantly differently at various SoC levels. Considering the parameter k, for example, it can be seen when the battery is full, the charge transfer rate from the bound charge tank to the available charge tank is higher than when it is empty, thus faster energy recovery can be obtained at fully charge. Additionally, the trend in parameter c suggests that the bound charge tank is used to store the charge as a reserved energy unit which is released as the SoC drops. Trends in V_{OC} and R_s are as expected, and E_0 almost similar trend to V_{OC} , affecting the predicted voltage as a correction value for V_{OC} .

IV. CONCLUSION AND FUTURE WORK

The structural modeling that is proposed in this paper, enables the opportunity to utilize the hidden states of charge, namely y_1 and y_2 , in the optimal control applications in which short-term high power demands can be controlled w.r.t. the value of y_1 and long-term planing based on energy reserve is enabled with the knowledge of y_2 . Moreover, the optimal control algorithm can utilize these information to schedule a rest period after a demanding workload to recover the battery voltage level. Another intresting future direction of this work is to incorporate ambient temperature and battery aging into the model as two important factors that can majorly affect battery behavior. Additionally, since usually the initial state of the battery is unknown, there is need to apply a filtering method to estimate the internal hidden states of the model based on measurable input and output of the battery system. Extended Kalman filters are among the best candidates for such purposes.

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