

# An Elastic Mixed-Criticality Task Model and Its Scheduling Algorithm

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**Abstract**—To address the service abrupt problem for low-criticality tasks in existing mixed-criticality scheduling algorithms, we study an *Elastic Mixed-Criticality (E-MC)* task model, where the key idea is to have variable periods (i.e., service intervals) for low-criticality tasks. The minimum service requirement of a low-criticality task is ensured by its largest period. However, at runtime low-criticality tasks can be released early by exploiting the slack time generated from the over-provisioned execution time for high-criticality tasks to reduce their service intervals and thus improve their service levels. We propose an *Early-Release EDF (ER-EDF)* scheduling algorithm, which can judiciously manage the early release of low-criticality tasks without affecting the timeliness of high-criticality tasks. Compared to the state-of-the-art EDF-VD scheduling algorithm, our simulation results show that the ER-EDF can successfully schedule much more task sets. Moreover, the achieved execution frequencies of low-criticality tasks can also be significantly improved under ER-EDF.

## I. INTRODUCTION

In cyber-physical systems (CPS), which have been denoted as the next-generation engineering systems, the computation tasks can have different levels of importance according to their functionalities that further lead to different criticality levels [1]. To incorporate various certification requirements and enable efficient scheduling of such tasks, the *mixed-criticality* task model has been studied recently [3], [8], [10], where a task generally has multiple worst case execution times (WCETs) according to different certification levels.

Considering the increasing need to execute tasks with multiple criticality levels on a shared computing system, how to efficiently schedule such mixed-criticality tasks while satisfying their specific requirements has been identified as one of the most fundamental issues in CPS [3]. Note that, without proper provisions for such mixed-criticality tasks, traditional scheduling algorithms are likely to cause the so-called “*pri-*ority *inversion*” problems [8]. In [11], Vestal first defined and formalized the mixed-criticality scheduling problem with multiple certification requirements at different degrees of confidence and studied a fixed priority algorithm. For sporadic mixed-criticality tasks, a hybrid-priority algorithm that combines EDF and Vestal’s fixed priority algorithm was studied in [4].

From a different aspect of mixed-criticality tasks, De Niz *et al.* proposed a zero-slack scheduling approach, which works on the top of fixed-priority based preemptive scheduling algorithm (such as RMS) [8]. More recently, for the scheduling of sporadic mixed-criticality task systems, Baruah *et al.* proposed

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a more efficient scheduling algorithm, namely EDF-VD (virtual deadline), that assigns *virtual* (and smaller) deadlines for high-criticality tasks to ensure their schedulability in the worst case scenario [2]. In [10], Santy *et al.* studied an online scheme that calculates a delay-allowance for entering high-level state late and thus delays the cancellation of low-criticality tasks to improve their services.

Note that, most existing mixed-criticality scheduling algorithms guarantee the timeliness of high-criticality tasks in the worst case scenario at the expense of low-criticality tasks. For instance, when any high criticality task uses more time than its low-level WCET and causes the system to enter high-level execution mode, *all* low-criticality tasks will be discarded to provide the required computation capacity for high-criticality tasks [2], [3], [8], [10]. Such an approach can cause serious service abrupt and significant performance loss for low-criticality tasks, especially for control systems where the performance of controllers is mainly affected by the execution frequency and period of control tasks [12].

To address such service abrupt and provide minimal service guarantee for low-criticality tasks, we study in this work an *Elastic Mixed-Criticality (E-MC)* task model by adopting the idea of variable periods in elastic scheduling [6], [9]. Specifically, the largest period of a low-criticality task can be determined by its minimum service requirement. However, low-criticality tasks can be released early and more frequently at runtime to improve their service levels. We propose an *Early-Release EDF (ER-EDF)* scheduling algorithm that allows low-criticality tasks to release early without sacrificing the timeliness of high-criticality tasks. We analyze the schedulability of E-MC tasks under ER-EDF and evaluate its performance through extensive simulations.

The results show that, compared to the state-of-the-art EDF-VD mixed-criticality scheduling algorithm [2], the proposed E-MC task model and ER-EDF algorithm are much more effective in scheduling mixed-criticality tasks. First, when low-criticality tasks have reasonable minimum service requirements (with their periods being extended 2 to 5 times), ER-EDF can schedule more task sets than EDF-VD. In addition, the achieved execution frequencies (i.e., service levels) for low-criticality tasks with a few early-release points under ER-EDF are significantly better than those under EDF-VD.

The remainder of this paper is organized as follows. Section II presents the E-MC task model and a motivational example. The ER-EDF scheduling algorithm is presented and analyzed in Section III. The evaluation results are discussed in Section IV and Section V concludes the paper.

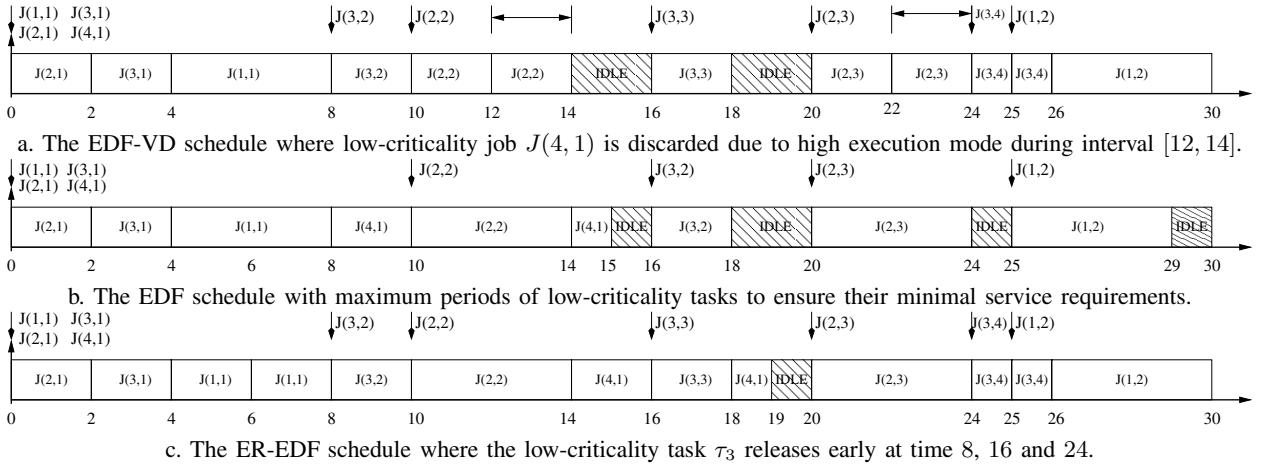


Fig. 1: Schedules for the example mixed-criticality task set within the interval  $[0, 30]$ ;  $J(i, j)$  denotes the  $j^{th}$  job of  $\tau_i$ .

## II. ELASTIC MIXED-CRITICALITY TASK MODEL

We consider a set of  $n$  tasks  $\Gamma = \{\tau_1, \dots, \tau_n\}$  running on a uniprocessor system, where the criticality level of a task  $\tau_i$  is denoted as  $\zeta_i$ . In this work, we focus on systems with two different criticality levels<sup>1</sup>, which are denoted as  $\zeta^{low}$  and  $\zeta^{high}$ , respectively. For a high-criticality task  $\tau_i$  (i.e.,  $\zeta_i = \zeta^{high}$ ), the same as in the traditional mixed-criticality task model, it has a period  $p_i$  and two different worst case execution time (WCETs), which are denoted as  $c_i^{low}$  and  $c_i^{high}$  and correspond to its low-level and high-level execution requirements, respectively. Similarly,  $u_i^{low} = \frac{c_i^{low}}{p_i}$  and  $u_i^{high} = \frac{c_i^{high}}{p_i}$  represent task  $\tau_i$ 's low-level and high-level task utilizations, respectively.

The **major difference** between the *Elastic Mixed-Criticality (E-MC)* task model and the traditional mixed-criticality task model is how to represent low-criticality tasks. Here, a low-criticality task  $\tau_i$  (i.e.,  $\zeta_i = \zeta^{low}$ ) with E-MC model has a *maximum* period  $p_i^{max}$  to reflect its *minimum* service requirements, in addition to a *desired* period  $p_i$  (which is mainly for existing mixed-criticality scheduling algorithms). Moreover, the low-criticality task  $\tau_i$  has a set of  $k_i$  possible *early-release points*  $P_i^{ER} = \{p_{i,1}, \dots, p_{i,k_i}\}$ , where  $c_i < p_{i,1} < \dots < p_{i,k_i} < p_i^{max}$  and  $c_i$  is  $\tau_i$ 's WCET. The early-release points of low-criticality tasks enable them to release early and improve their execution frequencies at runtime through appropriate slack reclamation. The desired and minimum utilizations of task  $\tau_i$  are defined as  $u_i = \frac{c_i}{p_i}$  and  $u_i^{min} = \frac{c_i}{p_i^{max}}$ , respectively.

A set of E-MC tasks is said to be **E-MC schedulable** if the high-criticality tasks' high-level execution requirements and low-criticality tasks' minimum service requirements can be guaranteed in the worst case scenario. Following the notations in [2], we define  $U(H, L) = \sum_{\tau_i \in \Gamma}^{\zeta_i = \zeta^{high}} u_i^{low}$  and  $U(H, H) = \sum_{\tau_i \in \Gamma}^{\zeta_i = \zeta^{high}} u_i^{high}$  as the low-level and high-level utilizations of high-criticality tasks, respectively. Similarly, for low-criticality tasks,  $U(L, L) = \sum_{\tau_i \in \Gamma}^{\zeta_i = \zeta^{low}} u_i$  and

$U(L, min) = \sum_{\tau_i \in \Gamma}^{\zeta_i = \zeta^{low}} u_i^{min}$  represent their desired and minimum utilizations, respectively. Based on the notations, we can easily get the following lemma.

**Lemma 1:** A set of E-MC tasks is E-MC schedulable under EDF if there is  $U(H, H) + U(L, min) \leq 1$ .

### A. A Motivational Example

We first show the effectiveness of E-MC in modeling and scheduling mixed-criticality tasks through a concrete example. There are four tasks in the example, where the first two are high-criticality tasks and the other two are low-criticality tasks. Their timing parameters are given in Table I.

	Basic MC Parameters			Parameters for E-MC		EDF-VD [2]
	$\zeta_i$	$c_i$	$p_i$	$p_i^{max}$	$P_i^{ER}$	
$\tau_1$	$\zeta^{high}$	{4, 10}	25	-	-	13.85
$\tau_2$	$\zeta^{high}$	{2, 4}	10	-	-	5.54
$\tau_3$	$\zeta^{low}$	2	8	16	{8}	8
$\tau_4$	$\zeta^{low}$	3	30	40	{30}	30

TABLE I: An Example Task Set

From the tasks' basic MC parameters, we can find that they are schedulable under EDF-VD [2], where the virtual deadlines of tasks are shown in the last column. Suppose that the second and third jobs of task  $\tau_2$  take its high-level WCET  $c_2^{high}$  while other high-criticality jobs take their corresponding low-level WCETs, the EDF-VD schedule (which relies on tasks' virtual deadlines) of the task set within the interval  $[0, 30]$  is shown in Figure 1a. The system enters high-level execution mode at time 12 and the active low-criticality job  $J(4, 1)$  during the high-level interval  $[12, 14]$  is discarded, which leads to service abrupt for  $\tau_4$  during its first period.

Note that there is  $U(H, H) + U(L, L) > 1$  for the example task set and it is impossible to guarantee the *desired* service levels for the low-criticality tasks  $\tau_3$  and  $\tau_4$ . However, if the minimum service requirements of  $\tau_3$  and  $\tau_4$  can be specified by their maximum periods, which are 16 and 40 respectively, such requirements can be guaranteed under EDF since there is

<sup>1</sup>We will study E-MC tasks with more criticality levels in our future work.

$U(H, H) + U(L, \min) = 1$ . The corresponding EDF schedule for interval  $[0, 30]$  is shown in Figure 1b.

Here, we can see that there are several idle intervals (i.e., slack) in the EDF schedule since not all high-criticality jobs take their high-level WCETs. Such slack can be exploited to improve the execution frequencies of low-criticality tasks. For instance, task  $\tau_3$  can release its second job  $J(3, 2)$  at time 8 (its early-release point) instead of waiting until time 16 (its maximum period) as shown in Figure 1c. It is clear that such early releases of low-criticality tasks have to be managed with great care to not affect the timeliness of high-criticality tasks.

### III. EARLY-RELEASE EDF SCHEDULING ALGORITHM

In this section, we study an *Early-Release EDF (ER-EDF)* scheduling algorithm, which enables low-criticality tasks to release earlier than their maximum periods and thus to improve their service levels. To prevent such early releases from affecting the execution of high-criticality tasks, there are a few key issues to address in ER-EDF. First and the most important is to determine *the deadline of a low-criticality job should it be released early*. Suppose that the first  $j$  jobs of a low-criticality task  $\tau_i$  have arrived regularly according to its maximum period  $p_i^{\max}$ , where the  $j^{\text{th}}$  job  $J(i, j)$  arrived at time  $r_{i,j}$  and has its deadline at time  $d_{i,j} = r_{i,j} + p_i^{\max}$ . Moreover, let's assume that  $J(i, j)$  finished its execution no later than  $r_{i,j} + p_{i,x}$ , where  $p_{i,x}$  ( $x = 1, \dots, k_i$ ) is one of  $\tau_i$ 's early-release points.

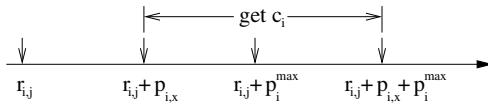


Fig. 2: The deadline of an early-release job.

As shown in Figure 2, if the next job  $J(i, j+1)$  is released at the early release point  $r_{i,j+1} = r_{i,j} + p_{i,x}$ , the new deadline for  $J(i, j+1)$  would be  $d_{i,j+1} = r_{i,j+1} + p_i^{\max}$ . That is, the *expected deadline* of a low-criticality job is always assigned according to the task's maximum period. First, such deadline assignment for early-released low-criticality jobs ensures their minimum service requirements. Moreover, it ensures the same number of early-release points before a job's deadline and enables uniform handling of future early releases.

Once we know how to assign deadlines for early-release jobs, the second important issue in ER-EDF is to determine whether it is *feasible* to release a job at a given early-release point. Intuitively, releasing a job at an earlier time point (with an earlier deadline) introduces extra workload to the system. Therefore, to avoid overload condition and affecting the execution of other (especially high-criticality) tasks, such early-release decisions require judicious slack allocation and demand efficient slack management [7], [13]. In this work, we adopt and extend the wrapper-task mechanism [13], which has shown to be an effective and efficient slack management technique. In what follows, we first discuss the early-release decision algorithm. The wrapper-task based slack management technique will be detailed in Section III-B.

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#### Algorithm 1 : Early-Release Algorithm of ER-EDF

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1: Input:  $\tau_i$ ,  $p_{i,x}$  and  $SlackQ(t)$  at current time  $t = r_i + p_{i,x}$ 
2:  $S_{\text{demand}} = c_i - p_{i,x} \cdot \frac{c_i}{p_i^{\max}}$ ; //calculate demanded slack
3: if ( $S_{\text{demand}} \leq CheckSlack(SlackQ(t), t + p_i^{\max})$ ) then
4:   //next job of  $\tau_i$  can be released earlier at time  $t$ 
5:    $ReclaimSlack(SlackQ(t), S_{\text{demand}})$ ;
6:    $r_i = t; d_i = t + p_i^{\max}$ ; //reset release time and deadline
7:    $Enqueue(ReadyQ, J_i)$ ; //add the job to ready queue
8: else
9:   //set the next early-release point for  $\tau_i$  (if  $x < k_i$ );
10:   $SetTimer(r_i + p_{i,x+1})$ ;
11: end if

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#### A. Early-Release Decisions

As the *centerpiece* of ER-EDF, the major steps for the early-release decision algorithm are shown in Algorithm 1. The algorithm will be invoked at any low-criticality task  $\tau_i$ 's early-release time point  $t = r_i + p_{i,x}$ , which is after the finish time of its current job that is released at time  $r_i$ . Moreover, we assume that  $SlackQ(t)$  contains the available slack at time  $t$  and  $ReadyQ$  holds all arrived ready jobs.

The amount of slack  $S_{\text{demand}}$ , which is needed for task  $\tau_i$  to *safely* release its next job at its early-release time point  $t$  ( $= r_i + p_{i,x}$ ), is first calculated (line 2). From previous discussions, the deadline of the to-be-released job of  $\tau_i$  will be  $t + p_i^{\max}$  (as shown in Figure 2). Note that, the minimum service requirement of  $\tau_i$  is assumed to be statically guaranteed. Therefore, during the interval  $[r_i + p_i^{\max}, t + p_i^{\max}]$ , which is between the current and new deadlines of  $\tau_i$  and has the length of  $p_{i,x}$ , the amount of time that can *safely* be allocated to  $\tau_i$  is  $p_{i,x} \cdot u_i^{\min} = p_{i,x} \cdot \frac{c_i}{p_i^{\max}}$  [5]. Here, the new job of  $\tau_i$  would need  $c_i$  time units within the interval  $[t, t + p_i^{\max}]$  (as shown in Figure 2) should it be released at time  $t$ . Hence, we can get  $S_{\text{demand}}$  as shown in the algorithm.

From [13], we know that slack has a deadline (i.e., its priority) in EDF-based scheduling and not all available slack can be utilized by a given task. Specifically, for task  $\tau_i$ 's new job that has a deadline at time  $t + p_i^{\max}$ , only the slack that has its deadline no later than  $t + p_i^{\max}$  can be reclaimed. Here, to find out the amount of *reclaimable* slack before a given time  $d$  in the current slack queue  $Q$ , a function  $CheckSlack(Q, d)$  is used that will be discussed in detail in the next section.

If there is enough amount of reclaimable slack,  $S_{\text{demand}}$  units of slack will be reclaimed by task  $\tau_i$  and be removed from the slack queue with the help of function  $ReclaimSlack()$  (line 5); Then, the new job of task  $\tau_i$  is released and put into the ready job queue (lines 6 and 7). Otherwise,  $\tau_i$  cannot release its job at time  $t$ . To give  $\tau_i$  a chance to release its job at future early-release points (if any), a new timer is set at  $\tau_i$ 's next early-release point (line 10). Clearly, the number of early-release points has a great impact on the execution improvement of low-criticality tasks. Our evaluation results (see Section IV) show that a few (e.g., 5) such points are effective enough to improve low-criticality tasks' executions.

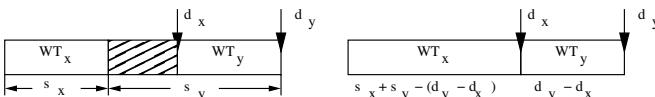
## B. Slack Management with Wrapper-Tasks

*Wrapper-task* has been studied as an efficient mechanism to manage slack for energy savings and reliability enhancements [13]. In this work, we extend and exploit the wrapper-task approach to safely provide the needed slack and enable low-criticality tasks release their jobs at their early-release time points. Essentially, a wrapper-task (*WT*) represents a piece of dynamic slack with two parameters  $(s, d)$ , where  $s$  denotes the amount of slack and  $d$  is the deadline that equals to that of the task giving rise to this slack [13].

Similar to ready tasks, which are kept in a *ready queue* (*ReadyQ*) in the increasing order of their deadlines (i.e., tasks with smaller deadlines are in front of *ReadyQ* and tie is broken to favor the task with smaller index), wrapper-tasks are kept in a *slack queue* (*SlackQ*) in the increasing order of their deadlines as well. At runtime, unclaimed wrapper-tasks compete for the processor with ready tasks. With EDF-based scheduling, when both queues are *not empty* and the header wrapper-task  $WT_h$  of *SlackQ* has earlier deadline than *ReadyQ*'s header task  $\tau_h$ ,  $WT_h$  will wrap  $\tau_h$ 's execution by lending its time to  $\tau_h$ . When the wrapped execution completes,  $\tau_h$  returns its borrowed slack by creating a new piece slack with the length of wrapped execution and  $\tau_h$ 's deadline (i.e., the slack is actually *pushed forward* with a later deadline). When *ReadyQ* is empty,  $WT_h$  executes no-ops and the corresponding slack is wasted. The basic operations of wrapper-tasks and *SlackQ* can be summarized as follows [13]:

- *GenerateSlack(SlackQ, s, d)*: Create a wrapper-task *WT* with parameters  $(s, d)$  and add it to *SlackQ* with increasing deadline order. Here, all wrapper-tasks in *SlackQ* represent dynamic slack with different deadlines. Therefore, the newly created *WT* may merge with an existing wrapper-task in *SlackQ* if they have the same deadline;
- *CheckSlack(SlackQ, d)*: Find out the amount of *reclaimable* slack before time  $d$  (i.e., the total size of all wrapper-tasks that have their deadlines no later than  $d$ );
- *ReclaimSlack(SlackQ, s)*: Reclaim the slack and remove wrapper-tasks from the front of *SlackQ*, which have accumulated size of  $s$ . The last wrapper-task may be partially removed by adjusting its remaining size.

**Push-Backward Slack:** At runtime, it is very likely that a job of high-criticality task  $\tau_j$  completes early and only takes its low-level WCETs  $c_j^{low}$ . Here, the over-provisioned time for the job will turn to be slack with the operation *GenerateSlack(SlackQ, c\_j^{high} - c\_j^{low}, d\_j)*, where  $d_j$  is the job's deadline. Moreover, it is highly possible for the slack to be pushed forward (and has a later deadline) with wrapped-executions. However, from previous discussion, we know that



a. Before slack push-backward      b. After slack push-backward;

Fig. 3: Push slack backward to make it more reclaimable.

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## Algorithm 2 : *CheckSlack(Q, d)* with slack push-backward.

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1: Input: Slack queue  $Q = \{WT_1, \dots, WT_m\}$  and time  $d$ ;
2: Output: the amount of reclaimable slack;
3: for  $WT_k: k = m \rightarrow 2$  do
4:   if  $(s_k > d_k - d_{k-1})$  then
5:      $s_{k-1} = s_{k-1} + s_k - (d_k - d_{k-1})$ ;
6:      $s_k = d_k - d_{k-1}$ ; //push slack backward
7:   end if
8: end for
9:  $k = 1; S_r = 0$ ; //initialize the amount of reclaimable slack
10: while  $(k \leq m \text{ AND } d_k \leq d)$  do
11:    $S_r = S_r + s_k; k += 1$ ; //accumulate reclaimable slack
12: end while
13: if  $(k \leq m \text{ AND } s_k > d_k - d)$  then
14:    $S_r = S_r + s_k - (d_k - d)$ ; //part of  $WT_k$  is reclaimable
15: end if

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low-criticality tasks prefer slack with earlier deadlines to obtain more reclaimable slack for their early releases.

Next, we show how slack can be pushed *backward* (with an earlier deadline) and become more reclaimable. Suppose that there are two pieces of slack represented by two wrapper-tasks  $WT_x : (s_x, d_x)$  and  $WT_y : (s_y, d_y)$  with  $d_x < d_y$  as shown in Figure 3a. Here, the amount of reclaimable slack before  $d_x$  and  $d_y$  are  $s_x$  and  $s_x + s_y$ , respectively.

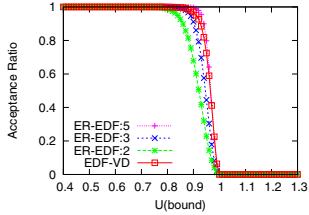
If there is  $d_y - d_x < s_y$ , for the slack represented by  $WT_y$ , at most  $(d_y - d_x)$  units can be consumed after time  $d_x$ . That is, for part of slack (with  $s_y - (d_y - d_x)$  units) represented by  $WT_y$  (dashed part in Figure 3a), we can safely push it backward and make that part has the deadline of  $d_x$ . After such transformation, the updated wrapper-tasks  $WT_x$  and  $WT_y$  are shown in Figure 3b. Although the amount of reclaimable slack before  $d_y$  remains the same, the one before  $d_x$  increases to be  $s_x + s_y - (d_y - d_x)$ . Note that, if there are wrapper-tasks with deadlines earlier than  $d_x$ , such push-backward can be performed iteratively.

Algorithm 2 details the steps of function *CheckSlack(Q, d)* with slack push-backward being considered. Here, all slack is pushed backward (if possible) to get more reclaimable slack before time  $d$  (lines 3 to 8). Then, the amount of reclaimable slack, including all completely reclaimable wrapper-tasks (lines 10 to 12) and the last partially reclaimable wrapper-task (lines 13 to 15), is accumulated.

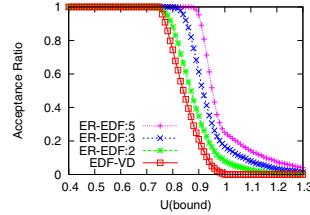
## C. Analysis of ER-EDF

From Section III-A, the time allocated to an early-release job of a low-criticality task comes from two parts: *its own contribution* and *reclaimed slack*. Based on the results in [5] and [13], both parts can be safely utilized by the early-release job without introducing extra workload and affecting other (current and future) tasks' allocations. Hence, we can have the following theorem regarding to the correctness of ER-EDF.

*Theorem 1:* An E-MC task set can be successfully scheduled under ER-EDF without causing any deadline miss if there is  $U(H, H) + U(L, min) \leq 1$ .



$$a. \frac{c_i^{low}}{c_i^{high}} \in [0.01, 0.3]$$



$$b. \frac{c_i^{low}}{c_i^{high}} \in [0.01, 0.9];$$

Fig. 4: Acceptance ratio;  $Prob(HIGH) = 0.5$ ;

#### IV. EVALUATIONS AND DISCUSSIONS

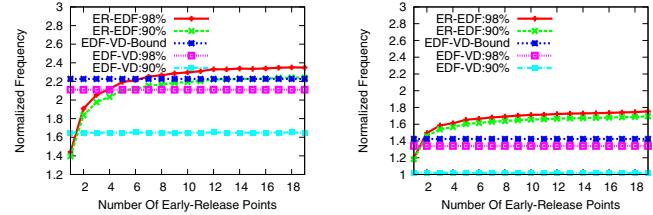
In this section, we evaluate the effectiveness of our proposed E-MC task model and ER-EDF algorithm through extensive simulations. For comparison, we also implemented EDF-VD [2], the state-of-the-art mixed-criticality scheduler. Here, whenever a high-criticality task uses more time than its low-level WCET and causes the system to enter the high-level execution mode at runtime, EDF-VD will discard all (current and future) low-criticality tasks until there is no more ready high-criticality tasks (at that time, the system can safely switch back to the low-level execution mode) [2].

##### A. Acceptance Ratio of Schedulable Task Systems

With the focus on the minimum service requirements of low-criticality tasks, which are represented by their maximum periods in the E-MC task model, we first show that more task sets can be scheduled under ER-EDF compared to that of EDF-VD. The synthetic tasks are generated following a similar workload generation scheme as used in [2]. Here, the parameter  $Prob(HIGH)$  denotes the probability of a generated task  $T_i$  being a high-criticality task. The periods of tasks are generated uniformly within the range of [10, 100]. The utilization of a task  $T_i$  is uniformly generated within the range [0.02, 0.2]. If  $T_i$  is a low-criticality task, it represents its normal utilization  $u_i$ . Otherwise, when  $T_i$  is a high-criticality task, it is actually task  $T_i$ 's high-level utilization  $u_i^{high}$ . The low-level WCET  $c_i^{low}$  and utilization  $u_i^{low}$  of a high-criticality task  $T_i$  is further obtained from its low-to-high execution ratio  $\frac{c_i^{low}}{c_i^{high}}$ , which is uniformly generated within a given range. Tasks are generated until the utilization bound  $U_{bound}$  is met, where  $U_{bound}$  is the larger of high-criticality utilization  $U(H, H)$  and low-criticality utilization  $U(H, L) + U(L, L)$  [2].

Figure 4 shows the acceptance ratio, which is the number of schedulable task sets over total number of task sets generated, under different scheduling algorithms when  $U_{bound}$  varies within the range of [0.4, 1.3]. Here, we consider balanced workload and set  $Prob(HIGH) = 0.5$ . Similar results have been obtained with other settings. Moreover, for each data point, 1000 task sets are generated. The results for EDF-VD are in line with what has been reported in [2], where more task sets can be schedulable when the low-to-high execution ratio for high-criticality tasks is smaller.

For ER-EDF, to specify a low-criticality task's minimum service requirement, its maximum period is obtained by ex-



$$a. \frac{c_i^{low}}{c_i^{high}} = 0.1;$$

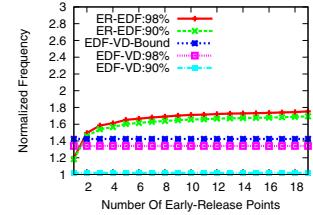


Fig. 5: Effects of early-release points;  $U(H, H) = 0.7$ .

tending its desired period upto  $k$  times (i.e., its minimum service requirement is no worse than one- $k^{th}$  of its desired service level under EDF-VD) and the corresponding ratio of schedulable tasks is denoted by  $ER\text{-}EDF:k$ . Here, we can see that, if low-criticality tasks' periods can be extended to 5 times, the number of scheduleable task sets under ER-EDF is comparable to that of EDF-VD even when the low-to-high execution ratio of high-criticality tasks is small (Figure 4a). For cases where the low-to-high execution ratio of high-criticality tasks is large (Figure 4b), extending the low-criticality tasks' periods to only 2 times can achieve more schedulable task sets under ER-EDF. In the following experiments, we set  $k = 3$ .

##### B. Effects of Early-Release Points

Next, we evaluate how the number of early-release points for low-criticality tasks can affect their performance under ER-EDF. The task sets are generated as described above with fixed  $U(H, H) = 0.7$  and  $\frac{c_i^{low}}{c_i^{high}} = 0.1$  or 0.5. The maximum periods of low-criticality tasks are set such that  $U(H, H) + U(L, min) = 1.0$ , which makes it schedulable under ER-EDF. The early-release points are assumed to uniformly distribute within the period of a low-criticality task.

The results are shown in Figure 5. Here, each data point represents the average of 100 task sets. The Y-axis is the normalized execution frequencies for low-criticality tasks with the one corresponding to their maximum periods (i.e., minimum service levels) being the baseline.  $ER\text{-}EDF:x\%$  and  $EDF\text{-}VD:x\%$  denotes the achieved frequencies, when the probability of high-criticality tasks taking their low-level WCETs is  $x\%$  (where  $x = 90$  and 98), under ER-EDF and EDF-VD, respectively. EDF-VD-Bound shows the highest frequency if no low-criticality task is discarded under EDF-VD.

Not surprisingly, having more early-release points generally results in better execution frequencies for low-criticality tasks under ER-EDF since it gives those tasks more opportunities to execute more (but with higher overhead). Compared to that of EDF-VD, having a few (e.g., 5) early-release points is sufficient for ER-EDF to obtain comparable (or better) execution frequencies for low-criticality tasks. Moreover, when high-criticality tasks are more likely to take their low-level WCETs, more slack can be expected at runtime and fewer low-criticality tasks may be discarded, which leads to better frequency improvements under both EDF-VD and ER-EDF.

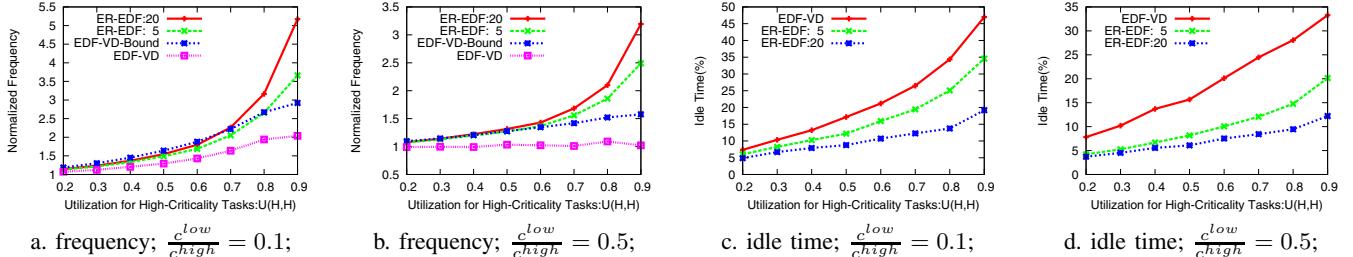


Fig. 6: The effects of  $U(H, H)$  on the execution frequencies for low-criticality tasks;  $Prob(c^{low}) = 90\%$  and  $\frac{p_i^{max}}{p_i} \leq 3$ .

### C. Effects of High-Criticality Task Utilization $U(H, H)$

To evaluate how ER-EDF performs under different workloads, Figure 6 shows the achieved execution frequencies for low-criticality tasks when  $U(H, H)$  varies from 0.2 to 0.9. Here, the low-to-high execution ratio of high-criticality tasks is set as 0.1 and 0.5. The probability of high-criticality tasks taking their low-level WCETs is 90%. We consider 5 or 20 early-release points for low-criticality tasks in ER-EDF, which are denoted as ER-EDF: 20 and ER-EDF: 5, respectively.

From Figure 6a (where  $\frac{c_{low}}{c_{high}} = 0.1$ ), we can see that higher normalized execution frequencies for low-criticality tasks can be obtained as  $U(H, H)$  increases. For EDF-VD, it comes from the fact that, to make the task set scheduleable under ER-EDF, the maximum periods of low-criticality tasks are relatively larger and the baseline frequencies become smaller at higher  $U(H, H)$ . For ER-EDF, it is mainly due to increased amount of available slack at runtime when there are more high-criticality tasks. When there are 20 early-release points, ER-EDF can obtain upto 2.5 times better execution frequencies for low-criticality tasks compared to that of EDF-VD. Similar results for the case of  $\frac{c_{low}}{c_{high}} = 0.5$  are shown in Figure 6b. The idle time in the result schedules is shown in Figure 6cd, which further illustrates that ER-EDF can make better use of slack and thus provide better service for low-criticality tasks.

Figures 7ab show the standard deviation and normalized maximum execution intervals between consecutive served jobs for low-criticality tasks, respectively. We can see that the low-criticality tasks are executed more smoothly under ER-EDF. The maximum interval under EDF-VD can be as large as 4 times compared to ER-EDF due to discarded jobs under EDF-VD at high-level execution mode.

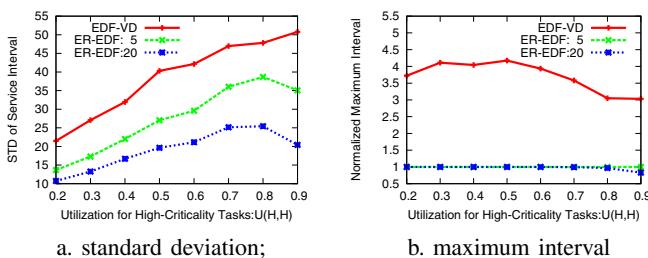


Fig. 7: Interval;  $\frac{c_{low}}{c_{high}} = 0.1$ ,  $Prob(c^{low}) = 90\%$ ,  $\frac{p_i^{max}}{p_i} \leq 3$ .

## V. CONCLUSIONS

In this work, we study an *Elastic Mixed-Criticality (E-MC)* task model to address the service abrupt problem for low-criticality tasks in conventional mixed-criticality scheduling algorithms. The central idea of E-MC is to have variable periods (i.e., service intervals) for low-criticality tasks, where the minimal service requirement is guaranteed by their largest periods. To improve their execution frequencies, we propose an *Early-Release EDF (ER-EDF)* scheduling algorithm that allows low-criticality tasks to release earlier than their largest periods at runtime by judiciously exploiting slack without affecting the timeliness of high-criticality tasks. Our simulation results confirm the effectiveness of the E-MC model and ER-EDF algorithm in scheduling mixed-criticality tasks when comparing to the state-of-the-art EDF-VD algorithm [2].

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