Statistical Model Order Reduction for Interconnect Circuits Considering Spatial Correlations *

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ABSTRACT

In this paper, we propose a novel statistical model order reduction technique, called statistical spectrum model order reduction (SSMOR) method, which considers both intra-die and inter-die process variations with spatial correlations. The SSMOR generates order-reduced variational models based on given variational circuits. The reduced model can be used for fast statistical performance analysis of interconnect circuits with variational input sources, such as power grid and clock networks. The SSMOR uses statistical spectrum method to compute the variational moments and Monte Carlo sampling method with the modified Krylov subspace reduction method to generate the variational reduced models. To consider spatial correlations, we apply orthogonal decomposition to map the correlated random variables into independent and uncorrelated variables. Experimental results show that the proposed method can deliver about $100\times$ speedup over the pure Monte Carlo projection-based reduction method with about 2% of errors for both means and variances in statistical transient analysis.

1. INTRODUCTION

The process-induced variability has huge impacts on the circuit performance in the sub-100nm VLSI technologies [12, 11]. The variational consideration of process has to be assessed in the various VLSI design steps to ensure robust circuit design.

Statistical modeling of RLC interconnects, which are typically treated as linear time-invariant (LTI) dynamic systems, has been studied intensively in the past and many research works have been reported [2, 4, 8, 7, 10, 14]. Fundamentally, the most common approach to statistical modeling and simulation is Monte Carlo based sampling method, which is the most flexible and trusted method. However, its high computing costs render its applications limited to very small circuits.

Statistical modeling methods of interconnects, based on the extracted parameters/variables, were proposed in [8, 2, 7]. The idea is to treat the variational variables as the global variables (parameters) of the circuits. The original circuits then can be represented by matrix polynomial forms in terms of those variables. Thus, the traditional model order reduction methods are applied to the coefficient matrices of the polynomials. Those methods are more suitable for the inter-die variations, because variations are treated as global variables. Interval-valued statistical modeling and model order reduction methods have been proposed recently [10, 9]. The idea is to approximate the variations as a finite interval and uses the interval arithmetic to generate the order reduced models in terms of variational poles/residues as well as order-reduced circuit matrices with interval-valued parameters. Interval methods in general suffer from the over-pessimism problem in spite of the recent improvement by using affine interval arithmetic. Also, the errors are accumulated with the arithmetic operations. Therefore, in [9], it was applied only to tree-like circuits, where solving the circuits was done with very few numerical operations by topology tracing.

Another approach to the statistical modeling and simulation of interconnect circuits is by means of statistical spectrum analysis [5, 14, 4], where statistical variations are presented by orthogonal polynomials. One only needs to solve for the coefficients of the polynomials deterministically in order to compute the variations of the responses or performance metrics. The major benefit of this method is the compatibility with current transient simulation framework: it requires to solve for some coefficients of the orthogonal polynomials. Thus, the computation of variances of node responses can be done by using transient simulations of given circuits with deterministic inputs. Ghanta [4] applied the statistical spectrum method to compute the timing delays, based on the moment methods in frequency domain.

In this paper, we propose a new statistical spectrum based method, called statistical spectrum model order reduction (SSMOR), to generate the order-reduced variational models, which in turn can be used to compute the variational responses and performance metrics with given variational inputs. The variational models are applied to fast statistical simulations of many interconnect circuits under various variations (both inter-die and intra-die). Specifically, our contribution includes a novel statistical model order reduction technique, which consists of the statistical spectrum method, the Krylov subspace based model order reduction technique, and Monte Carlo sampling method to generate order-reduced variational models. To consider the spatial correlation, we apply orthogonal decomposition via principal component analysis to map the correlated random variables into independent and uncorrelated variables.

The SSMOR follows similar reduction flow proposed previously in [9]. However, SSMOR uses the statistical spectrum method to compute the variational moments, which do not suffer the problems of over-pessimism, nor the accumulated inaccuracy in the interval-valued method. Also, the proposed method addresses the issue of spatial correlations, which were not considered in [9]. After variational moments are generated, Monte Carlo sampling method is applied by using modified Krylov subspace reduction approach to generate the variational order-reduced models. Since Monte Carlo only operates on the order-reduced space (namely, within a few moments), therefore the cost of high computing diminishes.

The rest of this paper is organized as follows: Section 2 presents statistical modeling problem to be solved. Section 3 reviews the orthogonal polynomial chaos based stochastic simulation methods and Section 4 reviews the principal component analysis method. Section 5 presents our new statistical model order reduction method. Section 6 presents the experimental results and Section 7 concludes this paper.

2. PROBLEM FORMULATION

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Considering the following state equation for a given RLC interconnect circuit using Modified Nodal Analysis (MNA) formulation:

$$Gv(t) + C\frac{\mathrm{d}v(t)}{\mathrm{d}t} = Bu(t) \tag{1}$$

where $G \in R^{n\times n}$ is the conductance matrix, $C \in R^{n\times n}$ the matrix resulting from storage elements. $v(t)$ is the vector of time-varying node voltages and branch currents of voltage sources. $u(t)$ is the vector of independent power sources, and B is the input selector matrix.

The G and C matrices and input currents $u(t)$ depend on the circuit parameters, such as metal wire width, length, thickness on interconnects, and transistor parameters, like channel length, width, gate oxide thickness, etc. In this paper, all the circuit parameter variations are treated as correlated Gaussian random variables, which differ from previous research to model the intra-die variations as uncorrelated in [5] . The spatial correlations are removed by using a set of independent random variables via orthogonal mapping technique, principal component analysis (PCA) [3].

In this paper, we assume that there are a number of dependent, correlated random Gaussian variables. After applying PCA, those correlated variables are transformed into independent, uncorrelated ortho-normal random Gaussian variables $\xi_i(\theta), i = 1,...,n$, which actually model the channel length and the device threshold voltage variations. Let Θ denote the process sampling space. Let $\theta \in \Theta$, $\xi_i : \theta \to R$ denote a normalized Gaussian variable, and $\xi(\theta) = [\xi_1(\theta),...,\xi_n(\theta)]$ is a vector of n independent Gaussian variables. So, the matrices G and C are functions of ξ, i.e. $G(\xi)$ and $C(\xi)$. Thus, equation (1) becomes

$$G(\xi)v(t) + C(\xi)\frac{\mathrm{d}v(t)}{\mathrm{d}t} = Bu(t) \tag{2}$$

Note that input $u(t)$ is also subject to variations. But here we focus on the variations of the interconnects for the sake of modeling. Thus, the problem is to develop a variational order-reduced system in terms of $\hat{G} \in R^{k\times k}$ and $\hat{C} \in R^{k\times k}$, where $k << n$,

$$\hat{G}v(t) + \hat{C}\frac{\mathrm{d}v(t)}{\mathrm{d}t} = \hat{B}u(t) \tag{3}$$

where $\hat{G}$ and $\hat{C}$ have variational matrix elements, which are treated as uncorrelated after applying PCA. The input sources may be variational, thus the reduced models can be combined with Monte Carlo method to compute the variational responses of interconnects, such as power grid and clock networks. In addition, the reduced system can be represented in terms of variational pole/residue forms. Therefore, transfer functions are evaluated through fast transient waveform computation by using the recursive convolution method.

3. STATISTICAL SPECTRUM ANALYSIS

In this section, we briefly review the statistical spectrum or orthogonal polynomial chaos (PC) based stochastic simulation methods.

3.1 Concept of Hermite Polynomial Chaos

In the following, a random variable $\xi(\theta)$ is expressed as a function of θ, which is the random event. Hermite PC utilizes a series of orthogonal polynomials (with respect to the Gaussian distribution) to facilitate stochastic analysis [15]. These polynomials are used as orthogonal basis to decompose a random process in a similar way that sine and cosine functions are used to decompose a periodic signal in a Fourier series expansion.

Given a random variable $v(t,\xi)$ with certain variance, where $\xi = [\xi_1, \xi_2, ...\xi_n]$ denotes a vector of ortho-normal Gaussian random variables with zero mean, the random variable can be approximated by truncated Hermite PC expansion as follows: [3]

$$v(t,\xi) = \sum_{k=0}^{P} a_k H_k^n(\xi) \tag{4}$$

where n is the number of independent random variables, $H_k^n(\xi)$ is n-dimensional Hermite polynomials, and a_k are the deterministic coefficients. The number of terms P is given by

$$P = \sum_{k=0}^{p} \frac{(n-1+k)!}{k!(n-1)!} \tag{5}$$

where p is the order of the Hermite PC. If only one random variable is considered, the one-dimensional Hermite polynomials are expressed as follows:

$$H_0^1(\xi) = 1, H_1^1(\xi) = \xi, H_2^1(\xi) = \xi^2 - 1, H_3^1(\xi) = \xi^3 - 3\xi, ... \tag{6}$$

Hermite polynomials are orthogonal with respect to Gaussian weighted expectation (the superscript n is dropped for simple notation):

$$< H_i(\xi), H_j(\xi) > = < H_i^2(\xi) > \delta_{ij} \tag{7}$$

where δ_{ij} is the Kronecker delta and $< *, * >$ denotes an inner product defined as:

$$< f(\xi), g(\xi) > = \frac{1}{\sqrt{(2\pi)^n}} \int f(\xi)g(\xi)e^{-\frac{1}{2}\xi^T\xi} d\xi \tag{8}$$

Thus, the coefficient, a_k, is found by a projection operation onto the Hermite PC basis:

$$a_k(t) = \frac{< v(t,\xi), H_k(\xi) >}{< H_k^2(\xi) >}, \ \forall k \in \{0,...,P\}. \tag{9}$$

3.2 Simulation Approach Based on Hermite PCs

In case that $v(t,\xi)$ is unknown random variable vector (with unknown distributions), such as node voltages in (1), then the coefficients can be computed by using Galerkin method. The principle of orthogonality states that the best approximation of $v(t,\xi)$ is obtained when the error, $\Delta(t,\xi)$, which is defined as

$$\Delta(t,\xi) = G(\xi)v(t) + C(\xi)\frac{\mathrm{d}v(t)}{\mathrm{d}t} - Bu(t) \tag{10}$$

is orthogonal to the approximation. That is

$$< \Delta(t,\xi), H_k(\xi) > = 0, k = 0, 1, ..., P \tag{11}$$

where, $H_k(\xi)$ are Hermite polynomials. In this way, we have transformed the stochastic analysis process into a deterministic form, where we only need to compute the corresponding coefficients of the Hermite PC. Once we obtain those coefficients, the mean and variance of the random variables can be easily calculated.

In the following section, we will show how to apply the statistical spectrum method to compute the variational circuit moments, which in turn are used to generate the variational reduced models via Krylov subspace reduction methods.

4. CONSIDERATION OF SPATIAL CORRELATION

In this section, we consider the spatial correlations among different variations. The spatial correlations exist in the intra-die variations and have been modeled for timing analysis [12, 1]. The general way to consider spatial correlation is by means of mapping the correlated random variables into a set of independent and uncorrelated variables. In this paper, we apply PCA method in our spectral statistical analysis framework for power/grid statistical analysis.

4.1 Concept of Principal Component Analysis

We first briefly review the concept of principal component analysis, which is used here to transform the random variables with correlation to uncorrelated random variables [6].

Suppose that x is a vector of n random variables, $x = [x_1, x_2, ..., x_n]^T$, with covariance matrix C and mean vector $\mu_x = [\mu_{x_1}, \mu_{x_2}, ..., \mu_{x_n}]$. To find the orthogonal random variables, we first calculate the eigenvalue and corresponding eigenvector. Arranging the eigenvectors in descending order based on corresponding eigenvalues, the orthogonal mapping matrix A is expressed as

$$A = [e_1^T, e_2^T, ..., e_n^T]^T \tag{12}$$

where e_i is the corresponding eigenvector to eigenvalue λ_i, which satisfies

$$\lambda_i e_i = C e_i, i = 1, 2, ..., n \tag{13}$$

and

$$\lambda_i < \lambda_{i-1},\ i = 2, 3, ..., n \tag{14}$$

With matrix A, we can perform the transformation to obtain the orthogonal random variables, $y = [y_1, y_2, ..., y_n]^T$, by using

$$y = A(x - \mu_x) \tag{15}$$

where, y_i is a random variable with Gaussian distribution. The mean, μ_{y_i}, is zero and the standard deviation, σ_{y_i}, is $\sqrt{\lambda_i}$ under the condition that [6]

$$e_i^T e_i = 1, i = 1, 2, ..., n \tag{16}$$

Here, due to the orthogonal property of matrix A

$$A^{-1} = A^T \tag{17}$$

Therefore, we may use the following equation to reconstruct the original random variables x:

$$x = A^T y + \mu_x \tag{18}$$

4.2 Spatial Correlation Analysis in Linear Dynamic Systems

Assuming $\Phi = [\Phi_1, \Phi_2, ..., \Phi_n]$ is a random variable vector, representing the correlated variations of certain sources of the circuit. Let $\Phi_i, i = 1, 2, ..., n$, be a random variable with Gaussian distribution. $\mu_\Phi = [\mu_{\Phi_1}, \mu_{\Phi_2}, ..., \mu_{\Phi_n}]$ is the mean vector of Φ and C is the covariance matrix of Φ.

Applying PCA, we can obtain the corresponding uncorrelated random variables $\phi = [\phi_1, \phi_2, ..., \phi_n]$ through the equation

$$\phi = A(\Phi - \mu_\Phi) \tag{19}$$

where, A is the orthogonal mapping matrix. Also, the original correlated random variables, Φ, can be expressed as

$$\Phi_i = \sum_{j=1}^{n} a_{ij}\phi_j + \mu_{\Phi_i}, i = 1, 2, ...n \tag{20}$$

where a_{ij} is the ith row, jth column element in the orthogonal mapping matrix as defined in equation (15). $\phi = [\phi_1, \phi_2, ..., \phi_n]$ is a vector with orthogonal (uncorrelated) Gaussian random variables. The mean of ϕ_j is zero and variance is λ_j, $j = 1, 2, ..., n$. The distribution of ϕ_i can be written as

$$\phi_i = \mu_{\phi_i} + \sigma_{\phi_i}\hat{\xi}_i, i = 1, 2, ..., n \tag{21}$$

where, $\hat{\xi} = [\hat{\xi}_1, \hat{\xi}_2, ..., \hat{\xi}_n]$ is a vector with ortho-normal Gaussian random variables. σ_{ϕ_i} is the standard deviation. Thus, the correlated random variable vector Φ_i can be expressed with uncorrelated orthogonal random variables, $\hat{\xi} = [\hat{\xi}_1, \hat{\xi}_2, ..., \hat{\xi}_n]$, as:

$$\Phi_i = \sum_{j=1}^{n} a_{ij}\sqrt{\lambda_j}\hat{\xi}_j + \mu_{\Phi_i}, i = 1, 2, ..., n \tag{22}$$

5. STATISTICAL SPECTRUM MODEL ORDER REDUCTION (SSMOR)

In this section, we first present our modified Krylov subspace model order reduction (MOR) framework, which is suitable for variational modeling, followed by the new variational moment computation method.

5.1 Modified Krylov Subspace Model Order Reduction

Krylov subspace based MOR method is to project the given circuit states into the dimension-reduced Krylov subspace of the circuit states. The Krylov subspace essentially is spanned by the dominant moment vectors of circuit transfer function. For a state space equation of an RLC circuit in equation (2), Krylov subspace is defined as

$$K_q(A, \mathbf{b}) = \mathrm{span}\{\mathbf{b}, A\mathbf{b}, A^2\mathbf{b}, ..., A^q\mathbf{b}\}, \tag{23}$$

where $A = G^{-1}C$ and $\mathbf{b} = G^{-1}B$ and q is some given positive integer. Note that $A^i\mathbf{b}$ is the i block moment defined as

$$\mathbf{m}_i = A^i\mathbf{b} = (-G^{-1}C)^i G^{-1}B, \tag{24}$$

of the circuit state transfer function, namely, $H(s) = (G + Cs)^{-1}B$. The block moment $\mathbf{m}_i$ can be directly computed in a recursive way

$$\begin{aligned} &\mathbf{m}_0 = G^{-1}B; \\ &\mathbf{m}_1 = -G^{-1}C\mathbf{m}_0; \\ &\cdots \\ &\mathbf{m}_i = -G^{-1}C\mathbf{m}_{i-1};\ \text{for } i > 0, \end{aligned} \tag{25}$$

One way to build the reduced model is by means of Pade approximation, which computes the poles/residues of the transfer functions by using the moment information directly, as shown in the classic AWE method [13]. However, this explicit moment matching method is not numerical stable for computing higher order models.

In our approach, we propose a modified Krylov subspace projection based MOR method to generate the reduced models. Specifically, we first define the moment matrix M as

$$M = [\mathbf{m}_0, \mathbf{m}_1, ..., \mathbf{m}_{q-1}] \tag{26}$$

The standard Krylov subspace projection method is to orthonormalize the vectors in M in order to generate a projection matrix V with the same dimension. Numerical methods like Arnoldi and Lanczos methods are typically used for the orthonormalization process, where the moment vectors are orthonormalized immediately after generation against all the previously-generated moment vectors.

Such orthonormalization process, however, is not suitable for our variational modeling process, as it is difficult to pass the variational information through the orthonormalization process using the aforementioned statistical spectrum method. Instead, we compute all variational moments by using statistical spectrum method. After all the block moments and associated variations are computed, we switch gears to the Monte Carlo sampling method to generate the variational reduced models. In each sampling run, we orthonormalize moment vectors in M by using Gram-Schmidt or modified Gram-Schmidt orthonormalization algorithms to compute projection matrix V. Once the projection matrix V is obtained, the original circuit matrix G and C are transformed to dimension-reduced matrices by *congruence transformation*:

$$\hat{G} = V^T G V; \hat{C} = V^T C V; \hat{B} = V^T B \tag{27}$$

Due to the nature of *congruence transformation*, the reduction process guarantees the passivity of all the reduced models. To compute the poles and residues, we can further perform eigen-decomposition of $\hat{G}^{-1}\hat{C}$

$$\hat{G}^{-1}\hat{C} = S\Lambda S^{-1}$$

where $\Lambda = diag(\lambda_1, \lambda_2, ...\lambda_i)$, which are the reciprocals of the dominant poles.

To find the residues, we solve for w in $\hat{G}w = V^T B$. Then the residues are simply the multiplications of $S^T V^T B$ and $S^{-1}w$. However, note that when generating the variational reduced models using Monte-Carlo method, we need to consider the variations in both moments (i.e. the projection matrix V) and the given G and C matrices in state equations. One important remark is that those variations are correlated, thus we need to treat them as correlated samples during the Monte Carlo method.

5.2 The New Statistical Model Order Reduction Flow

The proposed statistical model order reduction flow, SSMOR, is shown on the left hand side of Fig. 1. As comparison, we also show the pure Monte Carlo based MOR approach by using the traditional Krylov subspace projection MOR method. In the proposed flow, we use statistical spectrum method to compute the variational moments. Later on, we adapt the Monte Carlo sampling method to generate the variational reduced models by using the modified Krylov subspace method. The samplings are done based on the computed means and variances of Gaussian distributions for each corresponding moment. Since the Monte Carlo method is performed on the reduced models, it is obvious that we gain significant speedup over the pure Monte Carlo MOR method.

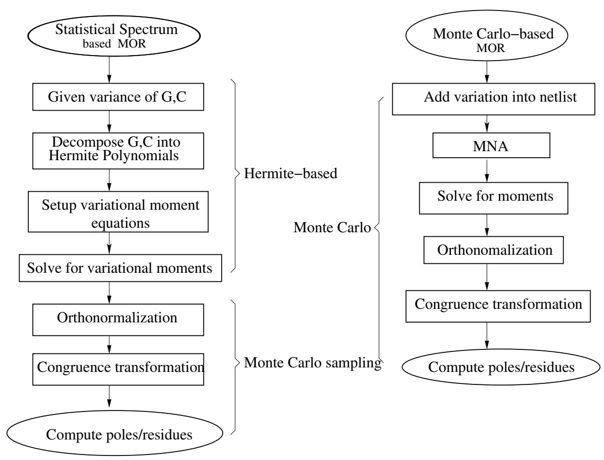

Figure 1: Flowchart of Statistical Spectrum and Monte Carlo Algorithms

5.3 Statistical Moment Computation with Multiple Random Variables

Since the correlations may be removed by PCA, let's consider n uncorrelated random variables. In this case, we use first order Hermite expansion. This is also a valid assumption practically, as first-order Hermite polynomials lead directly into Gaussian distributions. Given a Gaussian variations on the RLC elements, many interconnect timing performances are assumed to be Gaussian [1, 4]. The variational G and C matrices now become

$$G = g_0 + \sum_{i=1}^{n} g_i \xi_i \; ; C = c_0 + \sum_{i=1}^{n} c_i \xi_i$$

where, ξ_i is the random variable with Gaussian distribution with zero mean and standard deviation 1. g_0 and c_0 denote the means of G and C respectively. g_i and c_i are the variances of the associated ξ_i respectively.

For n random variables, it is clear that the basis of Hermite polynomials with expansion to first order is known as $[1, \xi_1, \xi_2, ..., \xi_n]$. Thus,

$$\begin{aligned} m_0 &= a_{m0} + \sum_{i=1}^{n} a_{mi}\xi_i \\ m_{2q} &= a_0 + \sum_{i=1}^{n} a_i \xi_i \\ m_{2q-1} &= b_0 + \sum_{i=1}^{n} b_i \xi_i \end{aligned} \tag{28}$$

where, $[a_{m0}, a_{m1}, ..., a_{mn}]$, $[a_0, a_1, ..., a_n]$, and $[b_0, b_1, ..., b_n]$ are coefficients with respect to the Hermite polynomial basis. Applying the principle of orthogonality and equalities of Gaussian distributions, the zero moment can be computed with the following equation:

$$\begin{bmatrix} g_0 & g_1 & g_2 & \dots & g_i & \dots & g_n \\ g_1 & g_0 & 0 & \dots & 0 & \dots & 0 \\ g_2 & 0 & g_0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_i & 0 & 0 & \dots & g_0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_n & 0 & 0 & \dots & \dots & \dots & g_0 \end{bmatrix} \begin{bmatrix} a_{m0} \\ a_{m1} \\ a_{m2} \\ \vdots \\ a_{mi} \\ \vdots \\ a_{mn} \end{bmatrix} - \begin{bmatrix} B \\ 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} = 0 \tag{29}$$

Once the zero moment is computed, the $(2q)$th moment can be evaluated from $(2q-1)$th moment recursively with the following equation:

$$\begin{bmatrix} g_0 & g_1 & g_2 & \dots & g_i & \dots & g_n \\ g_1 & g_0 & 0 & \dots & 0 & \dots & 0 \\ g_2 & 0 & g_0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_i & 0 & 0 & \dots & g_0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_n & 0 & 0 & \dots & \dots & \dots & g_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{bmatrix} + \begin{bmatrix} c_0 & c_1 & c_2 & \dots & c_i & \dots & c_n \\ c_1 & c_0 & 0 & \dots & 0 & \dots & 0 \\ c_2 & 0 & c_0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_i & 0 & 0 & \dots & c_0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_n & 0 & 0 & \dots & \dots & \dots & c_0 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{bmatrix} = 0 \tag{30}$$

Once all the moments and their variations are computed by statistical spectrum method, we proceed to compute the variational poles and residues via Monte Carlo methods by using modified Krylov subspace projection methods, as mentioned in the earlier part of this section.

6. EXPERIMENTAL RESULTS

This section describes the simulation results of circuits with variations in G and C in linear dynamic systems. The proposed method has been implemented in Matlab 7 and partially in Perl. All the experimental results are carried out in Linux system with dual Xeon CPU's with 3.06 GHz and 1 GB of memory.

Firstly, the magnitudes of variations are assumed to be the same without correlations for all nine random variables. These variations affect both G and C respectively in linear systems. Secondly, experimental results with nine random variables considering correlations are presented. Nine random variables are assumed to affect both G and C at the same time with predefined correlations among them. The correlated random variables are transformed into uncorrelated variables through PCA.

For pure Monte Carlo based model order reduction method, we perform the modified Krylov subspace method on the variational G and C matrices. Specifically, for each sampled linear dynamic circuit, we find the first qth order moments, say $q = 10$, of the system in our experiment in a recursive way. The next step is to compute the corresponding reduced circuit matrices, $\hat{G}$, $\hat{C}$ and $\hat{B}$. Finally, we find the pole and residues by eigen-decomposition. At least five poles are evaluated in our experiments.

In the SSMOR method, we compute variational moments by using the statistical spectrum method. After variational moments are calculated, we switch gears to Monte Carlo by using correlated samples to compute the variational poles and residues. The approach with modified Krylov subspace projection is employed to obtain the poles and residues.

For practical consideration, we select a small RLC network with 33 nodes and some variational current sources to test the proposed method. The small size of the circuit allows Mente Carlo simulations to finish within reasonable time. The variances with respect to R and C for nine random variables are set to be 0.005. The variances for current sources are set to be 0.01. Furthermore, larger-sized circuits are tested, as shown in Table 2, to study the scalability of the proposed method over the pure Monte Carlo method.

Fig. 2 shows the comparison between SSMOR approach and Monte Carlo simulation with nine independent and uncorrelated random variables in terms of pole variations. Given the same circuit for

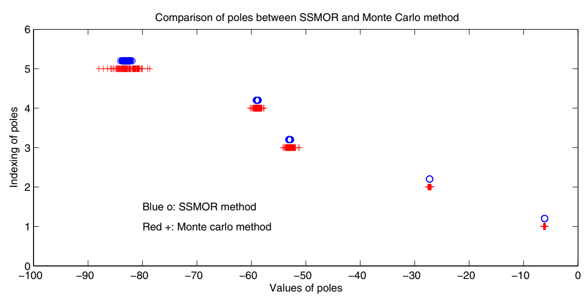

Figure 2: Comparison of poles between SSMOR and Monte Carlo methods (nine uncorrelated random variables).

both methods, the experiment is repeated for 2000 times in Monte Carlo method, such that there are 2000 sets of moments for each method. The number of samples is sufficient to guarantee 99% confidence level with 1% to 2% inaccuracy. The values of poles are derived from those 2000 sets of moments using the SSMOR method, the pure Monte Carlo MOR method. The values of poles are shown in x-axis with five pole indices shown in y-axis. We can see that the SSMOR method agrees pretty well with the pure Monte Carlo MOR method using Krylov subspace methods.

In addition, we compare the pole variations between the proposed SSMOR method and the pure Monte Carlo MOR method for nine correlated random variables. The results are shown in Fig. 3 and Fig. 4. Obviously, without applying PCA, the variational pole distribution does not match with the one obtained from pure Monte Carlo method as shown in Fig. 3. Applying PCA dramatically improves the accuracy of the pole distribution against Monte Carlo method, as shown in Fig. 4.

Finally, we use the reduced variational models to compute the transient responses with deterministic and variational power source inputs. The variational inputs are piecewise linear current sources.

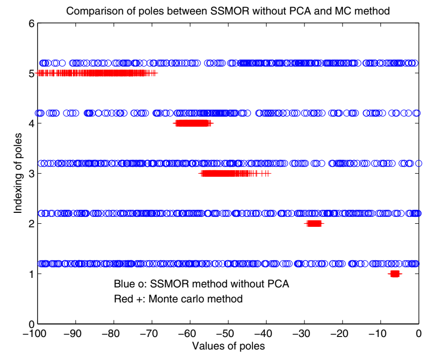

Figure 3: Comparison of poles between SSMOR without PCA and MC method (nine correlated random variables)

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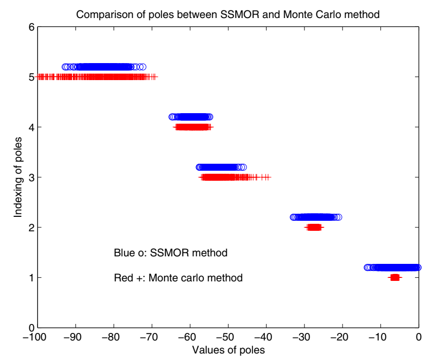

Figure 4: Comparison of poles between SSMOR with PCA and MC method (nine correlated random variables)

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Given a deterministic piecewise linear input, the comparison of voltage waveform in time domain at selected node between SSMOR and the Monte Carlo MOR method with 2000 samplings, is shown in Fig. 5. For the variational models, we use recursive convolution method to compute the transient responses after variational poles and residues are computed. The two waveforms are very similar. In the case of variational stimulus, the comparison be-

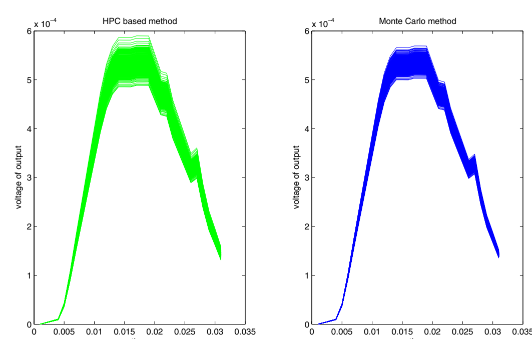

Figure 5: Comparison of PWL response between SSMOR reduced model and Monte Carlo method with deterministic stimulus (nine random variables).

Table 1: Voltage response comparison between SSMOR and Monte Carlo methods

Time instance (e-3) s	MC		SSMOR		% error	
	mean (e-5)	std (e-6)	mean (e-5)	std (e-6)	mean %	std %
3	0.673	9.24	0.674	9.27	0.15	0.32
5	3.94	9.22	3.95	9.28	0.25	0.65
30	17.63	15.35	0.1766	15.81	0.17	2.53

Table 2: Runtime comparison between SSMOR and Monte Carlo method

	#node	SSMOR	MC	Speedup
Ckt1	33	1	40.44	44 times
Ckt2	553	1	220.53	221 times
Ckt3	1720	1	338.34	338 times

tween SSMOR method and Monte Carlo MOR method is shown in Fig. 6. As it can be shown in both figures, the responses from our SSMOR method are almost identical to the ones using Monte Carlo method. To calculate the percentage of errors in SSMOR method

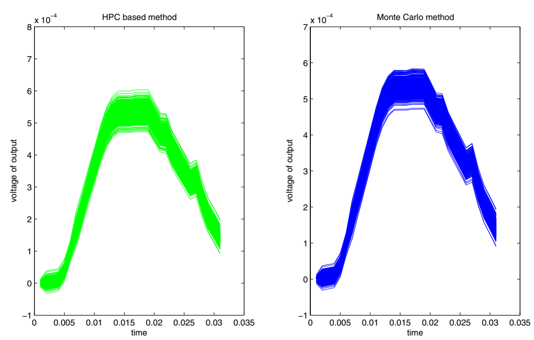

Figure 6: Comparison of PWL response between SSMOR reduced model and Monte Carlo method with stochastic stimulus (nine random variables)

against Monte Carlo method, we measure the transient waveforms at three different time instances at a randomly selected node, as shown in the first column of Table 1. The Table 1 shows the percentage of errors for the three time instances over 2000 samples. It can be seen that the transient errors between two methods are about 2% for both means and variances.

In consideration of the runtime speed between SSMOR and Monte Carlo MOR, the result of the speedup is shown in Table 2. Please note that the measurement of the speedup is based on the algorithms for SSMOR and Monte Carlo MOR. The benchmark includes the time for PCA transformation, and the computation of poles and residues. However, it does not include the time for transient analysis. The SSMOR shows about 100X of speedup over pure Monte Carlo method depending upon the size of the circuits.

7. CONCLUSION

We have proposed a new statistical model order reduction technique, named SSMOR, which considers both intra-die and inter-die process variations. The SSMOR generates order-reduced variational models from the given variational circuits with correlations. The reduced model can be used for fast statistical performance analysis of interconnect circuits with variational input sources. To consider spatial correlations, orthogonal mapping, based on principal component analysis, is applied to eliminate the correlations among variational random variables. The SSMOR method combines the statistical spectrum analysis method, Monte Carlo sampling method, and a modified Krylov subspace model order reduction technique, to generate the statistical reduced models. The proposed SSMOR method can deliver about 100× speedup over the pure Monte Carlo projection-based reduction method with about 2% of errors in statistical transient analysis.

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