Fast Positive-Real Balanced Truncation of Symmetric Systems Using Cross Riccati Equations

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Abstract

We present a computationally efficient implementation of positive-real balanced truncation (PRBT) for symmetric multiple-input multiple-output (MIMO) systems. The solution of a pair of algebraic Riccati equations (AREs) in conventional PRBT, whose complexity limits practical large-scale realization, is replaced with the solution of one cross Riccati equation (XRE). The cross-Riccatian solution then permits simple construction of projection matrices without actually balancing the system. The method encompasses passive linear networks, as commonly used in interconnect and package modelings, due to their inherent reciprocity and therefore symmetric transfer functions. Effectiveness of the proposed approach is verified by numerical examples.

1. Introduction

Parasitic extraction of packages and deep-submicron interconnects produces massively coupled RLC elements that prohibit direct computer simulation. Model order reduction (MOR) has become a standard routine whereby the initial model is approximated by a reduced-order model with little loss in time/frequency-domain accuracy. Moreover, the reduced-order model must preserve stability and passivity of the original model to ensure valid global simulation [3, 8, 9]. Specifically, a passive system is one that does not generate energy internally. A strictly passive system is dissipative and is automatically stable. In linear systems, passivity is equivalent to positive realness [3].

Among numerous MOR approaches, the control-theoretic balanced truncation (BT) schemes are known to have superior accuracy and closed-form error bounds [1–3, 9, 13–15]. The key idea of BT is to align and sort the internal states of the original model based on their participation in input-output state/energy transfer. The least important states are then truncated with little impact on the system responses. In standard BT, the bottleneck is the solution of two linear matrix equations, called *Lyapunov equations*, for obtaining the controllability and observability Gramians. The cross product of these Gramians is then used to obtain low-order projection matrices to reduce (truncate) the original system. However, standard BT does not necessarily preserve passivity. Positive-real balanced truncation (PRBT), also known as positive-real truncated balanced realization (PR-TBR) [9], is another BT approach that preserves both passivity and stability, and has no special structural restriction on the internal state space. However, it involves the solution of a pair of quadratic matrix equations, called *algebraic Riccati equations* (AREs), whose complexity is even higher than that of Lyapunov equations [13–15].

In the standard BT of symmetric multiple-input multiple-output (MIMO) systems[1], information in the cross product of controllability and observability Gramians can be directly extracted from one *Sylvester equation* which is a linear matrix equation [1, 2, 5]. As only one matrix equation is solved, computation is effectively halved, with further advantages like better consistency and numerical robustness [1, 2]. A quadratic counterpart of the Sylvester equation, called the *cross Riccati equation* (XRE), appeared in [6,10] on control topics like feedback control and discrete stochastic processes. Its integration with PRBT, however, has not been elaborated nor fully appreciated by the EDA community.

This paper generalizes the cross-Gramian framework in standard BT [1] to the cross-Riccatian framework for PRBT of symmetric MIMO systems. An invariant subspace method for solving the XRE is described. A Schur decomposition procedure, borrowed from the standard BT scenario [1], then allows simple construction of projection matrices to obtain equivalent PRBT-reduced models with-

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[1]Hence all single-input single-output (SISO) systems are automatically included.

out actually balancing the system. The scope of the algorithm encompasses passive linear circuits such as RLC networks, commonly used in interconnect and package modelings, which exhibit reciprocity and therefore have symmetric transfer functions [7, 11]. (A system is reciprocal if its admittance or impedance matrix is symmetric, i.e., the voltage and current at any two points in the network can be interchanged.) Numerical examples then confirm the remarkable efficiency of the proposed method over conventional PRBT realizations.

2. Background and Preliminaries

2.1. Positive-Real Balanced Truncation

Starting with the state space of a minimal (but not necessarily symmetric) MIMO square system

$$\dot{x} = Ax + Bu, \tag{1a}$$
$$y = Cx + Du, \tag{1b}$$

where $A \in \mathbb{R}^{n \times n}$, $B, C^T \in \mathbb{R}^{n \times m}$, $D \in \mathbb{R}^{m \times m}$, and u, y are power-conjugate: for every entry of u that is a node voltage (branch current), the corresponding entry of y is a branch current (node voltage) such that $u^T y$ represents a power metric. A is stable or its spectrum is in the open left half plane, denoted by $spec(A) \subset \mathbb{C}_-$. Let $M > 0 (\geq 0)$ denote a positive definite (positive semidefinite) matrix M, we assume without loss of generality that $D + D^T > 0$, otherwise the reduction technique in [12] is used to achieve this. Also, an impulse-free system in the *descriptor* representation [3] with a singular E before $\dot{x}$ can be put into the regular form in (1) [9]. The positive-real lemma [3] states that the linear system (1) is passive if and only if there exists an $X(\in \mathbb{R}^{n \times n}) \geq 0$ satisfying the linear matrix inequality

$$\begin{bmatrix} A^T X + XA & XB - C^T \\ B^T X - C & -(D + D^T) \end{bmatrix} \leq 0. \tag{2}$$

Applying Schur complement on (2), PRBT formulates and solves for the unique *stabilizing solutions*, $X_c(\geq 0)$ and $X_o(\geq 0)$, to the dual AREs

$$AX_c + X_c A^T + (X_c C^T - B)(D + D^T)^{-1}(CX_c - B^T) = 0, \tag{3a}$$
$$A^T X_o + X_o A + (X_o B - C^T)(D + D^T)^{-1}(B^T X_o - C) = 0. \tag{3b}$$

Existence of solutions is guaranteed by the passivity assumption. We call X_c the *controllability Riccatian* and X_o the *observability Riccatian*. Factoring out X_c and X_o in (3) for their coefficient matrices, we define

$$A_c = A - (B - X_c C^T)(D + D^T)^{-1}C, \tag{4a}$$
$$A_o = A - B(D + D^T)^{-1}(C - B^T X_o). \tag{4b}$$

Stabilizability of X_c and X_o implies $spec(A_c) = spec(A_o) \subset \mathbb{C}_-$ [15]. Let $X_c = L_c L_c^T$ and $X_o = L_o L_o^T$, where $L_c, L_o \in \mathbb{R}^{n \times n}$, be any matrix square-root decompositions. Compute the singular value decomposition (SVD)

$$L_c^T L_o = U \Sigma V^T , \Sigma = diag(\sigma_1, \cdots, \sigma_n). \tag{5}$$

Here Σ is a diagonal matrix and

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \gg \sigma_{r+1} \geq \cdots \geq \sigma_n. \tag{6}$$

Define the matrices

$$T_R = L_c U \Sigma^{-\frac{1}{2}} \text{ and } T_L = T_R^{-1} = \Sigma^{-\frac{1}{2}} V^T L_o^T, \tag{7}$$

and using '$\rightarrow$' to denote the corresponding similarity transform, we get the positive-real-balanced model

$$(A, B, C, D) \rightarrow (\hat{A}, \hat{B}, \hat{C}, D) := (T_L A T_R, T_L B, C T_R, D). \tag{8}$$

Such transform also results in simultaneously diagonalized Riccatians in the new state space, namely,

$$X_c \rightarrow T_L X_c T_L^T = \Sigma \text{ and } X_o \rightarrow T_R^T X_o T_R = \Sigma, \tag{9}$$

satisfying

$$\hat{A}\Sigma + \Sigma\hat{A}^T + (\Sigma\hat{C}^T - \hat{B})(D + D^T)^{-1}(\hat{C}\Sigma - \hat{B}^T) = 0, \tag{10a}$$
$$\hat{A}^T\Sigma + \Sigma\hat{A} + (\Sigma\hat{B} - \hat{C}^T)(D + D^T)^{-1}(\hat{B}^T\Sigma - \hat{C}) = 0. \tag{10b}$$

The states in $(\hat{A}, \hat{B}, \hat{C}, D)$ are aligned in descending importance in energy transfer [9]. Express $\Sigma = diag(\Sigma_b, \Sigma_s)$ where $\Sigma_b \in \mathbb{R}^{r \times r}$ holds the "bigger" singular values and $\Sigma_s \in \mathbb{R}^{(n-r) \times (n-r)}$ holds the "smaller" ones. Partition columns of T_R and rows of T_L accordingly so that with respect to (9),

$$X_c = T_R \Sigma T_R^T = \begin{bmatrix} T_{Rb} & T_{Rs} \end{bmatrix} \begin{bmatrix} \Sigma_b & \\ & \Sigma_s \end{bmatrix} \begin{bmatrix} T_{Rb}^T \\ T_{Rs}^T \end{bmatrix}, \tag{11a}$$

$$X_o = T_L^T \Sigma T_L = \begin{bmatrix} T_{Lb}^T & T_{Ls}^T \end{bmatrix} \begin{bmatrix} \Sigma_b & \\ & \Sigma_s \end{bmatrix} \begin{bmatrix} T_{Lb} \\ T_{Ls} \end{bmatrix}. \tag{11b}$$

The PRBT-reduced model is obtained from the rank-r subspace projection

$$(\hat{A}_r, \hat{B}_r, \hat{C}_r, D) := (T_{Lb} A T_{Rb}, T_{Lb} B, C T_{Rb}, D), \tag{12}$$

where $\hat{A}_r \in \mathbb{R}^{r \times r}$, $\hat{B}_r, \hat{C}_r^T \in \mathbb{R}^{r \times m}$. The system in (12) is passive and stable and the transfer matrix $G_r(s) = D + \hat{C}_r(sI - \hat{A}_r)^{-1}\hat{B}_r$ has an H_∞-norm error bound [9] with respect to $G(s) = D + C(sI - A)^{-1}B$.

2.2. Sylvester Equation in the Standard BT of Symmetric Systems

Standard BT requires solving a pair of dual Lyapunov equations

$$AW_c + W_cA^T + BB^T = 0, \qquad (13a)$$
$$A^TW_o + W_oA + C^TC = 0, \qquad (13b)$$

whose solutions, W_c and W_o, are the controllability and observability Gramians, respectively. The rest of the standard BT procedure is the same as in PRBT except that X_c and X_o are replaced with W_c and W_o, respectively. Previous work has studied the use of the *cross Gramian* in standard BT of symmetric MIMO (thus also SISO) systems [2]. Specifically, for a symmetric system, the *cross Gramian*, W_{co}, is solved from the Sylvester equation

$$AW_{co} + W_{co}A + BC = 0, \qquad (14)$$

where W_{co} satisfies the important property $W_{co}^2 = W_cW_o$. Since W_{co} contains both the controllability and observability information, standard BT can directly make use of the eigenvector bases of W_{co} without having to solve two Lyapunov equations [1].

3. PRBT of Symmetric Systems

We first define system symmetry. The MIMO system (A, B, C, D) in (1) is symmetric if $G(s) = G(s)^T$ or

$$D + C(sI - A)^{-1}B = D^T + B^T(sI - A^T)^{-1}C^T, \quad (15)$$

for all $s \in \mathbb{C} \setminus spec(A)$ where '\' denotes set subtraction. This necessitates $D = D^T$. Symmetry and minimality of (A, B, C) means that it is similar to (A^T, C^T, B^T) through a unique nonsingular $T \in \mathbb{R}^{n \times n}$ (e.g., [6]) such that

$$A^T = T^{-1}AT, \, C^T = T^{-1}B, \, B^T = CT. \qquad (16)$$

Analogous to (13) and (14), we formulate, with respect to (3), the *cross Riccati equation* (XRE)

$$AX_{co} + X_{co}A + (X_{co}B - B)(D + D^T)^{-1}(CX_{co} - C) = 0. \qquad (17)$$

Here X_{co} is called the *cross Riccatian* as in [10]. The following describes a two-step cross-Riccatian PRBT flow for symmetric systems.

3.1. Solving the Cross Riccati Equation

Based on (16) and the results in [6], we present a possible way of solving (17) associated with a symmetric system.

First, define the two *Hamiltonian* matrices, H_c and H_o, corresponding to (3a) and (3b), respectively,

$$H_c = \begin{bmatrix} A^T & 0 \\ 0 & -A \end{bmatrix} - \begin{bmatrix} C^T \\ B \end{bmatrix} (D + D^T)^{-1} \begin{bmatrix} B^T & -C \end{bmatrix}, \text{(18a)}$$

$$H_o = \begin{bmatrix} A & 0 \\ 0 & -A^T \end{bmatrix} - \begin{bmatrix} B \\ C^T \end{bmatrix} (D + D^T)^{-1} \begin{bmatrix} C & -B^T \end{bmatrix}. \text{(18b)}$$

Spectral structure of a Hamiltonian matrix and solutions of (3a) and (3b) by identifying the stable invariant subspaces of (18a) and (18b), respectively, are well studied, e.g., [15]. We note that H_c and H_o share the same spectrum since $H_c = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} (-H_o) \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}$. In [15] it has been shown that when the stable and unstable subspaces of H_o are separated (passivity implies no purely imaginary eigenvalues), i.e.,

$$H_o \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \begin{bmatrix} \Phi_- & 0 \\ 0 & \Phi_+ \end{bmatrix}, \text{(19)}$$

with $\Phi_-, \Phi_+ \in \mathbb{R}^{n \times n}$ corresponding to the stable and unstable parts, respectively, of $spec(H_o)$, then the stabilizing solution to (3b) is $X_o = X_{21}X_{11}^{-1}$ and that to (3a) is $X_c = X_{12}X_{22}^{-1}$. Now we show that this invariant subspace approach is also applicable to (17). First, define

$$H_{co} = \begin{bmatrix} A & 0 \\ 0 & -A \end{bmatrix} - \begin{bmatrix} B \\ B \end{bmatrix} (D + D^T)^{-1} \begin{bmatrix} C & -C \end{bmatrix}, \quad (20)$$

it can be seen that $spec(H_{co}) = spec(H_o) = spec(H_c)$ since

$$H_{co} = \begin{bmatrix} I & 0 \\ 0 & T \end{bmatrix} H_o \begin{bmatrix} I & 0 \\ 0 & T^{-1} \end{bmatrix}. \qquad (21)$$

Applying (21) to (19) we get

$$H_{co} \begin{bmatrix} X_{11} & X_{12} \\ TX_{21} & TX_{22} \end{bmatrix} = \begin{bmatrix} X_{11} & X_{12} \\ TX_{21} & TX_{22} \end{bmatrix} \begin{bmatrix} \Phi_- & 0 \\ 0 & \Phi_+ \end{bmatrix}. \qquad (22)$$

Then, noting $\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} H_{co} \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = -H_{co}$, (22) can be broken into two equations

$$H_{co} \begin{bmatrix} X_{11} \\ TX_{21} \end{bmatrix} = \begin{bmatrix} X_{11} \\ TX_{21} \end{bmatrix} \Phi_-, \qquad (23a)$$

$$H_{co} \begin{bmatrix} TX_{22} \\ X_{12} \end{bmatrix} = \begin{bmatrix} TX_{22} \\ X_{12} \end{bmatrix} (-\Phi_+). \qquad (23b)$$

It is easily shown that both $TX_{21}X_{11}^{-1} = TX_o$ and $X_{12}X_{22}^{-1}T^{-1} = X_cT^{-1}$ solve (17). From the Hamiltonian structure, we further have $spec(\Phi_-) = spec(-\Phi_+)$. Using $span(\circ)$ to denote the span (image) of a matrix, we have

$$span\left(\begin{bmatrix} X_{11} \\ TX_{21} \end{bmatrix}\right) = span\left(\begin{bmatrix} TX_{22} \\ X_{12} \end{bmatrix}\right), \qquad (24)$$

which implies $TX_o = X_c T^{-1} =: X_{co}$ is a solution to (17) found by identifying the stable invariant subspace of H_{co}. The choice of the actual basis is immaterial as long as the spectrum restricted to it is the same, and from this the uniqueness of X_{co} follows. Recalling (3) and (4), we generalize "stabilizability" in this symmetric-system cross-Riccatian sense: factoring out X_{co} from both sides in (17) for the associated matrices, and noting $X_{co} = X_c T^{-1} = TX_o$ and (16), we get

$$A - (B - X_{co}B)(D + D^T)^{-1}C = A_c, \quad (25a)$$

$$A - B(D + D^T)^{-1}(C - CX_{co}) = A_o. \quad (25b)$$

It is readily seen that $spec(A_c) = spec(A_o) = spec(\Phi_-) \subset \mathbb{C}_-$. Also, similar to results in Section 2.2, we have

$$X_{co}^2 = X_c X_o, \quad (26)$$

and the actual T in (16) is immaterial. With the same notion as in Section 2.1, the effect of similarity transform on X_{co} (in fact for arbitrary T_R and $T_L = T_R^{-1}$) is

$$X_{co} \to T_L X_{co} T_R, \quad (27)$$

so $spec(X_{co})$ is invariant. Since there exists a similar system in which X_c and X_o are both diagonal [c.f. (9)], we have

$$|\lambda_i(X_{co})| = \sigma_i\,, i = 1, 2, \cdots, n, \quad (28)$$

where σ_is are the singular values in (6) and $\lambda_i(\circ)$ denotes the ith eigenvalue in descending magnitude without loss of generality.

3.2. Constructing the Projection Matrices

The system balancing in PRBT, corresponding to (5)-(8), may sometimes be numerically inefficient and ill-conditioned. This is especially so for large-scale systems with nearly singular X_c and/or X_o, i.e., some states are nearly uncontrollable and/or unobservable [1, 2]. To avoid ill-conditional arithmetic, we borrow results from [1] on standard BT and present an alternative way for obtaining a reduced-order model with the same transfer function as the PRBT-reduced model. From (26),

$$span(X_{co}) = span(X_c), \quad (29a)$$

$$span(X_{co}^T) = span(X_o). \quad (29b)$$

Therefore, X_{co} contains both the controllable and observable subspaces in the cross-Riccatian sense. By (11) and (26),

$$X_{co}^2 = \begin{bmatrix} T_{Rb} & T_{Rs} \end{bmatrix} \begin{bmatrix} \Sigma_b^2 & 0 \\ 0 & \Sigma_s^2 \end{bmatrix} \begin{bmatrix} T_{Lb} \\ T_{Ls} \end{bmatrix}. \quad (30)$$

However, solution of X_{co} in (17) is generally not block-diagonal. To block-diagonalize X_{co}, we first compute an intermediate ordered real Schur form of it, namely,

$$X_{co} = \begin{bmatrix} Q_b & Q_s \end{bmatrix} \begin{bmatrix} X_{co}^b & \Omega \\ 0 & X_{co}^s \end{bmatrix} \begin{bmatrix} Q_b^T \\ Q_s^T \end{bmatrix}, \quad (31)$$

where $X_{co}^b \in \mathbb{R}^{r \times r}$ and $X_{co}^s \in \mathbb{R}^{(n-r) \times (n-r)}$ hold, in terms of magnitude, the bigger and smaller eigenvalues, respectively [c.f. (28)]. And X_{co}^b and X_{co}^s are block upper-triangular. Next, solve for $\Gamma \in \mathbb{R}^{r \times (n-r)}$ in the Sylvester equation

$$X_{co}^b \Gamma - \Gamma X_{co}^s + \Omega = 0. \quad (32)$$

It can be easily verified that

$$X_{co} = \begin{bmatrix} Q_b & Q_b\Gamma + Q_s \end{bmatrix} \begin{bmatrix} X_{co}^b & 0 \\ 0 & X_{co}^s \end{bmatrix} \begin{bmatrix} Q_b^T - \Gamma Q_s^T \\ Q_s^T \end{bmatrix}$$

$$=: \begin{bmatrix} V_b & V_s \end{bmatrix} \begin{bmatrix} X_{co}^b & 0 \\ 0 & X_{co}^s \end{bmatrix} \begin{bmatrix} W_b \\ W_s \end{bmatrix}, \quad (33)$$

where $\begin{bmatrix} V_b & V_s \end{bmatrix}^{-1} = \begin{bmatrix} W_b \\ W_s \end{bmatrix}$. Comparing (33) and (30), we have that T_{Rb} and V_b span the same (right) eigenspace corresponding to X_{co}^b, while T_{Rs} and V_s span the same (right) eigenspace corresponding to X_{co}^s. Therefore, there exist nonsingular $M_1 \in \mathbb{R}^{r \times r}$ and $M_2 \in \mathbb{R}^{(n-r) \times (n-r)}$ such that

$$\begin{bmatrix} T_{Rb} & T_{Rs} \end{bmatrix} = \begin{bmatrix} V_b & V_s \end{bmatrix} \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}. \quad (34)$$

Taking inverse on both sides, we also get

$$\begin{bmatrix} T_{Lb} \\ T_{Ls} \end{bmatrix} = \begin{bmatrix} M_1^{-1} & 0 \\ 0 & M_2^{-1} \end{bmatrix} \begin{bmatrix} W_b \\ W_s \end{bmatrix}. \quad (35)$$

Using (34) and (35), the reduced-order model by projection onto the "more significant" subspaces of (33),

$$(\tilde{A}_r, \tilde{B}_r, \tilde{C}_r, D) = (W_b A V_b, W_b B, C V_b, D), \quad (36)$$

is easily shown to be similar to (12), thereby producing the *same* transfer function as the PRBT-reduced model (so the H_∞-norm error bound [9] accompanying PRBT still applies). In other words, an equivalent PRBT-reduced model can be obtained from the block diagonalization of X_{co} without actually balancing the system.

4. Numerical Examples

We compare the proposed XRE-based PRBT method with conventional realizations in which two AREs are solved. All experiments are done in Matlab 7.0.4 on a 3GHz

	slcares	aresolv (eigen)	XRE
Spiral inductor order=500	$(4.33)^\dagger$ (3.89) (0.52) 48.45	(3.83) (3.98) (5.03) 78.13	(4.00) (0.56) 29.28
RLC ladder order=800	(182.61) (195.72) (17.02) 395.34	(193.58) (204.20) (124.38) 522.16	(194.11) (27.38) 221.48

† The brackets are time breakdowns. From top: time for solving each ARE (only one in the last column), the last bracket is the time for matrix factorizations.

Table 1. CPU times (sec) for different PRBT implementations.

PC with 3G memory. The AREs are solved with the Matlab routine aresolv with the eigen flag enabled (generally faster than its schur option) and also the SLICOT [4] FORTRAN routine slcares (which is faster than its generalized Schur counterpart slcaregs) invoked through a Matlab gateway. Matlab m-script files are coded for the proposed method. The XRE in (17) is solved by identifying the stable invariant subspace as in Section 3.1, and the projection matrices constructed through block diagonalization of X_{co} as in Section 3.2. Both steps essentially make use of the Matlab routine schur.

We try out two real-life symmetric-system benchmarks available from the literature [15]: the first one is a spiral inductor of order 500; the second one is an RLC ladder of order 800. The CPU times for PRBT are tabulated in Table 1. As expected, the cross-Riccatian approach easily outperforms the conventional ones. This is because the competing ARE solvers are also based on stable invariant subspace identification, and that instead of two AREs, only one XRE needs to be solved in the proposed method. Figs. 1(a) and 2(a) show the frequency responses of the original systems and the reduced-order models, while Figs. 1(b) and 2(b) show the relative errors. The PRIMA [8] curves are also shown to highlight the superior accuracy of PRBT. We note in passing that PRIMA constitutes a passivity-preserving projection-type MOR scheme and poses certain structural constraints on the internal state space for which PRBT does not [9]. It is seen that curves from the cross-Riccatian approach apparently overlap with those from conventional PRBT. This is expected because the system obtained in (36) is similar to that in (12), but without the need of the balancing operation.

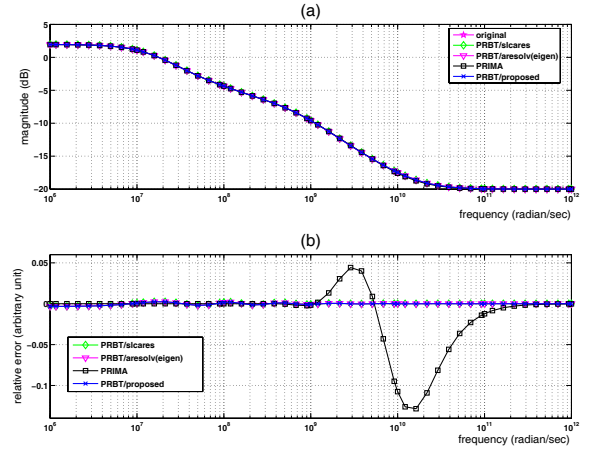Some additional remarks are in order:

Figure 1. (a) Frequency responses of the spiral inductor (order=500) and reduced models (order=7); (b) relative error from the original response.
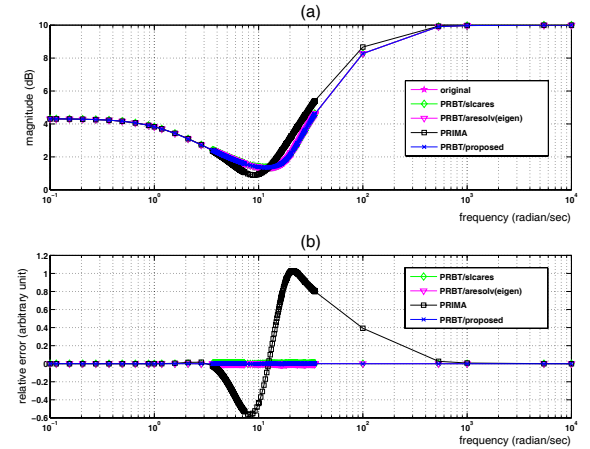

Figure 2. (a) Frequency responses of the RLC ladder (order=800) and reduced models (order=6); (b) relative error from the original response.

1. Referring to Sections 3.1 and 3.2, the invariant subspace approach for solving the XRE, without exploiting any matrix structure like sparsity or bands, has an $O(n^3)$ complexity and an $O(n^2)$ memory requirement. The workload arises mainly from the (ordered) Schur decompositions required in (23a) [or (23b)] and (31), and the solution of the Sylvester equation in (32).

2. With reference to Table 1, a recent algorithm in [15] can almost halve the time in solving two AREs through completely separating the stable and unstable invariant subspaces of a Hamiltonian matrix. However, such approach still requires the time-consuming full matrix factorizations afterward. The XRE method, besides practically halving the time in ARE solution, further saves time by replacing several large-size matrix factorizations with only one matrix block diagonalization. In fact, from other experiments not reported here, the latter PRBT approach is consistently about 40% faster than the former (with both algorithms coded and run in Matlab). Another merit gained from the XRE approach is the better numerical consistency and accuracy as noted in [1, 2].

3. The cross-Riccatian framework is generic and any fast solver, not necessarily using the invariant subspace approach, may be adopted for computing X_{co} in (17). For example, a recent ARE solver algorithm known as the quadratic alternating direction implicit (QADI) iteration [13, 14] may be modified to solve (17). A low-rank Cholesky-factor variant of QADI (called CFQADI) for XREs, if exists, can accelerate PRBT to a speed comparable to that of PRIMA [13, 14]. Research is being done along this direction and findings will be reported elsewhere.

5. Conclusion

This paper has presented an efficient PRBT implementation for symmetric MIMO systems. Instead of two AREs as in conventional PRBT, only one cross Riccati equation (XRE) needs to be solved. An invariant subspace method has been described for solving the XRE, followed by simple construction of projection matrices through block diagonalization. No positive-real balancing is required, thus avoiding the possible numerical ill-conditions in nearly uncontrollable/unobservable systems. The XRE-based PRBT framework is applicable to a large class of passive linear networks satisfying reciprocity. Numerical examples have demonstrated the significant computational savings from the proposed approach over conventional PRBT realizations.

References

[1] R. W. Aldhaheri. Model order reduction via real Schur-form decomposition. *Int. J. Control*, 53(3):709–716, 1991.

[2] A. C. Antoulas, D. C. Sorensen, and S. Gugercin. A survey of model reduction methods for large scale systems. *Contemporary Mathematics*, 280:193–219, 2001.

[3] Z. Bai, P. M. Dewilde, and R. W. Freund. Reduced-order modeling. *Numerical Analysis Manuscript No. 02-4-13, Bell Laboratories*, Mar. 2002.

[4] P. Benner, V. Mehrmann, V. Sima, S. Van-Huffel, and A. Varga. SLICOT-a subroutine library in systems and control theory. *Appl. and Comput. Contr., Signals, and Circuits*, 1:499–539, 1999.

[5] K. V. Fernando and H. Nicholson. On the cross-Gramian for symmetric MIMO systems. *IEEE Trans. Circuits Syst.*, 32(5):487–489, May 1985.

[6] L. Fortuna, A. Gallo, C. Guglielmino, and G. Nunnari. On the solution of a nonlinear matrix equation for MIMO realizations. *Systems and Control Letters*, 11:79–82, 1988.

[7] R. W. Freund and P. Feldmann. Reduced-order modeling of large passive linear circuits by means of the SyPVL algorithm. In *Proc. Int. Conf. Computer Aided Design*, pages 280–287, Nov. 1996.

[8] A. Odabasioglu, M. Celik, and L. T. Pileggi. PRIMA: Passive reduced-order interconnect macromodeling algorithm. *IEEE Trans. Computer-Aided Design*, 17(8):645–654, Aug. 1998.

[9] J. R. Phillips, L. Daniel, and L. M. Silveira. Guaranteed passive balancing transformations for model order reduction. *IEEE Trans. Computer-Aided Design*, 22(8):1027–1041, Aug. 2003.

[10] J. A. Ramos and E. I. Verriest. A note on a cross Riccatian and related properties for symmetric stochastic realizations. *IEEE Trans. Automat. Contr.*, 34(5):548–551, May 1989.

[11] B. N. Sheehan. ENOR: model order reduction of RLC circuits using nodal equations for efficient factorization. In *Proc. IEEE Design Automation Conf.*, pages 17–21, June 1999.

[12] H. Weiss, Q. Wang, and J. L. Speyer. System characterization of positive real conditions. *IEEE Trans. Automat. Contr.*, 39(3):540–544, Mar. 1994.

[13] N. Wong and V. Balakrishnan. Fast balanced stochastic truncation via a quadratic extension of the alternating direction implicit iteration. In *Proc. Int. Conf. Computer Aided Design*, pages 801–805, Nov. 2005.

[14] N. Wong and V. Balakrishnan. Multi-shift quadratic alternating direction implicit iteration for high-speed positive-real balanced truncation. In *Proc. IEEE Design Automation Conf.*, pages 257–260, June 2006.

[15] N. Wong, V. Balakrishnan, C.-K. Koh, and T. S. Ng. Two algorithms for fast and accurate passivity-preserving model order reduction. *IEEE Trans. Computer-Aided Design*, 25(10):2062–2075, Oct. 2006.