A Practical Method to Estimate Interconnect Responses to Variabilities

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Abstract

Variabilities in metal interconnect structures can affect circuit timing performance or even cause function failure in VLSI designs. This paper proposes a method to estimate the *difference* between the nominal and perturbed circuit waveforms by calculating the moments in frequency-domain via efficient iterative method. The algorithm can be used to accurately reproduce the differential waveforms, or to provide efficient early estimates on the timing impact of the variabilities for RC networks.

1 Introduction

Similar to active devices in VLSI designs, interconnect networks are also subject to variabilities. The variabilities include both manufacturing variations, which are static, and environmental perturbations, which are dynamic. For nanometer designs, the metal interconnect structures have large contribution to the total path delays. Consequently, interconnect network variabilities can have significant impact on the overall circuit timing performance and, in some extreme cases, correct functionality.

In this paper, we propose a method to construct a reduced-order model of the *time-domain difference* between the perturbed and the nominal circuit. The benefit of the differential response is that it can quickly tell us how the variation will affect the system response: a negative difference tells us that the variations will slow down the system, while a positive difference means that the system will be sped up. We achieve this goal by calculating the moments of the differential response and then matching those moments to a reduced-order model. It turns out that the moments of the differential responses can be calculated iteratively, without the need for any extra matrix factorization.

2 Differential Response and its Moments

Without loss of generality, the following MNA formulation can be used to describe a lumped interconnect network:

$$(\mathbf{G} + \mathbf{sC}) \cdot \mathbf{X(s)} = \mathbf{B} \tag{1}$$

The *moments* of the circuit response, which are Taylor expansion coefficients of $X(s)$, can be calculated as:

$$\begin{aligned} \mathbf{m_0} &= \mathbf{G^{-1}B} \\ \mathbf{m_k} &= -\mathbf{G^{-1}Cm_{k-1}} \end{aligned} \tag{2}$$

Once the moments are known, a Padé approximation can be used to construct a reduced-order model. Note that only one matrix factorization is required to calculate all the moments. This provides a huge runtime advantage compared to complete inclusion of the original netlist into a simulator[1].

When there are variations in the netlist, the perturbed MNA equations can be written as follows[2, 3]:

$$((\mathbf{G} + \mathbf{\Delta G}) + \mathbf{s}(\mathbf{C} + \mathbf{\Delta C}))\mathbf{\tilde{X}(s)} = \mathbf{B} \tag{3}$$

Define $\mathbf{\tilde{X}(s)} = \mathbf{X(s)} + \mathbf{\Delta X(s)}$ and substitute it into Eqn (3), we have:

$$\begin{aligned} &((\mathbf{G} + \mathbf{\Delta G}) + \mathbf{s}(\mathbf{C} + \mathbf{\Delta C}))\mathbf{X(s)} + \\ &+((\mathbf{G} + \mathbf{\Delta G}) + \mathbf{s}(\mathbf{C} + \mathbf{\Delta C}))\mathbf{\Delta X(s)} = \mathbf{B} \end{aligned} \tag{4}$$

Substituting Eqn (1) into Eqn (4), we have the following equation to describe the differential response $\mathbf{\Delta X(s)}$:

$$((\mathbf{G} + \mathbf{\Delta G}) + \mathbf{s}(\mathbf{C} + \mathbf{\Delta C}))\mathbf{\Delta X(s)} = -(\mathbf{\Delta G} + \mathbf{s\Delta C})\mathbf{X(s)} \tag{5}$$

The response of the nominal system $\mathbf{X(s)}$ is completely independent of the variations, which can be calculated a priori. Once the variational matrices $\mathbf{\Delta G}$ and $\mathbf{\Delta C}$ are known, theoretically we can use the above equation to calculate the differential response. Unfortunately, this brute-force approach requires one matrix factorizations for **each** combination of the variations. Therefore it is highly inefficient.

However, we can re-write Eqn (5) as an iterative procedure to calculate $\mathbf{\Delta X(s)}$:

$$(\mathbf{G} + \mathbf{sC})\mathbf{\Delta X^{(l)}(s)} = -(\mathbf{\Delta G} + \mathbf{s\Delta C})(\mathbf{X(s)} + \mathbf{\Delta X^{(l-1)}(s)}) \tag{6}$$

by defining the Taylor expansion as:

$$\mathbf{\Delta X(s)} = \mathbf{\Delta m_0} + \mathbf{\Delta m_1 s} + \mathbf{\Delta m_2 s^2} + \mathbf{\Delta m_3 s^3} + \cdots \tag{7}$$

By substituting the above equation into Eqn (5), we have:

$$\begin{aligned} \mathbf{\Delta m_0} &= -(\mathbf{G} + \mathbf{\Delta G})^{-1}\mathbf{\Delta G m_0} \\ \mathbf{\Delta m_k} &= -(\mathbf{G} + \mathbf{\Delta G})^{-1}(\mathbf{\Delta G m_k} + \mathbf{\Delta C m_{k-1}}) \\ &\quad -(\mathbf{G} + \mathbf{\Delta G})^{-1}(\mathbf{C} + \mathbf{\Delta C})\mathbf{\Delta m_{k-1}} \end{aligned} \tag{8}$$

A more useful equation is the iterative procedure of the above:

$$\begin{aligned} \mathbf{\Delta m_0^{(l)}} &= -\mathbf{G^{-1}}(\mathbf{\Delta G m_0} + \mathbf{\Delta G \Delta m_0^{(l-1)}}) \\ \mathbf{\Delta m_k^{(l)}} &= -\mathbf{G^{-1}}(\mathbf{\Delta G m_k} + \mathbf{\Delta G \Delta m_k^{(l-1)}} + \mathbf{\Delta C m_{k-1}}) \\ &\quad -\mathbf{G^{-1}}(\mathbf{C} + \mathbf{\Delta C})\mathbf{\Delta m_{k-1}} \end{aligned} \tag{9}$$

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Note that when the above procedure is implemented, at each iteration step we only have to carry out one sparse matrix-vector multiplication to calculate $\mathbf{\Delta G \Delta m_k^{(l-1)}}$ and one forward/backward substitution. The modern sparse solvers can carry out these operations very efficiently. In order to determine when the iteration should be stopped, the following relative error criterion can be easily calculated:

$$\epsilon_k = \|\mathbf{\Delta m_k^{(l)}} - \mathbf{\Delta m_k^{(l-1)}}\| / \|\mathbf{m_k}\| \tag{10}$$

Notice that for most of the interconnect networks, there is no DC-path to ground. In this case, it can be easily shown that the variational conductance matrix $\mathbf{\Delta G}$ is symmetric and the sum of each row is strictly zero. Also in this case, the first moment of the nominal system $\mathbf{m_0}$ has Vdd in node voltage entries and 0 in the KCL entries. It is trivial to show that the left-hand-side of the first equation in Eqn (8) is zero. Therefore, we have $\mathbf{\Delta m_0} = 0$. (This is just another way to say that any variability will not change the DC solution.) Then the differential response can be shown as:

$$\begin{aligned} \Delta X(s) &= \Delta m_1 s + \Delta m_2 s^2 + \Delta m_3 s^3 + \cdots \\ &= \Delta m_1 s \cdot (1 + \tfrac{\Delta m_2}{\Delta m_1} s + \tfrac{\Delta m_3}{\Delta m_1} s^2 + \cdots) \end{aligned} \tag{11}$$

The above equation tells us that the first moment of the differential response is actually a good indicator on the impact of the variations. Since $\mathbf{\Delta m_1}$ can be calculated by only one matrix factorization and a few iterations once the variations are given. We can use Δm_1 as a metric to construct a first-order response surface model. Since $\mathbf{\Delta m_0 = 0}$, in order to calculate $\mathbf{\Delta m_1}$, the iterative procedure outlined in Eqn (9) can be simplified as:

$$\mathbf{\Delta m_1^{(l)}} = -\mathbf{G^{-1}}(\mathbf{\Delta G m_1} + \mathbf{\Delta G \Delta m_1^{(l-1)}} + \mathbf{\Delta C m_0}) \tag{12}$$

With one matrix factorization and a couple of forward/backward solves, we can very quickly calculate $\mathbf{\Delta m_1}$. Recall in the traditional timing estimation, Elmore delay (or $\mathbf{m_1}$) provides quite accurate timing estimates for the far-end nodes in an RC network. Intuitively the difference in Elmore delay would also be a good indicator of the impact of the the variabilities. Our above analysis is another way to confirm this. We summarize the procedure of calculating moments of differential responses below.

Algorithm to calculate differential moments
1. Calculate the nominal moments m_0, m_1, m_2,...
2. Set $k = 0$, $l = 0$.
3. Set initial value $\Delta m_k^{(l)} = 0$.
4. Use Eqn 9 to calculate $\Delta m_k^{(l+1)}$.
5. Calculate error by using Eqn 10.
6. If converged or reached limit, exit
7. Goto 4.
8. Increase k. Goto 3.

Figure 1: Algorithm to calculate differential moments

3 Approximate Response Waveforms

Once the moments of the differential response are known, we can construct a reduced-order model of the differential response by using classic Padé approximation:

$$\begin{aligned} \Delta X(s) &= \Delta m_1 s + \Delta m_2 s^2 + \Delta m_3 s^3 + \cdots \\ &= \textstyle\sum_{i=0}^{n} \frac{r_i}{s - p_i} \end{aligned} \tag{13}$$

Fig. 2 shows the differential waveforms of the reduced-order model for an RC circuit. There are two independent variation sources in the circuit with 3σ value of $\pm 70\%$.

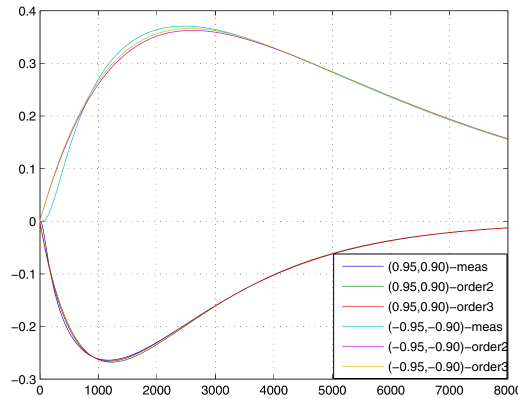

Figure 2: Waveforms of the reduced-order model of differential response.

Fig. 3 shows the error of using first differential moment to estimate the RC response, compared to full simulation. The differential moments can also used to construct response surface models to predict circuit behavior.

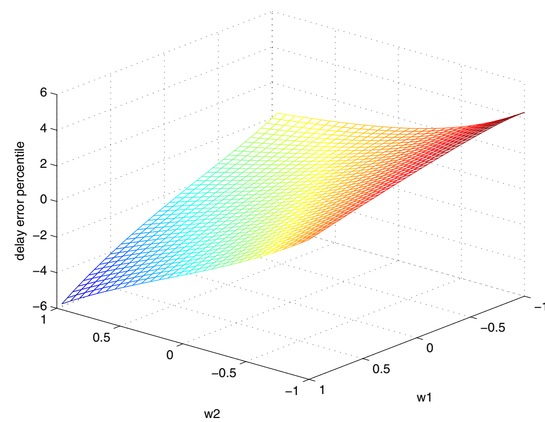

Figure 3: Error of using Δm_1 to estimate the delay for RC circuit.

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