Lifetime Modeling of a Sensor Network

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I. Introduction

In this paper, we study a communication/sensing network that comprises of large number of radio enabled sensors. These sensors are either randomly or deterministically placed within a certain region to monitor events that are spatially and temporally independent of each other. Possible applications include: habitat and climate monitoring, diagnosing faults in industrial supply lines, measuring data such as traffic-intensity, detecting human/vehicular intrusion, etc. The sensor nodes in these networks are powered by a battery with limited power, which is dissipated during the data transmission/reception. A cheap and effective approach is to replace the sensor nodes in due course instead of replenishing of their batteries. Thus, the objective is to find the replacement time T_r such that none of the sensor nodes run out of their batteries (disconnected) before T_r. An alternative way of formulating this problem is to find the *lifetime* T of the network, which is defined as the time after which the first node in the network disconnects. Studies evaluating the lifetime model of the sensor networks have been done before in [1], [2], [4]. However, the primary difference between previous approaches and our work is that we specifically model a data generation process at an individual sensor node, where each node covers certain area and the amount of data generated at a node is proportional to its coverage area.

II. Model Preliminaries

We present a routing abstraction where the network is divided into n number of bins of equal size. Further, we impose a restriction that nodes that are placed in the i^{th} bin can forward traffic to the nodes that are within their transmission radius and are nodes in the $i-1^{\text{th}}$ bin. The length of the bin b is chosen such that $r = b(1+\epsilon)$, where ϵ depends on the node density δ and is given as follows.

Lemma 1: The density δ of nodes in any bin in the network should atleast be equal to $\frac{1}{(1-\varepsilon)A}$ for the network to be connected with probability equal to $e^{-\frac{\varepsilon^2}{2(1-\varepsilon)}}$, where ϵ lies between 0 and 1.

A. Mathematical Formulation

Consider that the density of the nodes in the i^{th} bin is equal ω_i : $\sum_{i=1}^{n} \omega_i = 1$. Let m denote the total number of nodes in the network, each with initial battery energy equal to B. Then the total initial energy is denoted by P, which is equal to mB. Therefore, the energy in the i^{th} bin is equal to $\omega_i P$.

With the passage of time, energy is dissipated at regular intervals from each energy-unit until a routing hole appears after time T, which is known as the disconnection time or the lifetime of the network. Let γ be the energy dissipated per data transmission and $R_i(t)$ be the residual energy at the i^{th} energy-unit EU$_i$, which is given as:

$$R_i(t) = \omega_i P - \gamma_i \sum_{j=i}^{n} e_i(t), \tag{1}$$

where $e_i(t)$ represents the amount of data generated at the EU$_i$ by

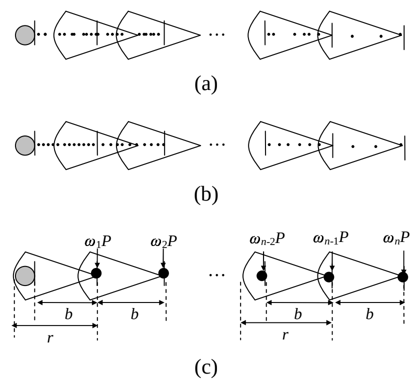

Fig. 1. A linear network of sensor nodes, when the node placement is (a) random, (b) deterministic, and (c) when each bin is considered a separate energy unit. The variable ω_i represents the fraction of nodes in the i^{th} bin and P represents the sum of initial energy at all the nodes in the network.

time t. Let T_i denote the disconnection time of individual energy-units. The time when the residual energy at EU$_i$ reaches zero, which can be written as:

$$T_i = \inf(t : R_i(t) = 0). \tag{2}$$

and the lifetime of the network, which is equal to the time when the residual energy at any one of the energy-units reaches zero is given as:

$$T = \min(\{T_i\}_{i=1}^{n}). \tag{3}$$

For the planar network, we consider a circular region divided into n annular regions and the DCR is placed at the center. Now, each annular region can be considered as a bin. Unlike linear network, the size of the bins unequal and is proportional to the area of the i^{th} annular region. Thus, if the size of the first bin is s, then the size of the i^{th} bin is equal to $i^2 - (i-1)^2 = 2i - 1$ times s.

B. Data Generation Model

Remember that each sensor node periodically senses for information within its coverage area and that data is generated only when a relevant event occurs within the coverage area of that node. We assume that an event can occur at any point in the network with equal probability, which implies that the rate of data generation at each point in the network is same. In addition, we assume that the occurrence of every event is both temporally and spatially independent of all the other events in the network. Under these assumptions, the data generation process at an individual sensor node is Poisson and hence the time interval between two successive data generation events is an exponentially distributed random variable.

Lemma 2: The inter-data generation time at each sensor node is an exponentially distributed random variable with mean proportional to its area of coverage.

Lemma 3: The load $\mu_i(t) = \sum_{j=i}^{n} e_i(t)$ at the i^{th} energy unit is a Poisson process, which is given as:

$$\Pr(\mu_i(t) = x) = \frac{e^{-\mu_i}(\mu_i)^x}{x!}, \tag{4}$$

where $\mu_i = \sum_{j=i}^{n} \lambda_j$ represents the rate of traffic forwarding at EU_i.

III. Lifetime Model

In this section, we formulate the node placement problem as the lifetime optimization problem via optimal energy allocation. We then solve the optimization problem to derive expected lifetime under an optimal allocation scheme and thereafter calculate the CDF of lifetime to derive the probability of disconnection.

A. Problem Formulation

Lemma 4: The residual energy $R_i(t)$ at the i^{th} energy-unit EU_i is a Pure-Death Continuous Time Markov Chain (CTMC) [3] and the rate of energy dissipation is equal to $\mu_i = \sum_{j=i}^{n} \lambda_j$

Lemma 5: The probability distribution of the hitting time T_i of the CTMC $R_i(t)$ is an Erlang distribution, where the PDF is given as:

$$f_i(x) = \frac{\mu_i^{k_i} e^{-\mu_i x} x^{k_i - 1}}{(k_i - 1)!}, \tag{5}$$

and the CDF is given as:

$$\Pr(T_i < x) = F_i(x) = 1 - \sum_{j=o}^{k_i - 1} \frac{e^{-\mu_i x}(\mu_i x)^j}{j!}, \tag{6}$$

where $k_i = \frac{\omega_i P}{\gamma}$.

B. Expected Lifetime

Lemma 6: The energy allocation vector $\{\omega_i\}_{i=1}^{n}$ in proportional allocation strategy for a linear network is given as:

$$\omega_i = \frac{2(n - i + 1)}{n(n+1)}, \tag{7}$$

and for a planar network is given as:

$$\omega_i = \frac{6(n^2 - (i-1)^2)}{n(n+1)(4n-1)} \tag{8}$$

Next, a fundamental question that needs to be answered is whether the proportional allocation strategy is an optimal allocation strategy: Does proportional allocation maximizes $E[T]$? To answer this question, we will introduce a random variable T' that is stochastically larger than T, written $T' \geq_{\text{st}} T$,which implies that:

$$\Pr\{T' > x\} \geq \Pr\{T > x\} \quad \text{for all } x$$

Lemma 7: The expected value of the stochastically larger random variable T' in a linear network is given as:

$$\text{E}[T'] = \frac{2P}{n(n+1)\gamma\lambda} \tag{9}$$

Lemma 8: The expected value of the stochastically larger random variable T' in a linear network is given as:

$$\text{E}[T'] = \frac{P}{n^3\lambda\gamma} \tag{10}$$

We validated our results with simulations shown in Figure 2, where we show that even though our model is upper-bound on the expected

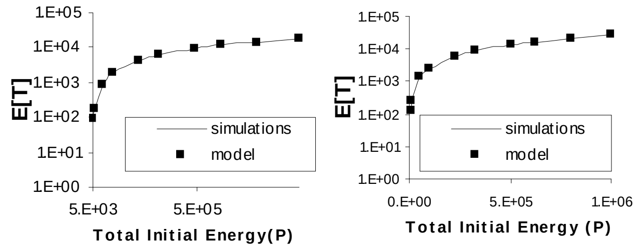

Fig. 2. Comparison of analytical model with simulation results of expected lifetime for a linear network (right) and a planar network (left).

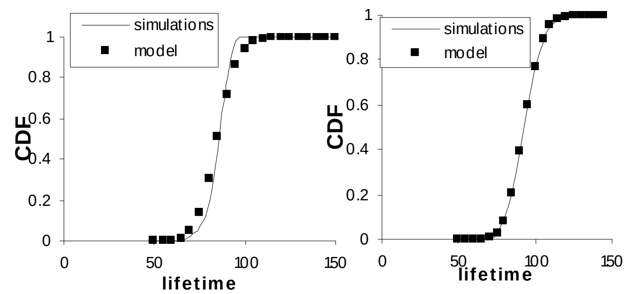

Fig. 3. Comparison of simulation results with model of CDF $G(x)$ of lifetime in a linear network (left) and in a planar network(right).

lifetime, it closely follows the simulation results. This allows us to assume that proportional allocation is a near optimal allocation policy.

C. Disconnection Probability

Note that even if the replacement time T_r is set equal to the expected lifetime $E[T]$, there is always a positive probability that the network may be disconnected before replacement. To evaluate such probability, we need to evaluate the probability distribution of T.

Lemma 9: The CDF of the lifetime T is approximated as:

$$\Pr(T < x) = G(x) = 1 - \sum_{j=o}^{\frac{\omega_n P}{\gamma} - 1} \frac{e^{-\lambda_n x}(\lambda_n x)^j}{j!}, \tag{11}$$

where $\lambda_n = \lambda$ and $\omega_n = \frac{2}{n(n+1)}$ for a linear network; and for a planar network $\lambda_n = (2n-1)\lambda$ and $\omega_n = \frac{6(2n-1)}{n(n+1)(4n-1)}$.

We verified the approximate CDF, as given in (11) with the sample CDF based on our simulation results. Figure 3 (left) depicts the results for a linear network and Figure 3 (right) for a planar network.

IV. Conclusion and Future Work

In this paper, we provide a mathematical analysis for the lifetime of a sensor network, when data-generation at individual sensor node is a random process. We showed that the mathematical results for expected lifetime and its probability distribution closely validate the simulations results, both in linear and planar networks.

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