Physics-based Electromigration Modeling and Assessment for Multi-Segment Interconnects in Power Grid Networks

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***Abstract*—Electromigration (EM) is considered to be one of the most important reliability issues for current and future ICs in 10nm technology and below. In this paper we focus on the EM stress evaluation for one-dimensional multi-segment interconnect wires in which all the segments have the same direction, which is a common routing structure for power grid networks. The proposed method, which is based on integral transform technique, could efficiently calculate the hydrostatic stress evolution for multi-segment metal wires stressed with different current densities. The new method can also naturally consider the pre-existing residual stresses coming from thermal or other stress sources. Based on this new transient EM assessment method, a full-chip assessment algorithm for power grid networks is then proposed. The new algorithm is also based on the IR-drop metrics for failure assessment of the power grid networks. However, it finds the precise location and time of EM-induced void nucleation by directly checking the time-changing hydrostatic stresses of all the wires. The resulting EM assessment method can ensure sufficient accuracy of the EM verification for large scale power grid networks without sacrificing the efficiency. The accuracy of the proposed transient analysis approach is validated against the numerical analysis. Also the resulting EM-aware full-chip power grid reliability analysis has been demonstrated and compared with existing methods.**

I. Introduction

VLSI chip reliability is vital to many mission-critical systems such as medical electronics, avionics, train control systems and so on. However, the reliability threats become more and more significant as the complexity of modern chips increase drastically. Electromigration (EM) has become one of the most critical design issues and limiting factors for nanometer VLSI designs because of the shrinking size and increasing current density of the interconnects as technology scals down to deep nanometers. As a result, electromigration verification became a critical step for chip sign-off.

The EM problem is specifically severe for power grid networks as they conduct large unidirectional currents and thus are more susceptible to EM failure than signal interconnects characterized by bidirectional currents. Traditionally a mean time to failure (MTTF) of the weakest segment is accepted as a measure for the life-time of interconnect network. This results in a very conservative design since modern power grids have mesh structures with high levels of redundancy, which could effectively tolerate the failures of some interconnect segments [1]. Therefore, the EM induced failure is happening mostly when the degrading power grid networks suffer a severe IR drop problem and fail to maintain the required voltage [2]. Another challenge in verification of EM reliability for power grids is the computational cost which is enormous due to the huge scale of the modern power grid networks. Full-chip EM assessment for large power grid networks requires very efficient EM simulation algorithms.

Traditionally, the approximate statistical Black's equation [3] and Blech limit [4] are employed to predict the EM-caused time to failure, which calculate the MTTF and the immortality for individual branches characterized by known current densities and temperatures. However, these methods are subjects of growing criticism due to their empirical nature and lack of consideration of residual stress [5], [6]. Modern interconnect networks such as power grids consist of interconnect trees representing continuously connected, highly conductive metal wires within one layer of metallization, terminated by diffusion barriers. Recent study shows that the stress evolution in each individual segment in an multi-segment interconnect wires are not independent as EM-induced migrations will take place in the whole interconnect tree [2]. To consider this effects, some physics-based EM analysis methods for the TSV and power grid networks have been proposed based on solving the basic mass balance equations [7]. However, these proposed methods can only solve small structures such as one TSV because the mass balance equations are solved by the expensive finite element method. To mitigate this problem, a novel compact physics-based EM model was proposed recently in [2], [8]. The proposed model is based on the diffusion-like continuity equation describing the kinetics of hydrostatic stress evolution [9]. Although the new EM assessment methods have been developed to deal with

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a multiple branch interconnect tree, they mainly focus on the steady-state hydrostatic stress instead of transient hydrostatic stress. Thus these methods cannot provide the accurate time evolution of the hydrostatic stress, which ultimately determines the failures (such as nucleation and void growth) for multi-branch interconnect wires. Recently an analytical modeling was proposed to provide exact expressions describing the hydrostatic stress evolution in several typical interconnect trees, namely the straight-line 3-terminal wires, the T-shaped 4-terminal wires and the cross-shaped 5-terminal wires [10]. This approach applies Laplace transformation technique to solve the stress evolution in a multi-branch tree by decoupling the individual segments through the proper boundary conditions accounting the interactions between different branches. However, this method only works for a few specific wire structures, not for general multi-branch or multi-segment metal wires.

In this paper, we developed the analytic and transient solution of the stress diffusion equations for general interconnect multi-segment wires. In section II, the EM models and analysis methods are reviewed, where the limitations of the existing method are also investigated. In order to mitigate the issues of existing methods, we proposed an accurate transient analysis method in section III. Based on this method, an effective method is proposed for power grids EM assessment in section IV. The experimental results are shown and discussed in section V. Finally, we give the conlusion.

II. Review of Physics-based EM Models and Analysis Methods

EM is a physical phenomenon of the migration of metal atoms along a direction of applied electrical field. The momentum exchange between atoms and the conducting electrons results in metal density depletion at the cathode and a corresponding metal accumulation at the anode ends of the metal wire. Since the thin layers of refractive metals form diffusion barriers for Cu atoms preventing them from diffusion into inter-layer (ILD) and inter-metal dielectrics (IMD), the EM occurs primarily on the interconnect tree, which is a continuously connected, highly conductive metal with one layer of metallization, terminated by diffusion barriers. When metal wire is embedded into a rigid confinement, the wire volume changes induced by the atom depletion and accumulation due to migration create tension at the cathode end and compression at the anode end of the wire. The lasting electrical load increases these stresses, as well as the stress gradient along the metal wire. The stress generated inside the embedded metal wire is a prime cause of the void and hillock formation at the opposite ends of the wire. Degradation of the electrical resistance of interconnect segment can be derived from the solution of kinetics equation describing the time evolution of stress in the interconnect segments [9], [11]–[13]. The void nucleation time could be obtained when stress reaches the critical value σ_{crit} and extracted kinetics of the void volume evolution governs the evolution of wire resistance. A finite-element analysis (FEA) modeling cannot be employed for the analysis of stress evolution caused by the current load in hundreds of millions interconnects because of the computational problem [2]. Therefore, the physics-based analytical compact model considering the void nucleation time and kinetics of void size evolution should be developed.

The physics-based analytical model considering the void nucleation and kinetics of void size evolution was proposed by Korhonen [9] and further developed by other researchers [11], [12]. For an one dimensional metal wire, the hydrostatic stress distribution $\sigma(x,t)$ along a single metal line is described as the diffusion-like equation (1) :

$$\frac{\partial \sigma(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[\kappa\left(\frac{\partial \sigma(x,t)}{\partial x} + \Gamma\right)\right] \tag{1}$$

where $\kappa = \frac{D_a B \Omega}{k_B T}$ is the "diffusivity of stress", $\Gamma = \frac{Eq^*}{\Omega}$ is the EM driving force and D_a is effective atomic diffusion coefficient, defined as (2)

$$D_a = D_0 \exp\left(-\frac{E_a}{k_B T}\right) \tag{2}$$

Here, D_0 is the pre-exponential factor, E_a is the activation energy, B is the effective bulk elasticity modulus, Ω is the atomic lattice volume, k_B is the Boltzmanns constant, T is temperature, E is the electric field, q^* is the effective charge, x is the coordinate along the line, and t is time. From Ohm's law, the electric field E could be replaced by the product of resistivity ρ and current density j, i.e. $E = \rho j$. The effective charge $q^* = |Z^*|e$ is a known quantity, where e is the elementary charge and $Z = |Z^*|$ is the effective charge number. As a result, the EM driving force Γ could be calculated by equation (3) as a function of current density.

$$\Gamma = \frac{eZ}{\Omega}\rho j \tag{3}$$

Under the effect of EM-induced driving force, the hydrostatic stress will build up as tensile stress (i.e. positive stress) or compressive stress (i.e. negative stress). As long as the tensile stress exceeds the critical stress σ_{crit}, the void nucleates. After the void nucleation, the wire comes to the void growth phase, in which the void would enlarge in size as a result of the atom depletion caused by current density. In the growth phase, the wire resistance starts to increase over the time accordingly. The drift velocity ϑ of the void edge is actually proportional to the current density, as shown in equation (4).

$$\vartheta = \frac{D_a}{k_B T} eZ\rho j \tag{4}$$

Then the wire resistance change can be approximately described as :

$$\Delta r(t) = \vartheta \cdot (t - t_{nuc}) \cdot \left[\frac{\rho_{Ta}}{h_{Ta}} \cdot \frac{1}{2H+W} - \frac{\rho_{Cu}}{HW}\right] \tag{5}$$

Besides the resistance change of wire, the void nucleation also causes significant change to the hydrostatic stress [14]. The tensile stress on the void nucleation will be effectively released, which usually leads to the tension release all over the wire. In addition, the void saturation also affects the kinetics of the void growth [8]. In this paper, we focus on the void nucleation phase, which is not affected by void saturation. Therefore we will ignore effects of void saturation and consider it in future work.

From kinetics of EM-induced void described above, it's clear that finding out the void nucleation time t_{nuc} is a key step for EM reliability assessment. In the case of a single-segment wire, the transient stress could be solved analytically by the technique of separation of variables. The analytical solution could be used to decide the void nucleation time by examining when the hydrostatic stress exceeds the critical stress σ_{crit}. However, in the case of interconnect tree, atoms migrating across branches of tree will eliminate the stress buildup at the unblocked branch ends. In other words, the stress will be effectively distributed all over the interconnect tree according to the current density. So it is important to figure out the hydrostatic stress regarding the interconnect as a whole. In order to calculate the hydrostatic stress on a interconnect tree, tree-based analysis methods [2] were proposed. According to these methods, the projected steady state distributions of hydrostatic stress (i.e. stress when atom diffusion stops) was first calculated instead of transient stress distribution and then the void nucleation time was estimated by the projected steady stress distribution.

However, it is known that the steady state analysis of the hydrostatic stresses distribution across the tree can provide just potential locations for void nucleation, where these stresses exceed the critical one. In reality, when the stress evolution kinetics is considered, these void nucleation site locations could be quite different. To mitigate this problem, an approach for calculating the hydrostatic stress evolution in the multi-branch interconnect trees during the void nucleation phase was recently proposed [10]. By using Laplace transformation technique, this method derived analytical expressions to describe the hydrostatic stress evolution in several typical interconnect trees: the straight-line 3-terminal wires, the T-shaped 4-terminal wires and the cross-shaped 5-terminal wires. The analytical solutions then were obtained in terms of a set of auxiliary basis functions using the complementary error function for each interconnect trees. Although both the steady state and transient hydrostatic stress could be analyzed by previous approaches, the existing methods are not yet sufficient for EM assessment of complex interconnect networks such as power grid networks, for following reasons. First, if only projected steady state stress is calculated and void nucleation time is estimated from the steady state stress distribution, then the calculated time and location of void nucleation are still estimations, the interplay between the void nucleation, void growth and current redistribution after resistance changes in segments can't be precisely determined and more transient stress analysis is required in this case. Second, the Laplace transformation based transient stress analysis method could obtain analytical solution of transient stress for interconnect trees. However, this method so far can only work for a few wire structures, not for general multi-segment interconnect wire structure.

Fig. 1. Multi-segements wire.

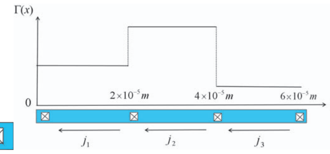


Fig. 2. EM driving force $\Gamma(x)$.

In this paper, we try to mitigate the mentioned issues of existing methods by seeking the solutions to the following problems: First, we propose an accurate transient analysis method for hydrostatic stress evolution using the integral transform technique to evaluate EM stress on the one-dimensional multi-segment interconnect wires. Second, based on proposed transient analysis of hydrostatic stress, we further propose an algorithm for effective EM assessment for power grid networks. These methods will ensure sufficient accuracy of the EM verification without sacrificing the efficiencies.

III. Accurate Transient Analysis of Hydrostatic Stress Evolution

In this paper, an accurate transient hydrostatic stress analysis method is proposed for one-dimensional multi-segment wire, as shown in the Fig. 1, which is a special case of interconnect tree. Since many on-chip networks, such as power grid networks, could be decomposed or approximately decomposed to multi-segment wires, the precise transient hydrostatic stress solutions for multi-segment wire lay the ground work for the EM reliability assessment for these networks. Based on the transient analysis method proposed in this section, a novel algorithm of EM assessment for power grid networks will be proposed in the next section.

For a multi-segment metal wire, the segments on wire can have multiple voltage input/output and current source ports represented by interlayer vias and contacts. Therefore, if we apply the Korhonen's equation (1) to the entire wire, then the EM "driving force" function $\Gamma(x)$ has varying values from segments to segments according to the current density of the segments, as illustrated by the Fig. 2 . It is also noticeable that other parts of the Korhonen's equation (1) remains unchanged while the boundary conditions are slightly different. Since the confined metal line has two blocked ends, the boundary conditions reflect the fact that the atom flux at the blocked ends are 0. In summary, the physics-based model for a confined multi-segment wire could be shown as Korhonen's equation (1) with EM "driving force" function as (6)

$$For\ x \in [x_k, x_{k+1}], \Gamma(x) = \Gamma_k = \frac{eZ}{\Omega}\rho j_k, k = 1, \cdots, n \quad (6)$$

and the boundary conditions as (7)

$$\begin{aligned} J_a(x,t)|_{x=0} &= \kappa\left(\frac{\partial\sigma(0,t)}{\partial x} + \frac{eZ}{\Omega}\rho j_1\right) = 0 \\ J_a(x,t)|_{x=L} &= \kappa\left(\frac{\partial\sigma(L,t)}{\partial x} + \frac{eZ}{\Omega}\rho j_n\right) = 0 \end{aligned} \quad (7)$$

where $J_a(x,t)$ represents the transient atom flux and j_1, j_n are current densities on the first and the last segment, respectively.

Lastly, the initial condition is specified as $\sigma(x, t = 0) = \sigma_0(x)$, where $\sigma_0(x)$ is a prescribed initial stress distribution.

To solve the Korhonen's equation (1) with the EM driving force function (6) and the boundary conditions (7), the integral transform technique is employed, which transforms the transient solution $\sigma(x, t)$ to a space-variable-free solution $\bar{\sigma}(\lambda_m, t)$ by using the integral transform (8).

$$\bar{\sigma}(\lambda_m, t) = \int_{\chi=0}^{L} \psi_m(\chi) \cdot \sigma(\chi, t) d\chi \tag{8}$$

where λ_m and $\psi_m(x)$ with $m = 1, 2, \cdots, \infty$ are the eigenvalues and eigenfunctions which are the solutions of the Sturm–Liouville problem corresponding to the diffusion equation (1) and the boundary conditions (7). For the hydrostatic stress over a multi-segment wire, the eigenvalues and eigenfunctions are determined by the geometry of the wire and the boundary conditions. With the confined metal boundary conditions (7), the eigenvalues and eigenfunctions are shown as following equation (9),

$$\begin{aligned} \lambda_m &= \frac{m\pi}{L} \\ \psi_m(x) &= \cos\frac{x}{L} m\pi \end{aligned} \tag{9}$$

where L is the length of the wire.

By the application of the integral transform technique , the Korhonen's equation (1) will be transformed to a first-order ordinary differential equation (ODE) for $\bar{\sigma}(\lambda_m, t)$, which is only function of time variable t. Then this ODE is solved for transformed solution $\bar{\sigma}(\lambda_m, t)$, which is shown as equations (10) and (11).

$$\bar{\sigma}(\lambda_m, t) = e^{-\kappa\lambda_m^2 t} \left[\bar{F}(\lambda_m) + \int_{\tau=0}^{t} e^{\kappa\lambda_m^2 \tau} A(\lambda_m) d\tau \right] \tag{10}$$

each parts of which are :

$$\begin{aligned} A(\lambda_m) &= \kappa \left[\bar{g}(\lambda_m) + \frac{\psi_m(x=0)}{k_1} f_1 + \frac{\psi_m(x=L)}{k_2} f_2 \right] \\ \bar{F}(\lambda_m) &= \int_{\chi=0}^{L} \psi_m(\chi) \cdot \sigma_0(\chi) d\chi \\ \bar{g}(\lambda_m) &= \int_{\chi=0}^{L} \psi_m(\chi) \cdot \Gamma'(\chi) d\chi \\ \Gamma'(\chi) &= \frac{\partial \Gamma(\chi)}{\partial \chi} \end{aligned} \tag{11}$$

where $f_1 = \frac{eZ}{\Omega}\rho j_1$, $f_2 = -\frac{eZ}{\Omega}\rho j_n$ and $k_1 = k_2 = 1$. Also notice that since $\Gamma(x)$ is combination of step functions, its derivative consist of Dirac δ-functions (12).

$$\Gamma'(x) = \frac{\partial \Gamma(x)}{\partial x} = \frac{eZ}{\Omega} \sum_{k=1}^{n-1} \rho(j_{k+1} - j_k) \cdot \delta(x - x_k) \tag{12}$$

Substitute the EM driving function (12) to (10) and (11), the transformed transient solution could be calculated as (13).

$$\begin{aligned} &\bar{\sigma}(\lambda_m, t) = \bar{F}(\lambda_m) e^{-\kappa\lambda_m^2 t} + \\ &\frac{1}{\lambda_m^2}(1 - e^{-\kappa\lambda_m^2 t}) \cdot \sum_{k=1}^{n} \Gamma_k \left(\cos\frac{x_{k-1}}{L} m\pi - \cos\frac{x_k}{L} m\pi \right) \end{aligned} \tag{13}$$

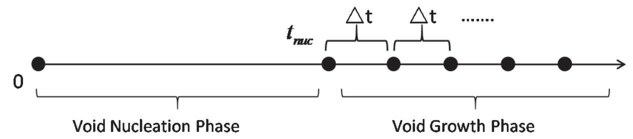


Fig. 3. Simulation phases and time steps for power grids.

Once the transformed solution $\bar{\sigma}(\lambda_m, t)$ is obtained, the transient solution of hydrostatic stress $\sigma(x, t)$ can be calculated by inverse integral transformation. Utilizing the orthogonality of the eigenfunctions $\psi_m(x)$, the transient hydrostatic stress could be easily expressed as infinite series, as shown by (14),

$$\sigma(x, t) = \sum_{m=1}^{\infty} \frac{\psi_m(x)}{N(\lambda_m)} \bar{\sigma}(\lambda_m, t) \tag{14}$$

where the norm of eigenfunctions $N(\lambda_m)$ is calculated as (15).

$$N(\lambda_m) = \int_{\chi=0}^{L} [\psi_m(\chi)]^2 d\chi \tag{15}$$

Once we know the stress change over time for all the wire segments of interest, we can determine the nucleation time of the wires when the stress of one segment hits the crtical stress and then we compute the resistance change over time for this segment in the void growth phase using (5).

IV. Effective EM Assessment Method for Full-chip Power Grid Networks

Based on the proposed transient analysis method for hydrostatic stress, the EM reliability of power grid networks could be assessed effectively. For every power grid metal wire, the transient hydrostatic stress at void nucleation phase is calculated accurately according to the equation (14), (13), (11) with eigenvalues and eigenfunctions (9). The time when the maximum tensile stress exceeds the critical stress σ_{crit} is determined as the nucleation time t_{nuc}. From then on, the void nucleates at the location of the maximum tensile stress and the metal wire comes into the growth phase.

In the growth phase, the void nucleation results in the resistance change of wire caused by the void size enlargement. Since the changing wire resistance results in current density redistribution over the power grid networks, the power grid networks become time-varying network and the corresponding time-varying systems 16 should be analyzed with the wire resistance updated according to equation (5).

$$G(t) \cdot V(t) = I(t) \tag{16}$$

The overall EM simulation process for power grid networks could be divided into two phases, as illustrated by the Fig. 3. In the void nucleation phase, none of the power grid wires have void nucleation. The current density remains unchanged in this phase, which implies that the parameters as well as the boundary conditions of the Korhonen's equation (1) are unchanged. Then the t_{nuc} could be solved by non-linear equation solving methods such as Newton's method or bisection method, given the transient solution of hydrostatic stress (14).

Algorithm 1: New transient full-chip EM assessment flow for power grid networks

Input: Power grid networks with current inputs, time step and technology parameters.
Output: The time exceeding the maximum voltage drop tolerance.

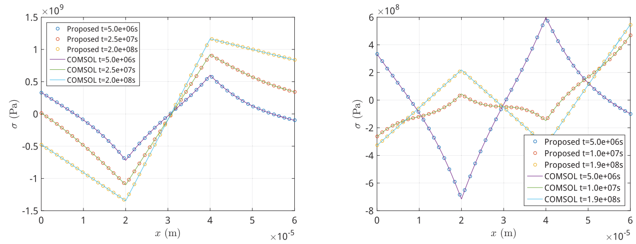
Compute the initial current density and the corresponding EM driving force $\Gamma(x)$;
Compute the steady state hydrostatic stress distribution;
For those wires of which steady state hydrostatic stress exceeds σ_{crit}, calculate the transient hydrostatic stress distribution and solve the nucleation time t_{nuc} using bi-section search method, given the initial stress distribution $\sigma_0(x)$. Compute the first nucleation time $t_0 = \min\{t_{nuc}\}$;
Start the analysis from time $t = t_0$. Initialize the wire set in growth phase with wires whose $t_{nuc} \geq t_0$;
while *the largest voltage drop* $\leq$ *threshold* **do**
    Move to next simulation time instance $t := t + \Delta t$. Update the wire resistance for wires in the growth phase;
    Perform the static analysis of the power grid networks. Update the current density distribution and the EM driving force function $\Gamma(x)$;
    Update the initial conditions $\sigma^k(x, 0) = \sigma^{k-1}(x, \Delta t)$. Perform the transient analysis of hydrostatic stress for wires in nucleation phase by solving the diffusion equation (1) with pre-voiding boundary conditions (7).;
    Find the new void nucleations where $\sigma(x, t) \geq \sigma_{crit}$ and add those wires with new void nucleation to the wire set in growth phase;
end
Output t as time to failure and the failed segment;

In the void growth phase, the EM simulation has to be divided to small time steps because the growing void nucleations leads to time-varying current density redistribution on the power grid networks, which changes the EM driving force function $\Gamma(x)$. Therefore a small time step Δt has to be taken to simulate the EM in this phase. We assume that the driving force function $\Gamma(x)$ stays unchanged in one time step while it varies from one time step to another. In consequence, the transient analysis of hydrostatic stress has to be carried out in each time step because of the changing $\Gamma(x)$. In order to push the simulation forward , the alorithm has to specify the initial conditions for each time step, which should be the transient stress distribution of the last time step, i.e. $\sigma^k(x, t = 0) = \sigma^{k-1}(x, t = \Delta t)$. The resulting algorithm flow for the full-chip EM assessment for power grid networks is presented in Algorithm 1.

V. Experimental Results and Discussions

A. Validation of Transient Analysis of Hydrostatic Stress

The proposed transient analysis method based on integral transform technique is implemented in Matlab. The proposed method is then validated by comparison with results of the recognized FEA tool COMSOL. The accuracy is measured as the difference between stress distribution calculated by the proposed method and COMSOL. i.e. $\frac{\|\sigma - \sigma_{COMSOL}\|}{\|\sigma_{COMSOL}\|}$. Fig. 4 shows the transient hydrostatic stress distribution over the multi-segments wire shown by the Fig. 2, presenting the results both from COMSOL and the proposed method. Fig. 4(a) shows the hydrostatic stress evolution for the three-segments wire with current densities as $j_1 = 0.8 \times 10^{10} A/m^2$, $j_2 = -2.3 \times 10^{10} A/m^2$, $j_3 = 0.3 \times 10^{10} A/m^2$ and Fig. 4(b) shows the stress evolution with current density redistribution



(a) (b)

Fig. 4. The hydrostatic stress evolution without current density redistribution (a) and with current density redistribution at $t = 5 \times 10^6 s$ (b).

to $j_1 = -0.5 \times 10^{10} A/m^2$, $j_2 = 0.5 \times 10^{10} A/m^2$, $j_3 = -0.8 \times 10^{10} A/m^2$ at time $t = 5 \times 10^6 s$. In both cases, the proposed method is shown to be accurate enough comparing to the results from COMSOL (with maximum discrepancy 0.31%) and could handle the current density redistribution correctly.

As a matter of fact, the accuracy of proposed method depends on the number of eigenfunctions used to represent the transient solution. The more eigenfunctions used, the more accurate the solution is. The sufficient number of eigenfunctions depends on the spatial variance of current density. The number of eigenfunctions is chosen to be 200 in these experiments.

B. EM Assessment for Power Grid Networks

The presented EM assessment algorithm is implemented in Matlab and validated by the IBM power grid benchmarks [15] on a linux workstation with a 3.6 G dual-core CPU and 8 GB memory. The power networks of the benchmarks are used to test our method and their source currents are scaled to ensure the initial voltage drop to be smaller than the threshold value. In this work, the interconnect material is assumed to be copper and the voltage drop tolerance is chosen to be 10% V_{DD}. Parameters used in the experiments follow those in [2]. Table I shows the power grid MTTF obtained from the existing methods and our proposed approach, where the column 1 to 3 are the name, number of nodes and number of wires of the benchmarks, respectively; column 4, 5 and 6 are the MTTF obtained from Black's equation as presented in [2], the method in [2] and the proposed method, all using the mesh modeling; column 7 and 8 are the total run-times of the method in [2] and the proposed method. Since in this work the void saturation is not considered, MTTF without void saturation obtained in [2] is chosen for comparison.

From the experimental results, we can observe similar characteristics as shown by [2] : the Black's equation based mesh model is more conservative because it assumes infinite resistance after the predicted TTF of each segment of wire while actually the metal line continues to conduct current after void nucleation with increasing resistance.

As we can see from Table I, the efficiency of the proposed algorithm is satisfactory. Actually, the run-time of the algorithm depends on the number of eigenfunctions used when solving the transient hydrostatic stress. More eigenfunctions

TABLE I
POWER GRID MTTF USING BLACK'S EQUATION AND PROPOSED EM ASSESSMENT METHOD

Power Grid			Time to Failure (yrs)			Runtime	
Name	Nodes	Wires	Black's Equation (mesh)	Method in [2] (no void sat)	Proposed Method	Method in [2]	Proposed Method
IBMPG2	61797	462	12.83	16.85	15.08	6.36 min.	0.96 min.
IBMPG3	407279	8683	17.90	23.56	27.00	5.83 hr.	0.16 hr.
IBMPG4	474836	11267	22.27	26.97	28.20	14.71 hr.	0.33 hr.
IBMPG5	497658	2500	12.34	19.13	23.72	40.64 min.	8.38 min.
IBMPG6	807825	10510	10.89	14.62	13.92	1.75 hr.	0.42 hr.
IBMPGNEW1	715022	20022	13.96	18.84	25.20	16.78 hr.	0.5 hr.
IBMPGNEW2	715022	20022	13.84	15.60	25.20	15.32 hr.	0.59 hr.

to use, more accurate the transient solution is, while more time the algorithm costs. The sufficient number of eigenfunctions depends on the current density distribution on power grid networks. In experiments of this work, the number of eigenfunctions is chosen to be 200, which is quite adequate for power grid testcases. From the run-time of the experiments, we can see that even with a large number of eigenfunctions, the time-cost of the proposed EM assessment method is still low.

VI. CONCLUSION

In this paper, we have proposed an accurate transient analysis method for the hydrostatic stress evolution for a multi-segment one-dimensional interconnect tree for fast electromigration failure assessment. The proposed method, which is based on integral transform technique, solves the stress diffusion equation for multi-segment wires. The new method overcomes the difficulties of previous methods by improving the numerical accuracy and efficiency. Based on this transient analysis method, an effective algorithm has been proposed for full-chip EM assessment of power grid networks. Instead of estimating the void nucleation time from steady state hydrostatic stress, the proposed algorithm determines the void nucleation by directly checking the transient stress distribution, which is more precise and reliable. Because of the benefits of the proposed methods, the EM verification for large scale power grids could be carried out accurately without sacrificing the efficiency.

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