Shape optimization of a Power MOS device under uncertainties

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***Abstract*—In this paper we focus on a shape/topology optimization problem of a power MOS transistor under geometrical and material uncertainties to reduce the current density overshoot. This problem, occurring in the automotive industry, yields a stochastic electro-thermal coupled problem. Its solution enables to investigate the propagation of uncertainties through a 3-D model, which affect yield and performance of a power transistor. In our work, the Stochastic Collocation Method (SCM) has been used for this purpose. In particular, uncertainties, which result from imperfections of an industrial production, are modeled by random variables with known a priori probability density distributions, for example, a Gaussian or uniform type. Then, the Polynomial Chaos Expansion (PCE) with the basis associated to the assumed distribution can be used to construct numerical methods for a stochastic representation of the random-dependent solutions. Furthermore, this optimization is formulated in terms of statistical moments such as the mean and the variance. The gradient directions of a bi-objective cost functional is calculated using the Continuum Design Shape Sensitivity and the PCE in conjunction with the SCM. Finally, the optimization results for a relevant nanoelectronics problem demonstrate that the proposed method is robust and efficient.**

I. Introduction

The use of the power transistor devices has gained large interests in several field of applications. For instance, they play an important role in energy harvesting and its distribution in order to control overall energy efficiency. Another field, which requires to handle demanding electro-thermal operational constraints with respect to both components and electronic systems, is the automotive industry. Especially, in this application, there is a strong demand for the continuous improvement of performance of a power MOS transistor, since they are widely used, for example, as a switching device in electronic control units. Consequently, they have found a broad application in the field of the Switch-Mode Power Supplies (SMPS) such as servers, solar and desktop, AC/DC converters, battery charges, etc., due to its several attractive features including the minimized gate charge, high transconductance, high speed switching and low static drain-source on resistance [2].

The basic structure of a power MOS transistor, shown schematically on Figs. 1 and 2, is comprised of several thousands of elementary transistor cells connected in parallel in to meet technical requirements related to the current handling capability. In this respect, due to the imperfections in manufacturing processes such as sub-wavelength lithography, lens aberration and chemical-mechanical polishing [17], the physical domain of power devices is affected by relatively large geometrical and material uncertainties. On the one hand, this has impact on the yield and performance of a power transistor and in general on the reliability of power electronic systems, especially in the context of the continuous improvement and the higher complexity of the smart power IC technology [37]. On the other hand, in particular the localized imperfections in the die inside can cause the formation of the current density overshoots [1] and lead to the unexpected thermal instability of power devices [28]. More specifically,

[1]According to IEEE Trans. Power Electr. 15, 575-581, 2000, it refers to the formation of a hot-spot phenomenon [10].

under an overload condition, the non-uniformity of the current distribution may result in a non-homogeneous electrical power dissipation, which, in consequence, yields an unstable self-heating effect [2], [24]. Another key issue for many applications is the ruggedness, which determines the capability of power devices to handle the high avalanche currents during the applied stress [25]. In addition, many reliability failure mechanisms strongly accelerate at high temperature, including, for example, the voltage breakdown [6]. Therefore, there is a need for a robust optimization method aimed at reducing the heat dissipation, which utilize an accurate and reliable simulation of electro-thermal behavior of power devices [14].

A lot of investigations devoted to this topic have been presented in recent years. For instance, in [5] and [31] two traditional countermeasures to the elimination of the hot-spot such as the ballasting of emitter resistance at the metal finger and a top copper spreader have been thoroughly studied. A thermal-aware exploration framework at the micro-architectures level has been analyzed in [8] and used for temperature hot-spots reduction by the selective resource replication. Authors of work [27], in turn, dealt with the temperature reduction at the gate level in order to eliminate a local temperature uprising and the current density accumulation. In [10], based on [6] the deterministic evolutionary framework has been used to eliminate a thermal instability by optimizing a power MOS layout.

In our approach we propose to optimize a topology of power devices under geometrical and material uncertainties to eliminate the current density overshoots. Thus, the novelty consists mainly in incorporating the industrial imperfections measure into the optimization flows, which results in a robust design of a power transistor device. However, our formulation involves also the random-dependent voltage source optimization problem, which has not been studied yet in the stochastic framework, neither in [21] or [23]. This allows us to investigate the influence the shape variations of voltage sources (pads of drain and source) on the drain-source on resistance and optimize its value in order to eliminate the current density accumulation phenomenon. Thus, in order to assess the reliability and robustness of a design with respect to uncertain parameters, the SCM [35] with the PCE has been applied. The technique yields a response surface model, which can be further incorporated into a topology optimization method.

II. Device design

Fig. 2 presents a typical layout (stretched in the vertical direction) of a special construction of a power device with three metal layers, and used in our research as a case study. It is a multi-finger MOSFET power transistor with a stripe cell structure, which consist of several thousands of parallel channel devices. The source and drain contacts are located on the top metal finger of the design, as shown in Fig. 1. A series of metal stripes and complex via patterns transport the current to drain and away from the sources of the individual channels. Consequently, the multi-dimensional current flow

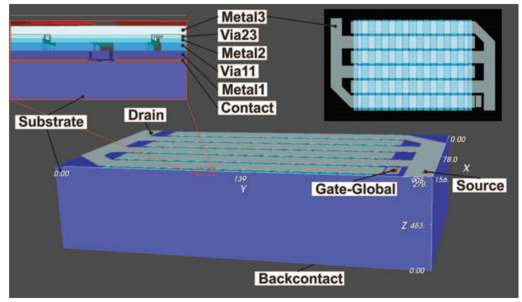

Fig. 1. Topology of a power transistor device.

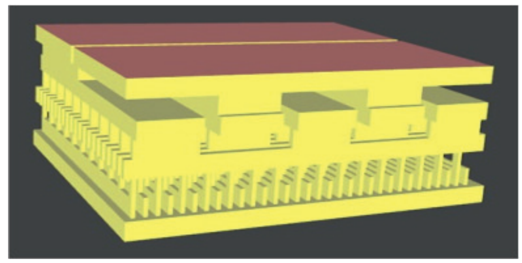

Fig. 2. Typical layout of a power transistor with its complex geometry (vertically stretched).

is governed by coupled time-dependent system of stochastic Partial Differential Equations (PDEs) of the form

$$\begin{cases} \nabla \cdot [\epsilon(\theta)\nabla V(\theta)] = \rho(\theta), \\ \partial_t \rho(\theta) + \nabla \cdot \vec{J}(\theta) = 0, \\ \vec{J}(\theta) = \sigma(\theta)\nabla V(\theta), \\ \partial_t U(\theta) = \nabla \cdot \vec{Q}(\theta) + Q_{\mathrm{e}}(\theta), \end{cases} \tag{1}$$

endowed with suitable initial and boundary conditions, where $\theta := (\boldsymbol{x}, t, \boldsymbol{\xi}) \in D \times (0, T] \times \Xi$ with D being a bounded domain in $\mathbb{R}^3, t \in (0, T]$ and Ξ a multidimensional parameter domain. The electric conductivity σ, the permittivity ϵ, independent of V (linear materials). $\rho, V, T, (T_0), U = C_v(\theta)(T(\theta) - T_0), Q_{\mathrm{e}} = \sigma(\theta)|\nabla V(\theta)|^2$ are real-valued function such as the charge density, the electric scalar potential, the temperature, the environment temperature, the heat flux and self-heating due to Joule's law, whereas $\vec{Q}(\theta) = -\kappa(\theta)\nabla T(\theta)$ and $\vec{J}(\theta)$ are the heat flow and the current density, respectively. The thermal conductivity κ and the thermal capacitance C_v are real-valued functions of space.

In order to solve the electro-thermal coupled problem, defined by the system of Eq. (1), the MAGWEL software has been applied [14]. The simulator uses a well-adopted mesh for substrate, which is crucial for the accurate simulation of the temperature distribution. Joule self-heating and the heat flow in a metal is modeled together with the linear temperature-dependent electrical conductivity ($\sigma = W_k\sigma_k$ with W_k being the layer size), thermal conductivity and thermal capacitance. As a consequence of the used self-consistent approach, every electric transport inside the MOS channels is dealt with a compact model, i.e, the drain to source current flows are described by $I_{\mathrm{DS}} = f(V_{\mathrm{DS}}, V_{\mathrm{GS}})$. Correspondingly, the heat generated in the channel is also calculated from the channel resistance using $Q_\mathrm{e} = V_\mathrm{DS} \cdot I_\mathrm{DS}(V_\mathrm{DS}, V_\mathrm{GS})$, where V_DS and V_GS denote the drain-source and the gate-source voltage, respectively. By this powerful approach it is possible to solve such a big system, which will be a time-consuming task when the drift diffusion model is applied.

III. Uncertainty Quantification for the stochastic forward problem

Recently the SCM combined with the PCE technique has attained some interest in electrical engineering to assess the reliability and robustness in the design of electric devices with respect to uncertain parameters, see, e.g., [3], [22], [26], [19]. Following the methodology proposed in [35], we substitute the parameters $\boldsymbol{\xi} \in \Xi$ in the model (1) by random variables $\boldsymbol{\xi} : \Omega \to \Xi$ on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$ using independent probability distributions. The availability of a joint density function $g : \Xi \to \mathbb{R}$ is assumed. Since we consider standard distributions such as Gaussian and uniform type in our simulations, the Hermite and the Legendre polynomials are required as orthogonal basis functions in the PCE.

Given a function $y : \Xi \to \mathbb{R}$, we define the expected value (provided the integral exists) as

$$\mathbb{E}\left[y(\boldsymbol{\xi})\right] = \int_\Omega y(\boldsymbol{\xi}(\omega))\,\mathrm{d}\mathbb{P}(\omega) = \int_\Xi y(\boldsymbol{\xi}) g\left(\boldsymbol{\xi}\right)\,\mathrm{d}\boldsymbol{\xi}, \quad (2)$$

Furthermore, let y be square integrable. It follows that a response surface model of y can be obtained by a truncated series of the PCE, see [36],

$$y\left(\mathbf{s}, t, \boldsymbol{\xi}\right) \doteq \sum_{i=0}^{N} v_i\left(\mathbf{s}, t\right) \Phi_i\left(\boldsymbol{\xi}\right) \quad (3)$$

with a priori unknown coefficient functions v_i and predetermined basis polynomials Φ_i with the orthogonality property $\mathbb{E}\left[\Phi_i \Phi_j\right] = \mathbb{E}\left[\Phi_i^2\right]\delta_{ij}$ (Kronecker delta). Therein the deterministic variables $\mathbf{s}$ denote design variables. In our work, a pseudo-spectral approach with the Stroud formula of order 3, see [30], also used in [4], [35], has been applied for the calculation of the unknown coefficients v_i. The basic idea of this technique relies on repetitive runs of the deterministic problem, defined by the system of Eq. (1), to obtain the solution at each quadrature node $\boldsymbol{\xi}^{(k)}$, $k = 1, \ldots, K$. Then, the multi-dimensional quadrature rule with associated weights w_k yields

$$v_i(\mathbf{s}, t) \doteq \sum_{k=1}^{K} y\left(\mathbf{s}, t, \boldsymbol{\xi}^{(k)}\right) \Phi_i\left(\boldsymbol{\xi}^{(k)}\right) w_k, \quad (4)$$

which represents an approximation of the exact projection of $\mathbf{y}$ onto the basis polynomials. Finally, statistics like the mean and the variance can be approximated as follows, cf. [34],

$$\mathbb{E}\left[y\left(\mathbf{s},\ t,\ \boldsymbol{\xi}\right)\right] \doteq v_0(\mathbf{s}, t) \quad (5)$$

and

$$\mathrm{Var}\left[y\left(\mathbf{s},\ t,\ \boldsymbol{\xi}\right)\right] \doteq \sum_{i=1}^{N} |v_i(\mathbf{s}, t)|^2. \quad (6)$$

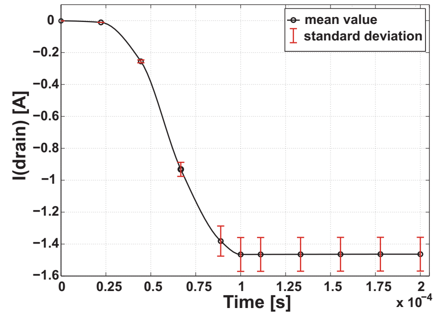

Fig. 3. UQ of I(drain) due to variation of the Metal3 thickness, modeled by a Gaussian distribution with 10% variation around a mean of 1μm.

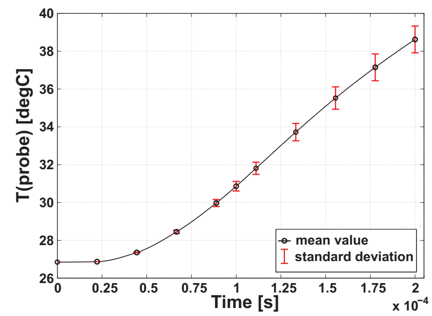

Fig. 4. UQ of T(probe) due to the variation of σ of Metal3, modeled by a uniform distribution with 15% variation around a mean of 20 MS/m.

The exemplary result for the UQ analysis, shown in Fig. 3 and 4 for $y = I(\mathrm{drain})$ and $y = T(\mathrm{probe})$, respectively, allows us to assess quantitatively and qualitatively the impact of the chosen input random variables on the output characteristics of the power devices. In this case, relatively large input variations influence moderately but significantly the behavior of power transistors. However, even more accurate information about influence of input variations can be provided by a sensitivity analysis. When using the response surface model (3), it can be easily carried out, see, e.g., [9], [33] and references therein. In this respect, based on the sensitivity analysis [22], we have chosen the random variables such as the conductivity of Metal3 layer σ_3, the thickness of Metal2 W_2 and the thermal capacitance of the Via12 C_v. Since we deal with the stochastic voltage source problem, the drain and source contacts are also considered as two additional random variables, thus finally we define $\boldsymbol{\xi}(\omega) = [\sigma_3(\omega), W_1(\omega), C_v(\omega), V_\mathrm{D}(\omega), V_\mathrm{S}(\omega)]$. Correspondingly, in this work shapes of Metal3 fingers and shapes with placement of drain and source pads are analyzed during the topology optimization process under uncertainties.

IV. Random-dependent cost functional

Let us consider a steady-state counterpart of the system Eq. (1) in the formulation of the stochastic optimization problem. Then, the random-dependent electro-thermal coupled problem is described by system of PDEs in the form

$$\left\{\begin{array}{l} \nabla \cdot \left[\epsilon\left(\chi\right) \nabla V\left(\chi\right)\right] = \rho, \\ \nabla \cdot \left[\sigma\left(\chi\right) \nabla V\left(\chi\right)\right] = 0, \\ \nabla \cdot \left[\kappa\left(\chi\right) \nabla T\left(\chi\right)\right] = \sigma\left(\chi\right) |\nabla V\left(\chi\right)|^2, \end{array}\right. \quad (7)$$

equipped with random Dirichlet boundary conditions, such that $V_{\text{Dra}}(\chi) = V_{\text{D0}}(\boldsymbol{\xi})$ on Γ_{Dra} and $V_{\text{Sou}}(\chi) = V_{\text{S0}}(\boldsymbol{\xi})$ on Γ_{Sou}, which describes the potentials of the drain and source pads, respectively, where $\chi := (\boldsymbol{x}, \boldsymbol{\xi}) \in D \times \Xi$.

Furthermore, to reduce the current density overshoots in the area of the contact layer of power device, we formulate based on the weighted average method [15] a random-dependent cost functional as follows

$$F(\boldsymbol{v}) = w_1 \int_{\text{D}_1} Q_{\text{e}}[\boldsymbol{v}, V(\boldsymbol{v})] \, \mathrm{d}\boldsymbol{x} + w_2 \int_{\Gamma} h[V(\boldsymbol{v})] \, \mathrm{d}\boldsymbol{\gamma}, \quad (8)$$

where the dissipation power Q_{e} is analyzed in the area of Metal3 layer $\text{D}_1 \subset \mathbb{R}^3$, and the source voltage term h is represented by the random Dirichlet boundary condition in the area of the source and drain pads $\Gamma \subset \mathbb{R}^2$. The variable $\boldsymbol{v}$ is defined as $\boldsymbol{v} = (\boldsymbol{x}, \mathbf{s}, \mathbf{p}(\boldsymbol{\xi}))$, whereas the weights w_1 and w_2 refer to known a priori information about objectives.

Summarizing, we deal with a stochastic system of PDEs constrained optimization problem, defined by the system (7) and Eq. (8). It will be solved using the topological derivative method [7].

V. Topological Derivative Method

The topological derivative method (TD), which originally was invented by the authors of [7] and [29], provides sensitivity information of a design with respect to the topological change, such as an air hole inside a domain of interest. Its asymptotic expansion can be found for a wide class of 2D/3D linear/nonlinear problems, see, e.g., [1] and [16]. More recently, this method has found a broad application in electrical engineering to solve deterministic optimization and inverse problems e.g., [12], [13] and [20]. Lately, it has been also successfully used for a solution of a stochastic optimization problem [23].

Let $j(D) := F(V)$ be an arbitrary cost functional that is minimized on domain D, whereas V denotes a solution of the defined PDE problem. Moreover, let for some $d > 0$, $D_d := D \backslash B(\boldsymbol{x}_0, d)$ be the subset of D when removing a small hole $B(\boldsymbol{x}_0, d)$ with center $\boldsymbol{x}_0$ and a radius d. Then, the asymptotic expansion of the TD is given by [16]

$$\begin{aligned} & j(\text{D}_d) - j(\text{D}) = f(d) g(\boldsymbol{x}) + o(f(d)), \\ & \lim_{d \to 0} f(d) = 0, f(d) > 0. \end{aligned} \quad (9)$$

Thus, in order to minimize a cost functional defined by Eq. (8), small holes at some points x, where the topological derivative exhibits $g(\boldsymbol{x}) \leq 0$, need to be created when taking the optimality condition and mass/volume constraints during the optimization process into account [7]. More precisely, the TD can be defined based on [11] and [16] as

$$g(\boldsymbol{x}) = \begin{cases} 2\left(\beta_1^{(k)} E(V^{(k)}) E(\lambda^{(k)})\right), & \text{if } \boldsymbol{x} \in D_1, \\ 3\left(\frac{(\gamma_2^{(k)} - \gamma_1^{(k)})}{(\gamma_1^{(k)} + 2\gamma_2^{(k)})\gamma_2^{(k)}}\right) \nabla V^{(k)} \nabla \lambda^{(k)}, & \text{if } \boldsymbol{x} \in D_2. \end{cases} \quad (10)$$

under the assumption that, we analyze the problem defined by system (7) and Eq. (8) at the k-th grid point of the random parameters. Here, E and λ are a normal component of the electric field strength and the adjoint variable, respectively. The piecewise constant function such as $\beta(\boldsymbol{x})$ and $\alpha(\boldsymbol{x})$ have been expressed by

$$\beta(\boldsymbol{x}), \alpha(\boldsymbol{x}), = \begin{cases} \beta_1, \alpha_1; & \text{if } \boldsymbol{x} \in D_j \backslash B_j(\boldsymbol{x}, d), \\ \beta_2, \alpha_2; & \text{if } \boldsymbol{x} \in B_j(\boldsymbol{x}, d), \end{cases} \quad (11)$$

where a subscript j indicates domains D_1, D_2, D_3 and D_4. Here, D_1, D_2 denote the regions of metal3 and the lack of this material, while D_3 and D_4 refers to position of the Dirichlet boundary conditions, respectively. However, for the functional (8) a dual problem is the rescaled version of a direct problem, since both objective functions are self-adjoint. Hence, in order to calculate the derivative of the mean (5) and variance (6) with respect to the random parameters, the calculation of the response surface model for the topological sensitivity using (10) is required.

VI. Robust optimization problem

Finally, when considering the mean and the standard deviation, a stochastic system of PDEs constrained optimization problem can be further reformulated into the robust single objective optimization problem [21], [23] as

$$\begin{aligned} \min_{\boldsymbol{v}} \quad & \mathbb{E}[F_{\boldsymbol{v}}(\boldsymbol{v})] + \eta \sqrt{\mathrm{Var}[F_{\boldsymbol{v}}(\boldsymbol{v})]} \\ \text{s.t.} \quad & \begin{cases} \nabla \cdot \left[\epsilon\left(\boldsymbol{\xi}^{(k)}\right) \nabla V^{(k)}\left(\boldsymbol{\xi}^{(k)}\right) \right] = \rho, \\ \nabla \cdot \left[\sigma\left(\boldsymbol{\xi}^{(k)}\right) \nabla V^{(k)}\left(\boldsymbol{\xi}^{(k)}\right) \right] = 0, \\ \nabla \cdot \left[\kappa\left(\boldsymbol{\xi}^{(k)}\right) \nabla T^{(k)}\left(\boldsymbol{\xi}^{(k)}\right) \right] = Q_{\text{e}}^{(k)}\left(\boldsymbol{\xi}^{(k)}\right), \\ \qquad V_{\text{D}}\left(\boldsymbol{\xi}^{(k)}\right) = V_{\text{D0}}\left(\boldsymbol{\xi}^{(k)}\right), \\ \qquad V_{\text{S}}\left(\boldsymbol{\xi}^{(k)}\right) = V_{\text{S0}}\left(\boldsymbol{\xi}^{(k)}\right), \\ \\ \text{area/vol.}(D_{i,j}) \leq m_i, j = 1, \ldots, 4; i = 1, 2, \\ p_{\max_\ell} \leq p_\ell \leq p_{\min_\ell}, \ell = 1, \ldots, 3. \end{cases} \end{aligned} \quad (12)$$

where we use $\eta = 3$ and p_ℓ is related to geometrical constraints of layers. The stochastic forward problem defined by system (7) is calculated at the K quadrature grid points.

VII. Numerical example

As a numerical example, we analyze the topology of a power transistor device, shown on Fig. (1) and (2). In particular we deal with a stochastic shape and source optimization problem of both the metal3 layer (two fingers) and the area of the pads (source and drain). For the stochastic optimization, we considered five random input parameters modeled by uniform distributions: the electric conductivity of the metal3 layer $\sigma = \sigma_{03}(1 + \delta_1 \xi_1)$ with $\sigma_{03} = 2.0$ [MS/m] and $\delta_1 = 0.25$; the thickness of the Metal1 layer $W_1 = W_{01}(1 + \delta_2 \xi_2)$ with $W_{01} = 1.0$ [μm] and $\delta_2 = 0.2$; and the thermal capacitance of the Via12 $C_v = C_{v0}(1 + \delta_3 \xi_3)$ with $C_{v0} = 2.46$ [MJ/K] and $\delta_3 = 0.3$; the voltage of drain and source contacts $V_{\text{Dra}}(\boldsymbol{x}) = V_{\text{D0}}(1 + \delta_4 \xi_4)$ with $V_{\text{D0}} = 0.5$ [V] and $\delta_4 = 0.1$; $V_{\text{Sou}}(\boldsymbol{x}) = V_{\text{S0}}(1 + \delta_5 \xi_5)$ with $V_{\text{D0}} = 0.1$ [mV] and $\delta_5 = 0.05$ where $\xi_m \in [-1, 1]$ for $m = 1, \ldots, 5$. The initial and the optimized shapes of the metal3 layer as well as the drain and source pads are depicted on Fig.5, respectively. The current density overshoots have been completely removed for the optimized structure in 18th iteration of the stochastic optimization process. Both the current density (CD) as well as the violation are shown on Fig. 6. The hot spots are treated here as a violation of the CD in the contact layer calculated for the initial model. The result for the current and temperature distribution in the optimized structure has been shown in Fig. 7 under the assumption that $|\vec{J}| = 1.13 \cdot 10^{10}[\mathrm{A/m^2}]$ serves as the violation threshold in the contact layer. A decrease of temperature in the metal3 layer is about $32°\mathrm{C}$, while for the contact layer the temperature reduction became $8°\mathrm{C}$. Additionally, we present also the course of the total resistance, the course of the total power and the course of the total current during the optimization in Fig. (10), in Fig. (9) and in Fig. (8), respectively.

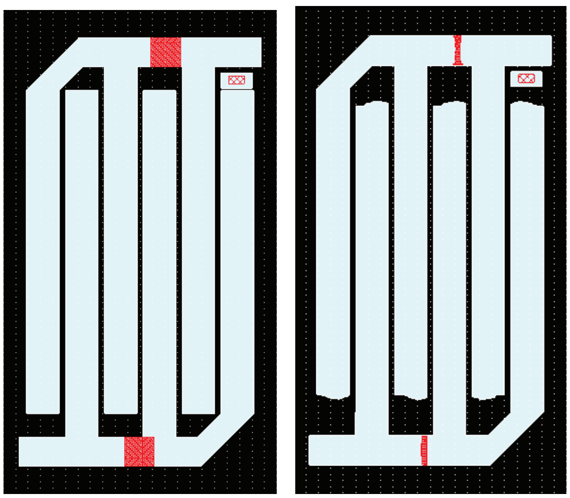

Fig. 5. Shapes of the matal3 layer as well as the drain and source pads (red color) for: the initial configuration (left), the optimized model in the 18th iteration (right).

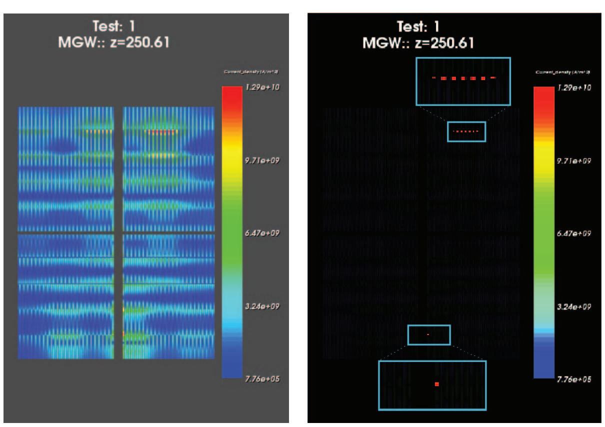

Fig. 6. The current density for the initial model (CD) in the contact layer (left) and hot spots represented by 8 red dots in the enlarged blue box (right).

VIII. Conclusion

In our work we investigated a stochastic optimization problem. We successfully implemented our algorithm in Python

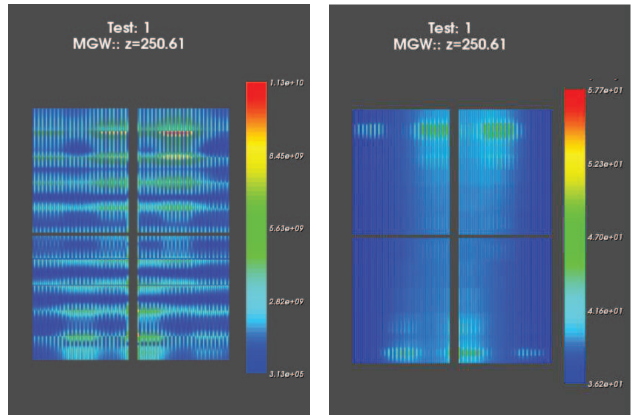

Fig. 7. The current density (left) and the temperature distribution (right) in the contact layer for the optimized model in 18th iteration.

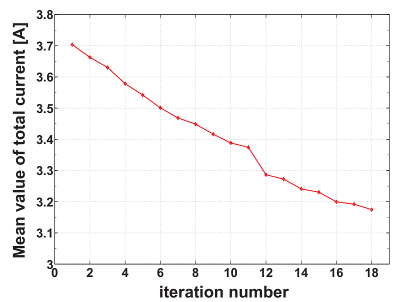

Fig. 8. The total current as a function of iteration.

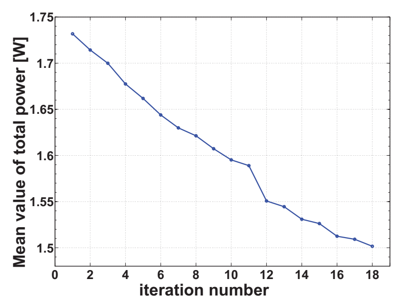

Fig. 9. The total power as a function of iteration.

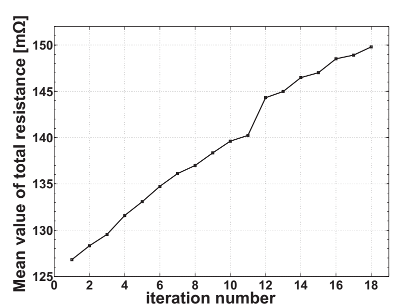

Fig. 10. The total resistance as a function of iteration.

using software from MAGWEL [14]. The current density overshoots have been completely removed by optimizing both the metal3 layers and shape of source and drain pads. However, the high temperature of the metal3 layer, which is rather related to the technical aspect of our test-case, might be considered as a drawback of the proposed method. As a result, also the temperature in the contact layer has been decreased significantly. In our opinion, this methodology for the stochastic optimization can be also used for different power device technologies like the low power MOS device.

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