# An Algorithm to Improve Accuracy of Criticality in Statistical Static Timing Analysis

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*Abstract*— **Statistical design approaches have been studied intensively in the last decade so as to deal with the process variability, and statistical delay fault testing is one of key techniques for the statistical design. In order to represent the distributions of timing information such as a gate delay, a signal arrival time, and a slack, various techniques have been proposed. Among them, Gaussian mixture model is distinguished from the others in that it can handle any correlation, non-Gaussian distributions, and slew distributions easily. However, the previous method of computing the statistical maximum for Gaussian mixture models has a defect such that it produces a distribution similar to Gaussian in a certain case, although the correct distribution is far from Gaussian. In this paper, we propose a novel method for statistical maximum (minimum) operation for Gaussian mixture models. It takes cumulative distribution function curve into consideration so as to compute accurate criticalities (probabilities of timing violation), which is important for detecting delay faults and circuit optimization. The proposed method reduces the error of criticality almost 80% from the previous method.**

***Keywords- criticality; probability of timing violation; statistical static timing analysis; Gaussian mixture model; cumulative distribution curve***

## I. INTRODUCTION

Due to the progress of nanometer process technologies, variability of circuit parameters is increasing and timing verification becomes a hard task[1]. To overcome the variability issue in the timing design, statistical static timing analysis (S-STA) has been studied intensively in the last decade[2]. The proposed S-STA algorithms are classified into two types; path-based approach and block-based approach[3-8]. Moreover, they can be classified into two types by the way to represent the distributions; one uses Gaussian distribution only[3,4] and the other uses non-Gaussian distribution[5-8]. Among them, the algorithms using Gaussian distribution can handle not only spatial correlation but also topological correlation caused by reconvergent paths easily. However, the distribution of the maximum (or minimum) of two Gaussian distributions is not Gaussian[2].

To handle non-Gaussian distribution, various methods have been proposed[5-8]. Among them, Gaussian mixture model proposed in [8] is distinguished from the others, in that it can handle not only the spatial correlation but also the topological correlation easily, because the distribution is represented by a probability weighted sum of Gaussian distributions. Moreover, slew distributions and the effects of slews to gate and interconnect delays can be handled[8].

The easiness of handling correlations is an important feature to select paths statistically for delay fault testing[9,10], because the path selection must be conducted by considering the criticality of a path and/or criticalities of an edge and a vertex. The criticality is defined by the probability for the slack $D_R - D_A$ to be negative, where $D_A$ and $D_R$ are the latest arrival time (or the maximum delay) and the required arrival time, respectively. Both $D_A$ and $D_R$ are random variables (RVs), unless the arrival times are considered at a primary output, and we must take correlations into account so as to find their accurate distributions.

Moreover, when calculating the conditional probability Pc = Pr[ $D_{P1} > D_{P2}$ | Max[$D_{P1}$,$D_{P2}$]>$\tau$ ] for the delay $D_{P1}$ of path P1 to be larger than the delay $D_{P2}$ of path P2 under the condition that Max[$D_{P1}$,$D_{P2}$] is larger than the cutoff delay $\tau$ (constant), we must take the correlation between $D_{P1}$ and $D_{P2}$ into account. Because even if the delays of all gates are independent each other, $D_{P1}$ and $D_{P2}$ may have a correlation, if P1 and P2 have a common gate. This probability Pc is useful in delay fault testing and in inserting a canary flip-flop[11], since if Pc is almost 1, we can get rid of path P2 from our consideration.

Although Gaussian mixture model has remarkable features, the maximum operation proposed in [8] has a defect such that it may produce a distribution like a Gaussian, when the correct distribution is quite different from Gaussian. Hence, the error of criticality may become large. Such a distribution is produced because the method in [8] considers only the mean and variance of the correct distribution but not its shape.

In this paper, we propose a novel method for statistical maximum (minimum) operation for Gaussian mixture models, which considers the shape of cumulative distribution function. The proposed method for the statistical maximum never produces a worse distribution shape than the method in [8], since the proposed method cover the method in [8], and we can attain almost 80% accuracy improvement from the method in

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[8]. Since the minimum operation is quite similar to the maximum operation, we explain the maximum operation only in this paper.

## II. Gaussian Mixture Model

The block based S-STA algorithm[3-8] represents a given circuit by a directed acyclic graph $G=(N,A)$, and processes each vertex $w \in N$ in the topological order. At a vertex w into which two edges $e_A=(v_A,w)$ and $e_B=(v_B,w)$ are incident, signal arrival time (or delay) $D(w) = Max[D(e_A), D(e_B)]$ is computed, where $D(e_A) = D(v_A)+d(e_A)$, $D(e_B) = D(v_B)+d(e_B)$, and $d(e_A)$ and $d(e_B)$ are the delays of edges $e_A \in A$ and $e_B \in A$, respectively. Since these delays $d$(e), $D$(e), and $D$(w) are all RVs, we need statistical maximum operation as well as statistical addition.

As is done in [8], we represent the distribution of delay $D$ by 2-GMM (Gaussian mixture model consisting of 2 Gaussian distributions), that is, we denote its probability distribution function (PDF) by the following form:

$$f(D) = P_1 \cdot \frac{1}{\sigma_{D1}} \cdot \phi\left(\frac{D-\mu_{D1}}{\sigma_{D1}}\right) + P_2 \cdot \frac{1}{\sigma_{D2}} \cdot \phi\left(\frac{D-\mu_{D2}}{\sigma_{D2}}\right) \quad (1)$$

Here, $\phi(\bullet)$ is the PDF of the standard Gaussian distribution N(0,1),

$$\phi(x) = \frac{1}{\sqrt{2\pi}} \cdot \mathrm{Exp}\left[-\frac{x^2}{2}\right] \quad (2)$$

and $P_1$ and $P_2$ are probabilities satisfying $P_1+P_2 = 1$, which are called mixture proportions of two Gaussian distributions $N(\mu_{D1},\sigma_{D1}^2)$ and $N(\mu_{D2},\sigma_{D2}^2)$. Henceforth, we assume $\mu_{D1} < \mu_{D2}$, and call the Gaussian distribution $N(\mu_{Di},\sigma_{Di}^2)$ ($i \in \{1,2\}$) the i-th component of 2-GMM. Moreover, we assume that the delay $d$(e) of an edge e does not have the second component ($P_2 = 0$), and denote cumulative distribution function (CDF) of N(0,1) by $\Phi(\bullet)$.

$$\Phi(x) = \int_{-\infty}^{x} \phi(z)\,\mathrm{d}z \quad (3)$$

As usually done[1], we assume that the i-th component of $D$ is represented by a first order canonical form of a local RV $x_{Di}$ and n global RVs $r_g$ ($1 \le g \le n$),

$$D_i = \mu_{Di} + s_{x,i}[D] \cdot x_{Di} + \sum_{g=1}^{n} s_{g,i}[D] \cdot r_g \quad (4)$$

where $D_i$ ($i \in \{1,2\}$) is the RV representing the i-th component of $D$, and $s_{x,i}[D]$ and $s_{g,i}[D]$ are the sensitivities of the i-th components of $D$ to $x_{Di}$ and $r_g$, respectively. We assume that $x_{Di}$ ($i \in \{1,2\}$) and $r_g$ ($1 \le g \le n$) are all N(0,1) and independent each other. Therefore, the mean $E[D_i]$ of $D_i$ is $\mu_{Di}$, and the variance $V[D_i]$ of $D_i$ is the following.

$$V[D_i] = s_{x,i}[D]^2 + \sum_{g=1}^{n} s_{g,i}[D]^2 \quad (5)$$

Let $D_{Aj}$ ($j \in \{1,2\}$) and $D_{Bk}$ ($k \in \{1,2\}$) be RVs representing the j-th component of delay $D_A$ and the k-th component of $D_B$, respectively, and $C_{jk}[D_A,D_B] = C[D_{Aj},D_{Bk}]$ be the covariance between $D_{Aj}$ and $D_{Bk}$ obtained by regarding each distribution of $D_{Aj}$ and $D_{Bk}$ as a single Gaussian. Moreover, let $x_{Aj} \sim N(0,1)$ and $x_{Bk} \sim N(0,1)$ be the local RVs of $D_{Aj}$ and $D_{Bk}$, respectively, and $C^L_{jk}[D_A, D_B]$ be the part of $C_{jk}[D_A,D_B]$ generated by these local RVs, which we call the local covariance of $C_{jk}[D_A,D_B]$. Namely, if we introduce the correlation coefficient $R[x_{Aj},x_{Bk}] = C[x_{Aj},x_{Bk}]$ between $x_{Aj}$ and $x_{Bk}$, then $C^L_{jk}[D_A, D_B]$ is the term written by the following equation.

$$C^L_{jk}[D_A, D_B] = s_{x,j}[D_A] \cdot s_{x,k}[D_B] \cdot R[x_{Aj}, x_{Bk}] \quad (6)$$

We keep the value of local covariance, and use it to compute covariance $C_{jk}[D_A,D_B]$ by the following equation.

$$C_{jk}[D_A, D_B] = C^L_{jk}[D_A, D_B] + \sum_{g=1}^{n} s_{g,j}[D_A] \cdot s_{g,k}[D_B] \quad (7)$$

## III. Maximum Operation

Let us consider the distribution of the maximum $D_M = Max[D_A,D_B]$ of RVs $D_A$ and $D_B$ whose distributions are represented by the following 2-GMMs, respectively.

$$f_A(D_A) = \sum_{j \in \{1,2\}} P_{Aj} \cdot \frac{1}{\sigma_{Aj}} \cdot \phi\left(\frac{D_A-\mu_{Aj}}{\sigma_{Aj}}\right) \quad (8)$$

$$f_B(D_B) = \sum_{k \in \{1,2\}} P_{Bk} \cdot \frac{1}{\sigma_{Bk}} \cdot \phi\left(\frac{D_B-\mu_{Bk}}{\sigma_{Bk}}\right) \quad (9)$$

In order to find the distribution of $D_M$, we introduce a JPDF of $D_A$ and $D_B$ represented by the following equation

$$\varphi_2(D_A, D_B) = \sum_{j,k \in \{1,2\}} P'_{jk} \cdot \frac{1}{\sigma_{Aj} \cdot \sigma_{Bk}} \cdot \phi_2\left(\frac{D_A-\mu_{Aj}}{\sigma_{Aj}}, \frac{D_B-\mu_{Bk}}{\sigma_{Bk}} : \rho_{jk}\right) \quad (10)$$

where $\phi(x,y:\rho)$ is the standard Gaussian JPDF

$$\phi_2(x, y: \rho) = \frac{1}{2\pi \cdot \sqrt{1-\rho^2}} \cdot \exp\left[-\frac{x^2-2\rho \cdot x \cdot y+y^2}{2 \cdot (1-\rho^2)}\right] \quad (11)$$

and $P'_{jk}$ ($j,k \in \{1,2\}$) is the mixture proportion of the j-k Gaussian JPDF satisfying $P'_{11} + P'_{12} = P_{A1}$, $P'_{21} + P'_{22} = P_{A2} = 1 - P_{A1}$, $P'_{11} + P'_{21} = P_{B1}$, and $P'_{12} + P'_{22} = P_{B2} = 1 - P_{B1}$. Fig. 1 shows an image of such a JPDF, where ellipses are contour lines of each Gaussian JPDF. Henceforth, the j-k Gaussian JPDF of the JPDF of $D_A$ and $D_B$ is called the j-k component of $JPDF_{AB}$.

The distribution of $D_M$ is composed of the distribution of $D_A$ in the area of $D_A \ge D_B$ and that of $D_B$ in the area of $D_A \le D_B$, and these distributions of $D_A$ and $D_B$ are composed from four j-k components of $JPDF_{AB}$. Therefore, the distribution of $D_M$ is approximated by an 8-GMM consisting of 8 Gaussian distributions. Namely, if we denote the distribution obtained from the area of $D_A \ge D_B$ by q = 1 and that from the area of $D_B \ge D_A$ by q = 2, then the PDF of 8-GMM of $D_M$ can be approximated by the following equation.

$$f_8(D_M) = \sum_{j,k,q \in \{1,2\}} P_{jkq} \cdot \frac{1}{\sigma_{jkq}} \cdot \phi\left(\frac{D_M-\mu_{jkq}}{\sigma_{jkq}}\right) \quad (12)$$

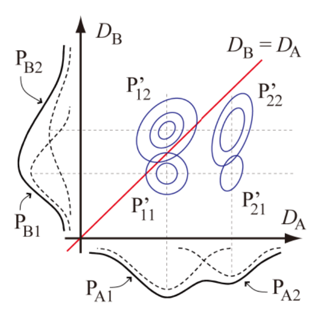


Figure 1. $JPDF_{AB}$ of $D_A$ and $D_B$.

As shown in [8], since the j-k component of $JPDF_{AB}$ is a single Gaussian JPDF, the mixture proportion $P_{jkq}$, the mean $E_{jkq}[D_M] = \mu_{jkq}$, and the second moment $E_{jkq}[D_M^2] = \sigma_{jkq}^2 + \mu_{jkq}^2$ of the j-k-q component of 8-GMM can be computed precisely. Moreover, the covariance $C_{jkqh}[D_M, Z]$ between the j-k-q component of 8-GMM of $D_M$ and the h-th component of other RV $Z$ can be computed precisely, too. Due to the limited space, we omit the equations to compute these values.

The problem to be solved is to represent the distribution of $D_M$ by a 2-GMM

$$f_2(D_M) = \sum_{i\in\{1,2\}} P_{Mi} \cdot \frac{1}{\sigma_{Mi}} \cdot \phi\left(\frac{D_M - \mu_{Mi}}{\sigma_{Mi}}\right) \quad (13)$$

such that the first moment $\sum_{i\in\{1,2\}} P_{Mi} \cdot \mu_{Mi}$ and the second moment $\sum_{i\in\{1,2\}} P_{Mi} \cdot (\sigma_{Mi}^2 + \mu_{Mi}^2)$ are matched to those of 8-GMM, respectively, and the shape of distribution is as close as possible to that of 8-GMM.

The method in [8] matched these moments, but composed a 2-GMM in a fixed way without taking the shape of the distribution into account. Namely, if probability $\Pr[D_A \geq D_B] = \sum_{j,k\in\{1,2\}} P_{jk1} \cong 0.5$ (case 1), then the first component of 2-GMM is composed from the four components of 8-GMM with q = 1, if $\Pr[D_A \geq D_B] \cong 1$ (case 2), then from those with j = 1, and if $\Pr[D_A \geq D_B] \cong 0$ (case 3), then from those with k = 1. The second component of 2-GMM is composed from the remaining four components of 8-GMM. Hence, in the case 1, it happens that a 2-GMM with a different distribution from the distribution of 8-GMM is produced. Thus, the criticalities computed from such a 2-GMM may have errors.

For the case 1, we propose a new method of generating a 2-GMM by taking the shape of CDF of 8-GMM into account. For the case 2 or 3, we use the method in [8], since 4 probabilities $P_{jkq}$ of j-k-q components with q=2 and q=1 are almost 0 in the cases 2 and 3, respectively, and the method in [8] can produce an appropriate 2-GMM.

The new method first approximates the CDF of 8-GMM by a piecewise linear function consisting of 17 lines with 16 joints, and then fits the middle 15 lines by 2 lines with one joint[12]. Finally, by using the coordinate of the joint of the 2 lines, we divide 8 components of 8-GMM into two groups each of which composes a component of 2-GMM. This is done as follows.

For each j-k-q component $N(\mu_{jkq}, \sigma_{jkq}^2)$ of 8-GMM, we introduce a uniform distribution $I_{jkq}$ with the mean $\mu_{jkq}$ and the variance $\sigma_{jkq}^2$. Let $\gamma_{jkq}(D_M)$ be the PDF of $I_{jkq}$, and $L_{jkq}$ and $R_{jkq}$ be the minimum and maximum boundaries of $I_{jkq}$. Then, we can approximate the PDF $f_8(D_M)$ of 8-GMM by $\gamma(D_M) = \sum_{j,k,q\in\{1,2\}} P_{jkq} \cdot \gamma_{jkq}(D_M)$. The CDF $\Gamma(D_M)$ of PDF $\gamma(D_M)$ is composed of 17 lines and has 16 points to joint two consecutive lines, unless two different uniform distributions have the same minimum and the maximum boundary (if it happens, the numbers of lines and joints decrease). We denote these joints by points $(\kappa_p, \lambda_p)$ on a 2-dimensional space with coordinates $\kappa$ and $\lambda$, where $\kappa$ corresponds to $D_M$ and $0 \leq \lambda \leq 1$.

In order to fit the middle 15 lines of CDF $\Gamma(D_M)$ by 2 lines[11], we concentrate on the part where the CDF curve changes steeply, by discarding the joints $(\kappa_p, \lambda_p)$ such that $\lambda_p$ is smaller than the threshold value $\lambda_T$ or greater than $1-\lambda_T$. In the experiment, we set $\lambda_T = 0.03$. Let $(\kappa_p, \lambda_p)$ $(p_L \leq p \leq p_R)$ be the remaining joints, for which we find two lines $\lambda = a_1 \cdot \kappa + b_1$ and $\lambda = a_2 \cdot \kappa + b_2$ with one joint $(\kappa_M, \lambda_M)$ by the least squares method. Namely, we try to hold $a_1 \cdot \kappa_M + b_1 = a_2 \cdot \kappa_M + b_2 = \lambda_M$ and to minimize the sum of square errors $(a_h \cdot \kappa_p + b_h - \lambda_p)^2$ at $\kappa_p$ $(p_L \leq p \leq p_R)$, where h = 1 if $\kappa_p \leq \kappa_M$ and h = 2 if $\kappa_M < \kappa_p$.

With the use of the joint coordinate $\kappa_M$, we divide 8 uniform distributions $I_{jkq}$ into U1 = { (j,k,q) | $\mu_{jkq} < \kappa_M$ }, U2 = { (j,k.q) | $\mu_{jkq} > \kappa_M$ }, and U3 = { (j,k.q) | $\mu_{jkq} = \kappa_M$ }. If U1 is empty, then there exist only minimum boundaries $L_{jkq}$ to the left of $\kappa_M$. Hence, we change U1 to the set of (j,k,q) such that the minimum boundary $L_{jkq}$ is located in the left one third interval of the range between the smallest boundary L = $\text{Min}[I_{jkq} \mid j,k,q \in \{1,2\}]$ and $\kappa_M$. If U2 is empty, we modify U2 similarly.

By using these sets U1, U2, and U3, we partition 8 components $N(\mu_{jkq}, \sigma_{jkq}^2)$ of 8-GMM into two, each of which composes a component of 2-GMM of $D_M$ as follows.

$$P_{Mi} \cdot \mu_{Mi} = \sum_{(j,k,q)\in Ui} P_{jkq} \cdot \mu_{jkq} + \frac{1}{2} \cdot \sum_{(j,k,q)\in U3} P_{jkq} \cdot \mu_{jkq} \quad (14)$$

$$P_{Mi} \cdot (\sigma_{Mi}^2 + \mu_{Mi}^2) = \sum_{(j,k,q)\in Ui} P_{jkq} \cdot (\sigma_{jkq}^2 + \mu_{jkq}^2) + \frac{1}{2} \cdot \sum_{(j,k,q)\in U3} P_{jkq} \cdot (\sigma_{jkq}^2 + \mu_{jkq}^2) \quad (15)$$

where i = 1 or 2, and

$$P_{M1} = \sum_{(j,k,q)\in U1} P_{jkq} + \frac{1}{2} \cdot \sum_{(j,k,q)\in U3} P_{jkq} \quad (16)$$

$$P_{M2} = 1 - P_{M1} \quad (17)$$

In order to calculate the covariance $C_{ih}[D_M, Z]$ between the i-th component of $D_M$ thus obtained and the h-th component of other RV $Z$, we match the covariances,

$$\sum_{i\in\{1,2\}} P_{Mi} \cdot C_{ih}[D_M, Z] = \sum_{j,k,q\in\{1,2\}} P_{jkq} \cdot C_{jkqh}[D_M, Z] \quad (18)$$

Hence, we determine $C_{ih}[D_M, Z]$ as follows:

$$P_{Mi} \cdot C_{ih}[D_M, Z] = \sum_{(j,k,q)\in Ui} P_{jkq} \cdot C_{jkqh}[D_M, Z] + \frac{1}{2} \cdot \sum_{(j,k,q)\in U3} P_{jkq} \cdot C_{jkqh}[D_M, Z] \quad (19)$$

Therefore, the sensitivity $s_{g,i}[D_M] = C_{i1}[D_M, r_g]$ of the i-th component of $D_M$ to each global RV $r_g$ is calculated from this equation.

On the other hand, the sensitivity $s_{x,i}[D_M]$ of the i-th component of $D_M$ to local RV $x_{Mi}$ of $D_M$ is calculated by

$$s_{x,i}[D_M]^2 = \sigma_{Mi}^2 - \sum_{g=1}^{n} s_{g,i}[D_M]^2 \quad (20)$$

If the right hand side of this equation becomes negative, we set $s_{x,i}[D_M] = 0$ and modify each $s_{g,i}[D_M]$ by multiplying the factor $\sigma_{Mi} / \sqrt{\sum_{g=1}^{n} s_{g,i}[D_M]^2}$ as is done in [4].

The local covariance $C_{ih}^{L}[D_M, Z]$ of $C_{ih}[D_M, Z]$ between the i-th component of $D_{Mi}$ and the h-th component of $Z$ is calculated by the following equation.

$$C_{ih}^{L}[D_M, Z] = C_{ih}[D_M, Z] - \sum_{g=1}^{n} s_{g,i}[D_M] \cdot s_{g,h}[Z] \quad (21)$$

However, as shown in Eq.(6), the local covariance $C_{ih}^{L}[D_M, Z]$

must satisfy $\left|\mathrm{C}^{\mathrm{L}}_{\mathrm{ih}}[D_{\mathrm{M}},Z]\right| \le \left|\mathrm{s}_{\mathrm{x,i}}[D_{\mathrm{M}}]\cdot\mathrm{s}_{\mathrm{x,h}}[Z]\right|$ . Therefore, if $\mathrm{C}^{\mathrm{L}}_{\mathrm{ih}}[D_{\mathrm{M}},Z]$ calculated by Eq.(21) does not satisfy this inequality, then $\mathrm{C}^{\mathrm{L}}_{\mathrm{ih}}[D_{\mathrm{M}},Z]$ is set to $\mathrm{s}_{\mathrm{x,i}}[D_{\mathrm{M}}]\cdot\mathrm{s}_{\mathrm{x,h}}[Z]$.

## IV. Experimental Results

In order to see the effect of the proposed methods, we considered RVs $D_A$ and $D_B$ whose PDFs are shown in Fig. 2 by a solid line and a dotted line, respectively. The correlation coefficient $\rho_{jk}$ and mixture proportion $P'_{jk}$ of the j-k component of the $JPDF_{AB}$ are $\rho_{jk} = 0.5$ and $P'_{jk} = 0.25$, respectively, for all j-k components. In the figure, the PDF of 8-GMM of $D_M = \mathrm{Max}[D_A,D_B]$ is also shown by a broken line.

Figure 3 shows various PDFs of $D_M$. In Fig.3(a), the broken line is the PDF of 8-GMM, and the dotted line is the correct PDF obtained by a numerical computation. Since they are almost same, the difference cannot be seen from this figure. The solid line (new 2GMM) is the PDF of 2-GMM obtained by the proposed method, and the same PDF is shown in Fig.3(b) by a solid line. In Fig.3(b), the broken line (old 2GMM) is the PDF of 2-GMM obtained by the method in [8], and the dotted line is the Gaussian with the same mean and the variance. From these figures, we can see that the proposed algorithm generates an accurate distribution of $D_M$.

In Fig. 4, the differences of CDFs of 2-GMMs from that of 8-GMM are shown, where the broken line is the difference of the method in [8] and the solid line is that of the proposed method. From this figure, we can see that more than 80% error reduction can be achieved.

## V. Conclusion

In this paper, we proposed a novel method of statistical maximum (minimum) operation for Gaussian mixture models. The method generates a 2-GMM such that the first two moments match those of 8-GMM and the CDF curve fits that of 8-GMM. By using this 2-GMM, we can increase the accuracy of criticalities. Therefore, the method is useful in delay fault testing and circuit optimization. We are now investigating the performance of the S-STA algorithm using the proposed method by ISCAS bench mark circuits.

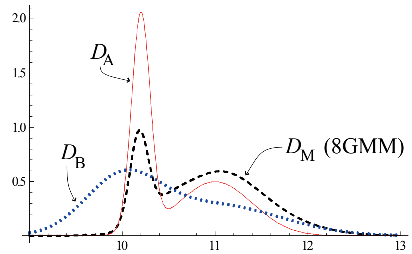


Figure 2. PDFs of $D_A$ (solid line), $D_B$ (dotted line), and $D_M = \mathrm{Max}[D_A,D_B]$ (broken line)

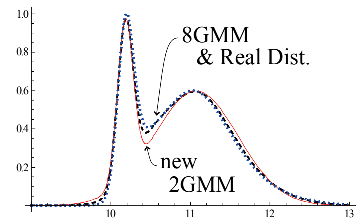


Figure 3 (a). Various PDFs of $D_M = \mathrm{Max}[D_A,D_B]$

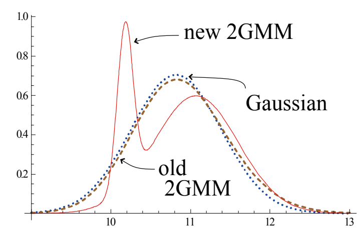


Figure 3 (b). Various PDFs of $D_M = \mathrm{Max}[D_A,D_B]$

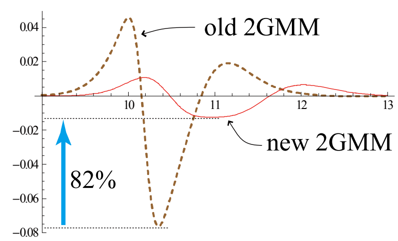


Figure 4. Errors from 8-GMM.

## References


[1] A.Srivastava, D.Sylvester, and D.Blaauw, Statistical Analysis and Optimization for VLSI: Timing and Power, Springer, 2005.

[2] D.Blaauw, K.Chopra, A.Srivastave, L.Scheffer, "Statistical timing analysis: From basic principles to state of the art," IEEE Trans. CAD/ICAS, vol.27, no.4, pp.589-607, 2008.

[3] C.Visweswariah, K.Ravindran, K.Kalafala, S.G.Walker, S.Narayan, D.K.Beece, J.Piaget, N.Venkateswaran, J.G.Hemmett, "First-order incremental block-based statistical timing analysis," IEEE Trans. CAD, Vol. 25, No. 10, pp. 2170-2180, 2006.

[4] H.Chang and S.S.Sapatnekar, "Statistical timing analysis considering spatial correlations using a single PERT-like traversal," Dig. Tech. Paper ICCAD, pp.621-625, 2003.

[5] Y.Zhan, A.J.Strojwas, X.Li, L.T.Pileggi, D.Newmark, M.Sharma, "Correlation-aware statistical timing analysis with non-Gaussian delay distributions," Proc. DA Conf., pp.77-82, 2005.

[6] K. Chopra, B. Zhai, D. Blaauw, D. Sylvester, "A new statistical max operation for propagating skewness in statistical timing analysis," Proc. ICCAD, pp. 237-243, 2006.

[7] X. Li, J. Le, P. Gopalakrishnan, L. T. Pileggi, "Asymptotic probability extraction for nonnormal performance distributions," IEEE Trans. CAD, Vol. 26, No. 1, pp. 16-37, 2007.

[8] S.Takahashi and S.Tsukiyama, "A new statistical timing analysis using Gaussian mixture models for delay and slew propagated together," IEICE Trans. Fundamentals, vol.E92-A, no.3, pp.900-911, 2009. (also see Proc. DA Conf., pp.110-115, 2009)

[9] L.C.Wang, J.J.Liou, and K.T.Cheng, "Critical path selection for delay fault testing based upon a statistical model," IEEE Trans. CAD/ICAS, vol.23, no.11, pp.1550-1565, 2004.

[10] V.Zolotov, J.Xiong, H.Fatemi, C.Visweswariah, "Statistical path selection for at-speed test," IEEE Trans. CAD/ICAS, vol.29, no.5, pp.749-759, 2010.

[11] T.Sato and Y.Kunitake, "A simple flip-flop circuit for typical-case design for DFM," Proc. Int. Symp. Quality Electronic Design, pp.539-544, 2007.

[12] B.Aronov, T.Asano, N.Katoh, K.Mehlhorn, T.Tokuyama, "Polyline fitting pf planar points under min-sum criteria," Int. J. of Computational Geometry: Theory & Applications, vol.16, no.2&3, pp.97-116, 2006.