Trigonometric Method to Handle Realistic Error Probabilities in Logic Circuits

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***Abstract*—We present a novel trigonometry-based probability calculation (TPC) method for analyzing circuit behavior and reliability in the presence of errors that occur with extremely low probability. Signal and error probabilities are represented by trigonometric functions controlled by their corresponding angles. By combining trigonometric identities and Taylor expansions, the effect of an error at a particular gate is simulated as a rotation. In addition, the correlations among signals caused by re-convergence are carefully handled. The TPC method is shown to be more scalable and accurate than prior approaches, especially for very low-probability errors. We measure the performance of TPC by applying it to the ISCAS and LGSyn-91 benchmark circuits. Experimental results show that TPC achieves near-linear runtime complexity even with the largest circuits, while the accuracy gradually increases with decreasing error probabilities.**

Keywords- Error modeling, logic circuits, probabilistic analysis, reliability, soft errors.

I. INTRODUCTION

Modern ICs are very sensitive to transient effects caused by high energy particles, thermal noise, and voltage over-scaling. These soft errors are a major cause of system failure [14], and their impact is usually described probabilistically in terms of circuit reliability or gate susceptibility to errors. The occurrence probability of such transient phenomena is usually very low, however. For example, several studies indicate that the probabilities of an erroneous bit in one month of operation for 90-nm FPGAs and 65-nm DRAMs are 1.38×10^{-10} and 8.28×10^{-10}, respectively [8, 14].

Analyzing such transient phenomena by physical simulation methods like radiation testing is extremely expensive [6]. Hence, computer simulation is widely used for this purpose. However, it can be very time-consuming, especially when the occurrence probabilities of the error phenomena of interest are so low. An alternative to simulation is probability analysis, which has been widely adopted in applications such as calculating testability and switching activity [12]. The accuracy and efficiency of probabilistic error estimation depends heavily on how errors are modeled and how correlation among signals is handled, two important issues we consider here.

Consider a combinational circuit *CC* whose input signals have certain known probabilities. The probability of a particular line or signal *s* being 1 is called its *signal probability*, and is denoted by $p(s)$. The occurrence of one or more errors *e* affecting *CC* can change $p(s)$ to $p(s^e)$. Typically, the signal probability of a line is also the output probability of gate *g*.

A. Signal Correlation

If a gate *g*'s inputs are not independently controllable from the primary inputs of *CC*, then the signal probability or error susceptibility of *g*'s output cannot be directly calculated from *g*'s inputs alone. Take the two circuits in Fig. 1 as examples. Since the circuit in Fig. 1(a) is fanout-free, $p(y_{3a})$can be easily and accurately expressed as $p(y_{3a}) = p(y_{1a})p(y_{2a})$. On the other hand, in Fig. 1(b), $p(y_{3b})$cannot be directly estimated from its local inputs; *i.e.*, $p(y_{3b}) \neq p(y_{1b})p(y_{2b})$ because both y_{1b} and y_{2b} are controlled by x_{4b} and are therefore correlated. Calculating signal probabilities without properly handling correlation may lead to huge computational inaccuracy. For example, if each input probability has signal probability $p(x_{ib}) = 0.5$, the values of $p(y_{3b})$ with and without considering correlation are 0.125 and 0.218, respectively. The computational inaccuracy can be up to 75%, even in this relatively small circuit.

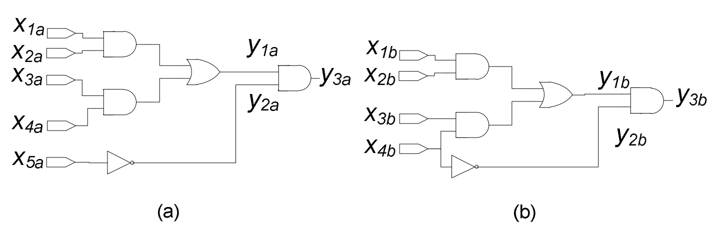

Figure 1. Five-gate circuit (a) without fanout, and (b) with fanout.

Signal correlation problems of this kind are caused by circuit connectivity and functionality issues associated with fanout and reconvergence. The study of signal correlation in logic circuits can be traced to Parker and McCluskey [10]. They proposed a set of calculation rules for analyzing signal probabilities, which can accurately handle all correlations in a circuit. However, the complexity of the Parker-McCluskey approach is exponential in circuit size, which implies that such exact calculations tend to be intractable. Therefore, several heuristic approaches such as the Correlation Coefficient Method (CCM) [5] and the Boolean Approximation Method (BAM) [12] have been proposed, which avoid the need to analyze all possible correlations, either by local estimation or by ignoring cases considered to have insignificant impact.

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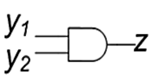
B. Error Modeling and Estimation

Error representation is also crucial to computational efficiency and accuracy in transient-error analysis, something that has not been generally recognized [13]. Two error types are used in probabilistic methods to describe the soft-error behavior of a gate or other components. A *conditional error* model allows an error probability to vary with respect to different input vectors, while an *unconditional* model implies that the error probability is constant, and therefore is independent of the input vectors. Thus, unconditional models ignore correlations among the gate's inputs. Figure 2 shows the two error models for a two-input AND gate. While the conditional model can handle more complicated error situations, the unconditional one is far more scalable to large circuits. In addition, measuring real error probabilities is much easier for unconditional error models at the circuit level [8].



(a)

Input y_1y_2	Output probability 0	1
00	$1-p_1$	p_1
01	$1-p_2$	p_2
10	$1-p_3$	p_3
11	p_4	$1-p_4$

(b)

Input y_1y_2	Output probability 0	1
00	$1-p_{err}$	p_{err}
01	$1-p_{err}$	p_{err}
10	$1-p_{err}$	p_{err}
11	p_{err}	$1-p_{err}$

(c)

Figure 2. (a) Two-input AND gate and its behavior with (b) a conditional error model, and (c) an unconditional error model.

In recent years, a number of probabilistic techniques for transient-error estimation have been proposed including the probabilistic transfer matrix (PTM) approach [7], Bayesian networks (BNs) [11], probabilistic decision diagrams (PDDs) [1], the Four-Event (FE) [2] and Single-Pass (SP) methods [3-4]. The main differences between these methods lie in their error models, the completeness of their correlation analysis, and their ability to simulate the situations where errors occur at multiple gates in one simulation cycle. For instance, the PTM and BN methods allow very general conditional error models, while the PDD, FE and SP methods employ unconditional ones. The PTM, BN and PDD methods preserve complete signal correlations by mapping circuits into complex data structures. As a result, these three methods are limited to relatively small circuits. For instance, the PTM and BN methods are only capable of processing circuits with fewer than one hundred gates [7, 11]. The FE method assumes that all error-free lines are uncorrelated, and only one error occurs in the circuit at any time, while the SP method only examines the correlations between consecutive levels of the target circuit. The FE and SP approaches are more scalable but much less accurate than the others and, as we show later, are less suitable for transient-error estimation than the method presented here.

As the preceding discussion suggests, many of the prior methods are relatively inefficient or inaccurate from a practical viewpoint. Other methods reduce computational effort by restricting the simulation process to one error at a time [2]; this can lead to inaccurate results. Some approaches [3-4] first calculate signal probabilities for error-free versions of individual gates, and then evaluate reliability or error susceptibility based on the error-free probabilities. Such calculations implicitly assume that the occurrence of errors does not affect the signal probabilities. As a result, the accumulated effects of multiple soft errors are underestimated, and the overall accuracy diminishes significantly with very small error probabilities. For instance, in [3] the average computational inaccuracy for error probabilities of 0.3 and 0.05 are 0.46% and 5.63%, respectively, which suggests that the inaccuracy grows rapidly with decreasing error probabilities.

C. Paper Outline

We present a new probabilistic calculation algorithm TPC (Trigonometry-based Probability Calculation) aimed at logic circuits employing the unconditional error model. Unlike previous methods, TPC formulates signal probability in terms of trigonometric functions controlled by angles. It models gate error probability in as a small angular deviation, which is treated as a parameter of the signal probability. Consequently, a gate's error-free signal probability and error probability are fully integrated. With the proposed trigonometric re-presentation, typical (Taylor) expansion techniques can be applied to probabilistic calculation. This enables efficiency and accuracy to be determined and controlled. Signal correlation is carefully managed by TPC using the BAM technique introduced in [12] for switching activity analysis.

TPC can be used to estimate circuit reliability and gate susceptibility to soft errors. It can also simulate the impact of multiple errors. Reliability is measured by the difference in primary output signal probabilities between the error-free and erroneous cases. Gate susceptibility is measured similarly. TPC is efficient because of the way the trigonometric model integrates signal and error probabilities, as well as its use of expansion techniques and a well-designed correlation handling method. Unlike all existing heuristics, TPC's accuracy gradually increases with decreasing error probabilities. Our experimental results show that TPC scales well to circuits with tens of thousands gates, and its average accuracy, even with error probabilities as small as 10^{-8}, is over 96%.

II. UNCONDITIONAL ERROR REPRESENTATION

We first derive a mathematical expression for unconditional errors from a typical conditional error model, and examine some basic properties of unconditional models. We then briefly discuss an error representation technique using exclusive-or (XOR) gates, which has been adopted in error-estimation methods. We explain why this technique can dramatically increase complexity, making it impractical for large circuits.

Suppose the probabilities of the four possible gate input vectors for the 2-input AND gate of Fig. 2 are $p(\overline{y_2}\overline{y_1})$, $p(\overline{y_2}y_1)$, $p(y_2\overline{y_1})$ and $p(y_2y_1)$. Let z and z^e be the output signals of the error-free and erroneous gates, respectively. Since the four probabilities are mutually exclusive, the exact signal probability of z with errors, $p(z^e)$ can be expressed as

$$p(z^e) = p(\overline{y_2}\overline{y_1})p_1 + p(\overline{y_2}y_1)p_2 + p(y_2\overline{y_1})p_3 + p(y_2y_1)(1-p_4)$$

Note that $p(z) = p(y_1y_2)$ so with each $p_i = p_{err}$, this reduces to

$$p(z^e) = p(z)(1-p_{err}) + \big(1-p(z)\big)p_{err} \qquad (1)$$

Equation (1) shows that for a particular gate, the erroneous version of its signal (output) probability $p(z^e)$ can be obtained

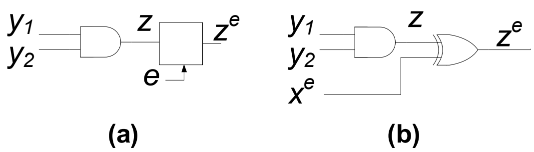


Figure 3. (a) Two-input AND gate with an unconditional error; (b) using a two-input XOR gate to model the error.

from its error-free signal probability $p(z)$ and the gate error probability p_{err}. The maximum and the minimum values of $p(z^e) - p(z)$ are p_{err} and $-p_{err}$ when $p(z) = 0$ and 1, respectively. Thus, for an arbitrary $p(z)$, $p(z^e)$ is closer to 0.5 than $p(z)$. In other words, the presence of errors increases the degree of uncertainty. Moreover, it is worth noting that $p(z^e) - p(z) = 0$ when $p(z) = 0.5$, which means, from the statistical point of view, errors have no significant impact on signals whose signal probabilities are close to 0.5.

We now sketch a widely-used method for error representation in simulation algorithms. Equation (1) is interpreted as defining the signal probability expression for a two-input XOR gate, as in Fig. 3(b). This XOR gate has two uncorrelated input signals z and x^e, and the function of x^e is a random variable whose probability of being 1 is p_{err}. The values of x^e form a stochastic binary sequence, whose bits are temporarily uncorrelated. Hence, the overall error effect of a circuit can be modeled by connecting each gate in the circuit to a new primary input via an XOR gate as in Fig. 3(b). Several error estimation algorithms [1, 9] explicitly or implicitly employ this technique. Clearly, this error representation method will double the number of gates and primary inputs in a circuit, so the complexity of processing a XOR-extended circuit may be much higher than that of the original version.

III. TRIGONOMETRIC PROBABILITY REPRESENTATION

We show next how signal probabilities can be efficiently described by the *trigonometric signal probability* (TSP) approach, where signal probabilities are represented by angles. We also introduce a corresponding representation for error probabilities, the *trigonometric error model* (TEM), which models a gate's error probability as a small positive or negative addition to its signal probability angle. Consequently, TEM calculations avoid both the insertion of XOR gates and the multiplication steps required by (1).

A. Trigonometric Signal Probability

Let s be a signal in a logic circuit. For any input probability distribution of s, $0 \leq p(s), p(\bar{s}) \leq 1$ and $p(s) + p(\bar{s}) = 1$. For a given pair $p(s)$ and $p(\bar{s})$, there exists an angle θ, where $0 \leq \theta \leq \pi/2$, such that $\sin^2\theta = p(s)$ and $\cos^2\theta = p(\bar{s})$, a consequence of the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$. This implies that a signal probability can be fully described in terms of certain trigonometric functions controlled by a single angle called the *signal probability angle* (SPA). The SPA θ of s can be expressed as

$$\theta = \cos^{-1}(\sqrt{p(s)}) \quad \text{or} \quad \theta = \sin^{-1}(\sqrt{p(\bar{s})}) \qquad (2)$$

For example, the SPA for $p(s) = 0.25$ is $\pi/3$.

B. Trigonometric Error Model

Unlike the XOR-based approach of (1), TEM formulates a gate's error probability as a linear rotation associated with its (error-free) signal probability angle by combining the TSP representation and Taylor expansion technique. TEM is more scalable because it maps probabilistic calculations into rotations without inserting extra gates or signals.

Recall that for a particular signal s, if $p(s) \leq 0.5$, then $p(s^e) \geq p(s)$; else if $p(s) \geq 0.5$, then $p(s^e) \leq p(s)$. In TSP terms, this means if an error-free SPA is less (greater) than $\pi/4$, then its erroneous SPA is greater (less) than the error-free version. Such small rotations reflect signal errors we call *error probability angles* (EPAs). With TEM, given a gate's error-free signal and error probabilities, the EPA is first calculated, and then the overall signal probability is obtained by adding or subtracting the EPA from the error-free SPA.

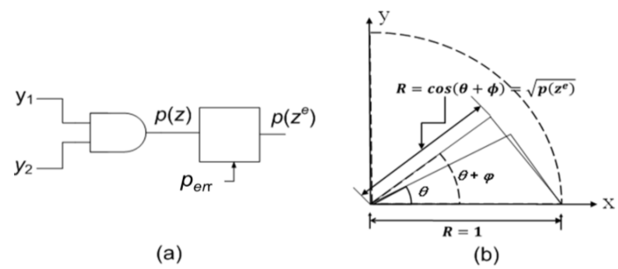


Figure 4. (a) Two-input AND gate with unconditional error; (b) its TSP representation where θ and ϕ are z's error-free SPA and EPA.

Figure. 4 illustrates how an EPA can be calculated from the corresponding error-free SPA and gate error probability. Assume that $p(z) \leq 0.5$, and $p(z) > p_{err}$. Then

$$\cos^2(\theta - \phi) = p(z)(1 - p_{err}) + \big(1 - p(z)\big)p_{err} \qquad (3)$$

where θ and ϕ $(\phi \geq 0)$ are the SPA and EPA of z, respectively. This equation can be re-written as

$$(\cos\theta\cos\phi + \sin\theta\sin\phi)^2 = p(z)(1 - p_{err}) + \big(1 - p(z)\big)p_{err}$$

If p_{err} is very small, then ϕ is also very small, so $\cos\phi$ and $\sin\phi$ can be approximated by $1 - \phi^2/2!$ and ϕ, the initial terms of their respective Taylor series expansions. By definition, $\cos^2\theta = p(z)$ and $\sin^2\theta = 1 - p(z)$, so

$$\big(1 - 2p(z)\big)\phi^2 + 2\sqrt{p(z)\big(1 - p(z)\big)}\phi - \big(1 - 2p(z)\big)p_{err} = 0$$

This has the solution

$$\phi = \frac{\sqrt{p(z)\big(1-p(z)\big) + \big(1-2p(z)\big)^2 p_{err}} - \sqrt{p(z)\big(1-p(z)\big)}}{1-2p(z)} \qquad (4)$$

(Note that the other solution is invalid because $\phi \geq 0$.) We can replace (4) by its Taylor series expansion at $p_{err} = 0$:

$$\phi = \frac{\big(1 - 2p(z)\big)}{2\sqrt{p(z)\big(p(\bar{z})\big)}} p_{err} + \frac{\big(2p(z) - 1\big)^3}{8\sqrt{\big(p(z)p(\bar{z})\big)^3}} p_{err}^2 + \cdots$$

Since p_{err} is very small, we can approximate ϕ by the first term of its Taylor series. Thus ϕ can be re-stated as

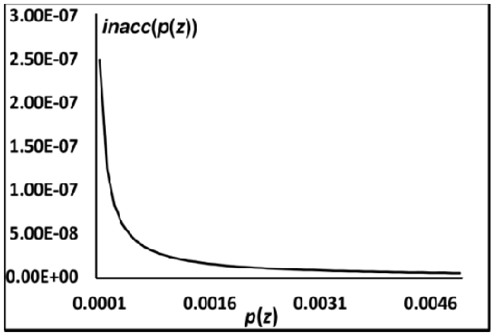


Figure 5. Computational inaccuracy *inacc*($p(z)$) for $p(z) = 10^{-4}$ to 5×10^{-3} with gate error probability $p_{err} = 10^{-5}$.

$$\phi \cong \left(1 - 2p(z)/2\sqrt{p(z)p(\bar{z})}\right)p_{err} = -\cot(2\theta)\,p_{err} \quad (5)$$

By combining (3) and (5), $p(z^e) \cong \cos^2(\theta + \cot(2\theta)p_{err})$, where $\theta = \cos^{-1}(\sqrt{p(z)})$ and $p(z) \le p_{err} \le 0$. If $p(z) \le p_{err}$, then z's EPA can be calculated in a similar way, which yields $\phi \cong \left(1 - 2p_{err}/2\sqrt{p_g(z)p_g(\bar{z})}\right)p(z)$. Note that if $p(z) \le p_{err}$, there is no need to evaluate signal correlation on z later, since z's signal behavior is not controlled by its input signals, but is mainly determined by "random noise". As can be seen from (5), given a gate's error-free SPA, the resulting EPA is a small rotation linear in the gate error probability. The rotation operations in (5) can be easily realized by additions. The XOR-based technique discussed in Sec. II models an unconditional error probability with multiplications and additions, while only additions are required in TEM. Clearly, with its trigonometric representation, TEM is computationally much simpler than the XOR-based technique.

Next we examine the computational inaccuracy of (5) with respect to $p(z)$. Given a gate error probability p_{err}, this inaccuracy can be measured by $|\text{ Eq. (1)} - \text{Eq. (3) }|$, where ϕ in (3) is replaced by (5), namely, $inacc(p(z)) = p_{err}^2(p(\bar{z}) - p(z))^3/4p(z)p(\bar{z})$. The worst case happens at $p(z) = p_{err}$ and the maximum of $inacc(p(z))$ is less than $0.25p_{err}$. Figure 5 shows a plot of $inacc(p(z))$ for $p_{err} = 10^{-5}$. As can be seen, the computational inaccuracy decreases sharply from 2.5×10^{-7} to 4.8×10^{-9} when $p(z)$ just slightly increases from 10^{-4} to 5×10^{-3}. Suppose that $p(z)$ is a uniformly-distributed random variable. The average inaccuracy of (5) is

$$inacc_{avg}(p(z)) = \int_{p(z)=p_{err}}^{p(z)=0.5}(0.5 - p_{err})^{-1} \times inacc(p(z))dp(z)$$

The average inaccuracy for $p_{err} = 10^{-5}$ and 10^{-6} are 4.56×10^{-10} and 5.71×10^{-12}, respectively. Thus the average inaccuracy is quadratic in the given gate error probability. As a result, the accuracy increases rapidly with decreasing error probabilities, thus making TEM particularly suitable for estimating practical non-deterministic phenomena.

IV. Probabilistic Calculation Algorithm

We now present the TPC algorithm for calculating signal probabilities in circuits with errors. TPC combines the TEM error representation and the efficient BAM heuristic [12] for

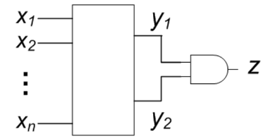


Figure 6. Single-output circuit with n inputs.

signal estimation. A useful feature of TPC is that its efficiency and accuracy can be controlled by carefully selecting terms that make a significant contribution to the results.

A. Correlation Handling

TPC incorporates BAM to account for correlation among signals because BAM can handle correlation more accurately and efficiently than other methods. BAM was developed by Uchino *et al.* [12] to estimate switching activity in error-free combinational circuits. Without losing generality, we again use a two-input AND gate to illustrate the basic concepts of BAM. In Fig. 6, z is the output of an AND gate whose inputs are y_1 and y_2, and x_1, x_2, ⋯, and x_n are the circuit's primary inputs, assuming the circuit is error-free. The Shannon expansion of z with respect to x_i is $z = x_i z_{x_i} + \bar{x}_i z_{\bar{x}_i}$ where z_{x_i} and $z_{\bar{x}_i}$ are cofactors with respect to x_i. The exact signal probability $p(z)$ of z can be written as

$$p(z) = p(x_i)p(z_{x_i}) + p(\bar{x}_i)p(z_{\bar{x}_i}).$$

Suppose that x_i is the only primary input shared between y_1 and y_2, then $p(y_1) = p(x_i)p(y_1) + p(\bar{x}_i)p(y_{1\bar{x}_i})$ and $p(y_2) = p(x_i)p(y_{2x_i}) + p(\bar{x}_i)p(y_{2\bar{x}_i})$. The exact signal probability of z can then be expressed as

$$p(z) = p(x_i)p(\bar{x}_i)\left(p\left(y_{1\bar{x}_i}\right) - p\left(y_{1x_i}\right)\right)\left(p\left(y_{2\bar{x}_i}\right) - p\left(y_{2x_i}\right)\right) + p(y_1)p(y_2) \quad (6)$$

The second term in (6) is the product of y_1 and y_2's signal probabilities without considering correlation, while the first term "compensates" for the signal correlation caused by the common input x_i shared between y_1 and y_2. The general form of (6) for n shared primary inputs can be approximated by

$$p(z) = \sum_{i=1}^{i=n} p(x_i)p(\bar{x}_i)\left(p\left(y_{1\bar{x}_i}\right) - p\left(y_{1x_i}\right)\right)\left(p\left(y_{2\bar{x}_i}\right) - p\left(y_{2x_i}\right)\right) + p(y_1)p(y_2) \quad (7)$$

which, for brevity, we write as $p(y_1)*p(y_2)$. Equation (7) only accounts for correlation of n major cases caused by individual primary inputs, whereas exact correlation calculation would require explicitly enumerating 2^n cases. In addition, z's cofactor probabilities associated with x_i are

$$p(z_{\bar{x}_i}) \cong p\left(y_{1\bar{x}_i}\right)p\left(y_{2\bar{x}_i}\right) \text{ and } p(z_{x_i}) \cong p\left(y_{1x_i}\right)p\left(y_{2x_i}\right) \quad (8)$$

The complexity of BAM is $O(nN)$ where n and N are the numbers of inputs and gates in the circuit, respectively. Generally, $n \cong N^{0.5}$, so BAM's complexity is about $O(N^{1.5})$ [12].

Other heuristics besides BAM could be used for the correlation analysis. Several soft-error estimation methods employ CCM for this purpose [3-4]. However, unlike BAM, which only analyzes gates with common primary inputs, CCM accounts for the correlations between any two gates at the same level, even if they have no common predecessor gates. This tends to make CCM's complexity quadratic in the size of a circuit. The simulation results in [12] show that BAM and CCM achieve similar accuracy, but BAM is generally faster by an order of magnitude.

B. Signal Probability Estimation

We use the two-input AND in Fig. 3(b) to explain how the signal probability of a gate with errors is calculated with TEM and BAM. Assume that y_1 and y_2 are uncorrelated. Let θ_i be the SPA of $p(y_i)$, and let $p_{err}(y_i)$ be the gate error probability associated with y_i. From (5), z's error-free signal probability is

$$p(z) = \prod_{i=1}^{2} \cos^2\left(\theta_i + \cot(2\theta_i)\, p_{err}(y_i)\right) \qquad (9)$$

In fully expanded form, expression (9) contains 12 terms. Of these 12 terms, some are higher-order terms in $p_{err}(y_i)$ e.g., $\sin^2\theta_1 \sin^2\theta_2 \sin^2(\cot(2\theta_1)p_{err}(y_1))\sin^2(\cot(2\theta_2)p_{err}(y_2))$, which is bounded above by $p^2_{err}(y_1)\ p^2_{err}(y_2)$. Such higher-order terms make an insignificant contribution to the overall result when $p_{err}(y_1)$ and $p_{err}(y_2)$ are very small. In that case, $p(z)$ estimated by removing $p_{err}(y_1)$ and $p_{err}(y_2)$'s higher-order terms remains very accurate. If we calculate $p(z)$ only using terms containing $p^i_{err}(y_1)\ p^j_{err}(y_2)$ where $i + j \leq 2$, we get:

$$\begin{aligned} p(z) \cong \prod_{i=1}^{2} p(y_i)\, (1 - 2\textstyle\sum_{i=1}^{2} p_{err}(y_i) + \sum_{i=1}^{2} \frac{p_{err}(y_i)}{p(y_i)} \\ + \textstyle\sum_{i=1}^{2} \frac{(1-2p(y_i))^2}{4p^2(y_i)} p^2_{err}(y_i) + 4\prod_{i=1}^{2} p_{err}(y_i)) \\ +(1 - 2\textstyle\sum_{i=1}^{2} p_{err}(y_i)) \prod_{i=1}^{2} p_{err}(y_i) \qquad (10) \end{aligned}$$

The maximum computational inaccuracy of (10) is $p^2_{err}(y_1)\ p^2_{err}(y_2)$. If each $p_{err}(y_i) = 10^{-8}$, this inaccuracy is less than 10^{-32} If we need to reduce computational overhead further, we can just drop terms containing $p^i_{err}(y_1)\ p^j_{err}(y_2)$ where $i + j \geq 2$, so (10) is further simplified to

$$p(z) \cong \prod_{i=1}^{2} p(y_i) \left(1 - 2\sum_{i=1}^{2} p_{err}(y_i) + \sum_{i=1}^{2} \frac{p_{err}(y_i)}{p(y_i)}\right) \qquad (11)$$

The maximum inaccuracy of (11) is $p_{err}(y_1)\ p_{err}(y_2)$. Again, if $p_{err}(y_i) = 10^{-8}$, this is less than 10^{-16}, which from the practical viewpoint is still good enough for most applications. Equations (10) and (11) can be easily extended to other gate types. The cofactor probabilities can be estimated in a similar way. For instance, z's positive cofactor probability associated with x_i is

$$p(z[x_j]) \cong \prod_{i=1}^{2} p(y_i[x_j]) \left(1 - 2\sum_{i=1}^{2} p_{err}(y_i) + \sum_{i=1}^{2} \frac{p_{err}(y_i)}{p(y_i[x_j])}\right)$$

Now we explain how the signal correlation is estimated. As noted in Sec. II, a gate's unconditional error probability is independent of its inputs and indeed, of all circuit signals. In this case, $p_{err}(y_1)$ and $p_{err}(y_2)$ are uncorrelated with y_1 and y_2 and any preceding gates of y_1 and y_2. Therefore, in (10) and (11) there is no need to consider signal correlation for any term in $p(y_i)$ and $p_{err}(y_i)$. The term $\prod_{i=1}^{2} p(y_i) = p(y_1)p(y_2)$is the only one requiring correlation analysis because it implicitly assumes y_1 and y_2 are uncorrelated. If y_1 and y_2 can be represented as functions of n primary inputs as shown in Fig. 6, the operation of $p(y_1)\ p(y_2)$ in (10) and (11) is replaced by $p(y_1)*p(y_2)$ using BAM. For instance, (11) becomes

$$p(z) \cong p(y_1) * p(y_2) \left(1 - 2\sum_{i=1}^{2} p_{err}(y_i) + \sum_{i=1}^{2} \frac{p_{err}(y_i)}{p(y_i)}\right)$$

Table 1. TPC probability estimation algorithm.

TPC_algorithm(combinational circuit *CC*, gate error probability configuration *ge_conf*, primary input signal probabilities *pi_sp*)
Levelize *CC*

Initialize all primary input *pi_sp* and gate error probabilities *gp_conf*

For each topological level *l* (from primary inputs)

For each gate z at the l^{th} level
Use TEM to model the error probabilities as rotations

Calculate *z*'s output probability using Eq.(10) or Eq.(11)
(Correlation is handled using BAM)

For each primary input *x*
Calculate *z*'s cofactor probabilities associated with *x*

End TPC_algorithm

The overall computational complexity of TPC is $O(nN)$, where n and N are the numbers of inputs and gates in the circuit, so TPC has near-linear runtime complexity in the general case.

V. EXPERIMENTAL RESULTS

To evaluate the performance of the proposed TPC method, and to check its accuracy, we used it to calculate signal probabilities for various benchmark circuits with three representative gate error probabilities $p_{err} = 10^{-3}$, 10^{-4} and 10^{-8} applied to all gates. All primary input signal probabilities are assumed to be 0.5. The selection of benchmarks is constrained by the published, and often incomplete, data available. Note that our TPC program is quite general and places no a priori restriction on the error probability values. The experiments were performed on an Intel Quad-Core, 2.35 GHz, 64-bit PC, with 4GB RAM, and running under Linux.

Runtime, memory usage and accuracy data for gate error probability 10^{-8} appear in Table 2. To measure the accuracy for such low error probabilities, we also used Monte Carlo (MC) sampling to estimate signal probabilities with 320 million random input vectors. The inaccuracy figures were determined by comparing the TPC and MC results, where the MC results are assumed to be accurate.

Table 2. Performance of TPC and MC for gate error probability 10^{-8} with selected ISCAS-85 benchmarks (including the largest ones).

Circuit	Runtime (sec.) TPC	Runtime (sec.) MC	TPC memory usage (MB)	Inaccuracy (%)
C1196	0.054	3508	1.03	6.038
C1238	0.071	3348	1.04	6.154
C1355	0.174	3612	2.23	0.189
C1908	0.462	5619	2.75	5.235
C2670	0.459	8609	3.49	6.090
C3540	1.454	11063	5.53	7.614
C5315	2.319	15283	6.39	7.989
C6288	11.290	15186	6.81	3.559
C7552	5.911	23096	10.06	7.507

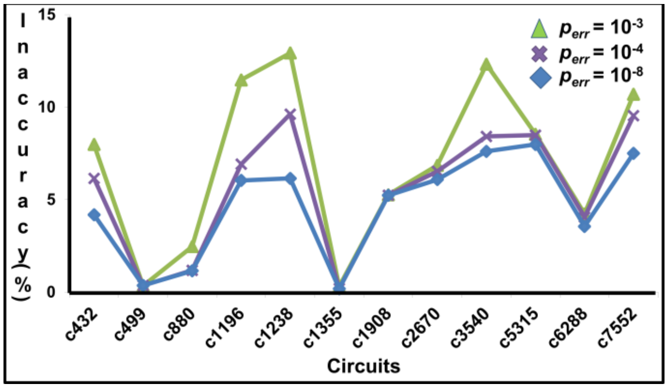


Figure 7. Average computational error of the TPC method for various gate error probabilities.

The maximum runtime, memory usage, and computational inaccuracy of our TPC program for gate error probability $p_{err} = 10^{-8}$ are 11.29 sec. (for C6288), 10 MB (for C7552), and 7.98% (for C5315), respectively. Figure 7 shows the inaccuracy defined by comparing the results of TPC and Monte Carlo sampling for the ISCAS-85 benchmarks and several values of p_{err} The average computational error with $p_{err} = 10^{-3}$ is about 9.27%, but it drops to 5.4% with error probability $p_{err} = 10^{-4}$. The runtime and accuracy data in Table 2 and Fig. 7 indicate that the TPC method can efficiently produce accurate results in cases with gate error probabilities of 10^{-4} or less. These simulation results also suggest that, unlike the FE method [2], TPC is capable of handling cases where errors simultaneously occur at multiple gates, which makes TPC more general than FE. Moreover, unlike the SP method, the accuracy of the TPC approach grows with decreasing gate error probabilities. As a result, TPC may provide less accurate results for high error probabilities like 10^{-3}; however, as discussed in Sec. I, such high probabilities are rarely encountered in real applications. More importantly, its accuracy with low error probabilities makes it particularly suitable for soft-error estimation.

Table 3. Runtime comparison between the TSP and SP methods for reliability estimation.

Circuit	No. of gates	Runtime (sec.)	
		TPC	SP [4]
C499	650	0.080	15.40
C1355	653	0.417	14.70
C1908	699	0.892	30.06
C2670	756	0.901	1.11
frg2	1024	4.000	0.30
C3540	1466	2.97	234.63
i10	2643	19.473	145.35

Table 3 compares the TPC and SP methods for evaluating circuit reliability. The SP data is taken from [4]. These results show that TPC is faster than SP by one or two orders of magnitude, mainly due to the differences between their respective use of BAM and CCM for analyzing signal correlation. Generally, the TPC method is efficient and accurate for practical applications such as transient-error estimation, whereas the SP method is suitable for applications with very high error probabilities.

VI. Conclusion

We have presented a new trigonometric framework TPC for probabilistic calculations intended to handle very-low-probability transient error events of the type widely found in practice. Prior methods for transient-error estimation are typically limited by error modeling and signal correlation considerations to relatively small circuits or to relatively inaccurate analysis of large circuits. The TPC approach, on the other hand, is able to produce accurate results, even in the case of large circuits, because of its novel use of trigonometric operations and its improved heuristics for handling correlation. An interesting feature of TPC is that its computational inaccuracy falls as gate error probabilities are reduced to levels that better reflect the actual frequency of transient errors in ICs. Finally, we note that the trigonometric approach described here can be used for other probability-based applications such as switching activity analysis and power estimation.

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