Extended Hamiltonian Pencil for Passivity Assessment and Enforcement for S-parameter Systems

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ABSTRACT

An efficient algorithm based on the Extended Hamiltonian Pencil was proposed in [1] for systems with hybrid representation. Here we further extend the Extended Hamiltonian Pencil method to systems described with scattering representation, i.e. S-parameter systems. The derivation of the Extended Hamiltonian Pencil for S-parameter systems is presented. Some properties that allow passivity enforcement based on eigenvalue displacement are reported. Experimental results demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

Rational fitting and vector fitting have been widely used in generating macro-models for passive systems such as spiral inductors, baluns, SAW filters, etc. The major concern in such techniques is the passivity issue.

Passivity of a system indicates the inability of the system to generate energy. From a mathematical point of view, passivity requires that the transfer matrix under investigation be positive real (in the case of hybrid representations) or bounded real (in the case of scattering representations). Existing techniques for passivity test include frequency sampling methods [2], which only test passivity on discrete frequencies and are very difficult to be complete. SDP based methods [3] provide global optimality, but they are too expensive.

Recently, methods based on the Hamiltonian matrix [4, 5, 6, 7, 8, 9] became more attractive. However, they still need to solve the eigenvalues of a dense matrix with high dimensionality. While heuristic methods exist to compute only a subset of the eigenvalues, it is very difficult (theoretically impossible) to avoid the risk of missing important ones in some extreme cases.

The method based on the Extended Hamiltonian Pencil [1] provides an efficient and reliable way to tackle this problem. The method is based on dealing with an equivalent and sparse form of the Hamiltonian matrix, called the Extended Hamiltonian Pencil (EHP). The sparse pencil allows very efficient eigensolving for all the eigenvalues, thus avoiding the risk inherent in traditional methods.

The method proposed in this work is an extension of the EHP method described in [1] in that it applies to systems described with scattering representation, while [1] only deals with systems with hybrid representation. It follows that the resulting pencils from both methods are non-trivially different. We also report in this paper some properties of the EHP, which allows passivity enforcement that is consistent to those in traditional Hamiltonian methods.

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The rest of this paper is organized as follows. In Section 2 we introduce the mathematical background of the problem, and review the derivation of Hamiltonian matrix. In Section 3, we propose the extended Hamiltonian pencil for the scattering case and discuss its properties in contrast to the regular Hamiltonian matrix. In Section 4, we describe how to do the passivity enforcement with the extended Hamiltonian matrix. Results and conclusions are given in Sections 5 and 6.

2. BACKGROUND

In this section we review some of the main procedures for passivity test. Here we consider both hybrid case and scattering case. The linear system under investigation is governed by the following state space equations.

$$\dot{x} = Ax + Bu \tag{1}$$
$$y = Cx + Du \tag{2}$$

where A, B, C, D are system matrices, and $u \in \mathcal{R}^m$ is the input to the system, $y \in \mathcal{R}^m$ is the output of the system and $x \in \mathcal{R}^n$ is the internal state. Generally $m \ll n$.

In this paper we assume that the system is strictly stable, and matrix A is *diagonal*. The first assumption is easily preserved and the second assumption is not generally true. However, if we focus only on systems generated from rational fitting or vector fitting, then it holds true naturally.

The frequency domain transfer function of the linear system is

$$H(s) = C(sI - A)^{-1}B + D \tag{3}$$

where $s = j\omega$ is the Laplace variable.

2.1 Passivity Conditions

Passivity means a system does not generate energy. For hybrid representation, mathematically it requires that the Hermitian part of the transfer function is always positively defined.

$$H(s) + H^H(s) \geq 0, \quad s = j\omega \tag{4}$$

For scattering representation, it requires that the transfer matrix be bounded real.

$$I - H(s)H^H(s) \geq 0, \quad s = j\omega \tag{5}$$

2.2 Hamiltonian Matrix and Extended Hamiltonian Pencil

Passivity conditions requires that the system satisfies (4) or (5) in the whole frequency domain. Thus a naive passivity test is to verify the positive realness or bounded realness at discrete frequencies. However, such test has been proven to be incomplete. A more preferable way is to do it in an algebraic way.

Hamiltonian matrix is one of the algebraic tool to assess the passivity of a system. Given a Hamiltonian matrix of a system, the passivity of the system can be tested by verifying the existence of purely imaginary eigenvalues of the Hamiltonian matrix.

For hybrid representation, the traditional Hamiltonian matrix is written as follows [5]

$$M = \begin{bmatrix} A + BR^{-1}C & BR^{-1}B^H \\ C^H R^{-1}C & -A^H + C^H R^{-1}B^H \end{bmatrix} \tag{6}$$

and the Extended Hamiltonian Pencil [1] has been given as

$$Mv = sNv \tag{7}$$

where

$$M = \begin{bmatrix} A & & B \\ & -A^H & -C^H \\ C & B^H & D + D^H \end{bmatrix}, \quad N = \begin{bmatrix} I & & \\ & I & \\ & & \end{bmatrix} \tag{8}$$

It has been proven that (6) and (7) have the same eigenvalues thus they can both be used to assess the passivity of a system.

For scattering representation, traditional Hamiltonian matrix is written as

$$\begin{bmatrix} A + BR^{-1}C & BR^{-1}B^H \\ C^H R^{-1}C & -A^H + C^H R^{-1}B^H \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = s \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{9}$$

Intuitively there should be a parallel version of EHP for scattering representation, however, this is not shown in literature. And it will be presented in the following sections.

3. EHP FOR SCATTERING SYSTEMS

In this section we will first give the derivation of the EHP for scattering systems. And then we will study some useful properties that builds the connections between the EHP and regular Hamiltonian matrices. And then we will discuss efficient eigenvalue computation for the EHP.

3.1 Derivation

In this section we will derive the Extended Hamiltonian Pencil for scattering systems. For a stable scattering system, the bounded real condition (5) implies that the H_∞ norm of the transfer function never exceed 1, where the H_∞ norm is defined as [10]

$$||H||_\infty = \sup_{\omega \in R} \sigma_{\max}(H(j\omega) \tag{10}$$

where $\sigma_{\max}(\cdot)$ denotes the maximum singular value of a matrix.

Suppose the singular value decomposition (SVD) of $H(j\omega)$ can be written as

$$H(j\omega) = U\Sigma V^H \tag{11}$$

and $u = U_k$, and $v = V_k$ are the k-th columns of U and V, and $\sigma = \Sigma_{kk}$ is the k-th singular value, where k is an arbitrary index, then we have

$$\begin{aligned} H(j\omega)v &= U\Sigma V^H v = \sigma u & (12) \\ H^H(j\omega)u &= V\Sigma U^H u = \sigma v & (13) \end{aligned}$$

Substituting matrices A, B, C, D into the above equations and letting $s = j\omega$ will result in

$$\begin{aligned} \left[C(sI - A)^{-1}B + D\right] u &= \sigma v & (14) \\ \left[B^H(-sI - A^H)^{-1}C^H + D^H\right] v &= \sigma u & (15) \end{aligned}$$

Let

$$\begin{aligned} r &= (sI - A)^{-1}Bu & (16) \\ t &= (-sI - A^H)^{-1}C^H v & (17) \end{aligned}$$

and substitute (16-17) into (14-15), resulting in

$$\begin{aligned} Cr + Du &= \sigma v & (18) \\ B^H t + D^H v &= \sigma u & (19) \end{aligned}$$

Multiplying (16-17) by $sI - A$ and $-sI - A^H$ respectively leads to

$$\begin{aligned} (sI - A)r &= Bu & (20) \\ (-sI - A^H)t &= C^H v & (21) \end{aligned}$$

Then (18-21) can be written in matrix as

$$\begin{bmatrix} A & & B & \\ & -A^H & & -C^H \\ & B^H & -\sigma I & D^H \\ C & & D & -\sigma I \end{bmatrix} \begin{bmatrix} r \\ t \\ u \\ v \end{bmatrix} = s \begin{bmatrix} I & & & \\ & I & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} r \\ t \\ u \\ v \end{bmatrix} \tag{22}$$

It is readily seen that this becomes a generalized eigenvalue problem.

$$Mx = sNx \tag{23}$$

where

$$M = \begin{bmatrix} A & & B & \\ & -A^H & & -C^H \\ & B^H & -\sigma I & D^H \\ C & & D & -\sigma I \end{bmatrix}, \quad N = \begin{bmatrix} I & & & \\ & I & & \\ & & & \\ & & & \end{bmatrix} \tag{24}$$

The pencil $\{M, N\}$ is called the extended Hamiltonian pencil (EHP) for S-parameter system. In [1] we have the parallel version of the EHP for hybrid system.

Given the definition of EHP, the passivity can be tested as stated in the following theorem:

THEOREM 3.1. *The scattering system $\mathcal{S} = \{A, B, C, D\}$ is non-passive if and only if the EHP $\{M_{\sigma=1}, N\}$ has purely imaginary eigenvalue.*

PROOF. Follows straightforward derivation. □

It would be interesting to see that if we introduce an auxiliary hybrid system $\mathcal{S}_a = \{A_a, B_a, C_a, D_a\}$ with system matrices written as

$$\begin{aligned} A_a &= A & (25) \\ B_a &= [B \;\; 0] & (26) \\ C_a &= \begin{bmatrix} 0 \\ C \end{bmatrix} & (27) \\ D_a &= \begin{bmatrix} -\sigma I & D^H \\ D & -\sigma I \end{bmatrix} & (28) \end{aligned}$$

then the EHP of the original scattering system S is the same as the EHP of the auxiliary hybrid system S_a. On the other word, the bounded realness of the original scattering system is equivalent to the positive realness of the auxiliary hybrid system.

3.2 Properties

It is clear that in constructing M a singular $DD^T - I$ is not an issue, since there is no need to invert $DD^T - I$, so this is an advantage compared to regular Hamiltonian matrix, where it can only deal with systems for which $DD^T - I$ is non-singular.

Following we will show that the Extended Hamiltonian Pencil shares many properties with the regular Hamiltonian matrix, among which the most important one is the eigenvalues, and thus it can be used for passivity assessment in the same way as the regular Hamiltonian matrix is.

Besides this, some other properties of the regular Hamiltonian matrix still hold for the Extended Hamiltonian Pencil. The following theorem reveals the J-symmetry property of the matrix M

THEOREM 3.2.

$$MJ = (MJ)^T \tag{29}$$

where the matrix J is defined as

$$J = \begin{bmatrix} & -I & & \\ I & & & \\ & & I & \\ & & & I \end{bmatrix} \tag{30}$$

PROOF. The proof is straightforward, since

$$MJ = \begin{bmatrix} & -A & & B \\ -A^H & & -C^H & \\ & -C & -\sigma I & D \\ B^H & & D^H & -\sigma I \end{bmatrix} \tag{31}$$

is symmetric. □

Parallel relation has been used in [5] to derive the passivity enforcement in the method based on regular Hamiltonian matrix, and it is used in Section 4 to derive the passivity enforcement with EHP.

The following theorem reveals the relations between the left- and the right-eigenvectors of EHP. These relations are also key to derive the passivity enforcement scheme.

THEOREM 3.3. *Suppose v is the left eigenvector as shown in (7) and w is the right eigenvector, i.e.*

$$M^T w = sN^T w \tag{32}$$

then there exists the following relation

$$w_+ = J^T v_- \tag{33}$$

where v_- is the left eigenvector for eigenvalue $-s$.

PROOF. Applying (29) to (32) and considering N is symmetric,

$$J^{-T} MJw = sNw \tag{34}$$

Now multiply both sides by J^T, and consider $J^T J = I$

$$MJw = s(J^T N J^T)Jw \tag{35}$$

and considering $J^T N J^T = -N$, we have

$$MJw = -sNJw \tag{36}$$

It is easily seen that $Jw = v_-$, and thus $w = J^T v_-$. □

In particular, when s is purely imaginary, i.e. $s = j\omega$, then $-s = s^*$ and $v_- = v^*$. Thus we have

$$w = J^T v^* \tag{37}$$

This relation is similar to the one derived for regular Hamiltonian matrix in [5].

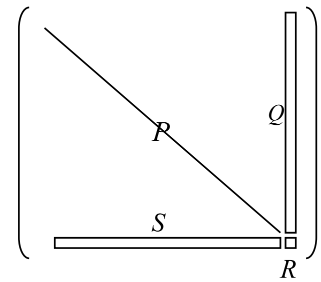
3.3 Eigensolving for Extended Hamiltonian Pencil

In [1], an efficient algorithm based on Laguerre's method [11] has been proposed for solving the following generalized eigenvalues problem

$$Mx = \lambda Nx \tag{38}$$

where the non-zero patterns of M and N satisfy the following structure

$M - \lambda N =$



in which λ is any scalar number, matrix P is diagonal, and matrix $Q \in \mathcal{C}^{n\times m}$, $S \in \mathcal{C}^{m\times n}$, $R \in \mathcal{C}^{m\times m}$, and $m \ll n$.

In this case, the eigensolving problem can be recast as the polynomial rooting finding problem for the determinant

$$p(\lambda) = |M - \lambda N| \tag{39}$$

The computational cost for solving all the eigenvalues is provably $O(n^2)$. For details please refer to [1].

4. PASSIVITY ENFORCEMENT

The passivity enforcement with EHP is similar to the enforcement with regular Hamiltonian matrix. Following is a derivation for the passivity enforcement for EHP, which is similar to that of [5], in which the passivity enforcement scheme based on regular Hamiltonian matrix is given.

Consider perturbing the system by modifying only the C matrix by $C_1 = C + \delta C$. This corresponds to a perturbation to matrix M by $M_1 = M + \delta M$, where

$$\delta M = \begin{bmatrix} & \delta C^T \\ \delta C & \end{bmatrix} \tag{40}$$

Suppose the eigenvalue and eigenvector are perturbed by $\delta\lambda$ and δv respectively, then the equation becomes

$$(M + \delta M)(v + \delta v) \quad = \quad (\lambda + \delta\lambda)N(v + \delta v) \tag{41}$$

Expanding both sides, and dropping high order terms $\delta M \delta v$ and $\delta\lambda\delta v$, and considering the cancellation due to (7), we have

$$M\delta v + \delta M v \approx \lambda N \delta v + \delta\lambda N v \tag{42}$$

Let w be the left eigenvectors and left-multiply both sides by w^T, we have

$$w^T M\delta v + w^T \delta M v \approx \lambda w^T N\delta v + \delta\lambda w^T N v \tag{43}$$

Since $w^T M\delta v = \lambda w^T N \delta v$ by definition of w, (43) can be reduced to (for the sake of simplicity, we change the symbol "$\approx$" to "$=$")

$$w^T \delta M v = \delta\lambda w^T N v \tag{44}$$

Thus

$$\delta\lambda = \frac{w^T \delta M v}{w^T N v} \tag{45}$$

Considering (37), w can be replace by $J^T v^*$, and the formula can be recast as

$$\delta\lambda = \frac{v^H J \delta M v}{v^H J N v} \tag{46}$$

Suppose v is partitioned as

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \tag{47}$$

and consider $J\delta M$ is

$$J\delta M = \begin{bmatrix} & & \delta C^H & \\ & & & \\ & & & \\ \delta C & & & \end{bmatrix} \tag{48}$$

The numerator of (46) can be written as

$$v^H j\delta M v = (v_4 - v_4^*)^T \delta C (v_1^* - v_1) \quad (49)$$
$$= (v_1^* - v_1)^T \otimes (v_4 - v_4^*) \mathrm{vec}(\delta C) \quad (50)$$

Thus we have

$$\delta\lambda = Z \cdot \mathrm{vec}(\delta C) \quad (51)$$

where

$$Z = \frac{(v_1^* - v_1)^T \otimes (v_4 - v_4^*)}{v^H J N v} \quad (52)$$

The remainder procedure of the enforcement scheme would be the same as the one in [5]. We omit it here and interested readers may refer to [5] for details.

5. RESULTS

This section illustrates the performance of the passivity check and enforcement using the proposed algorithms.

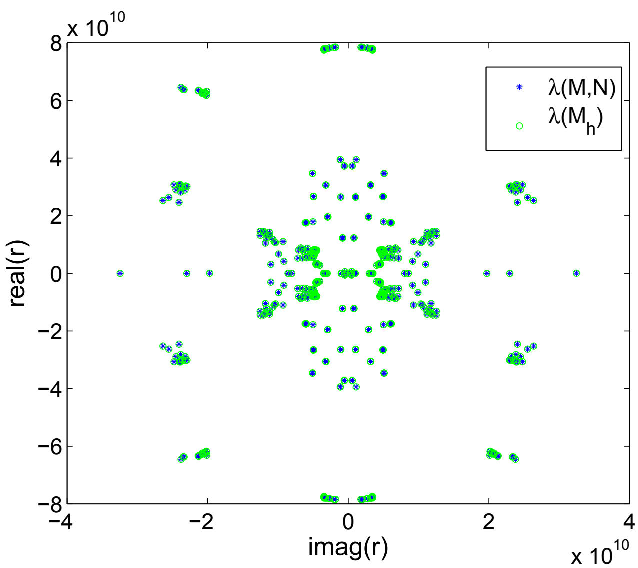


Figure 1: Eigenvalues of the EHP and regular Hamiltonian matrix for a DIP14 package. $\lambda(M,N)$ are the eigenvalues of the EHP, and $\lambda(M_h)$ are the eigenvalues of the Hamiltonian matrix.

The first example is done for a state space model generated for a DIP14 package. The number of ports of the system is 14. The total number of poles is 280. Fig. 5 shows the eigenvalues of the EHP as well as the eigenvalues of the regular Hamiltonian matrix. The eigenvalues of the EHP are identical to the eigenvalues of the Hamiltonian matrix.

The second example is a two-port system for a long transmission line. The eigenvalues of the EHP and the Hamiltonian matrix are shown in Fig. 5. Due to the distributed nature, the transmission line system requires many poles in order to capture the wide band behavior. Thus although the number of ports of the systems is small, the size of the system is still large. In this case, there are 200 poles, and thus the size of the Hamiltonian matrix is 400, and the size of the EHP is 404.

In the next experiment, the proposed method is tested for a set of large scale problems. The size of the problems are summarized in Table 1, where the number of poles of the tested problems n varies from 600 to 8000. The number of ports m varies from 3 to 20. The sizes of the Hamiltonian matrix and the extended Hamiltonian pencil are $2n$ and $2n + 2m$ respectively. The Hamiltonian matrices are solved by eigensolver from Lapack [12], while the extended Hamiltonian pencil is solved by the Laguerre's method proposed in [1].

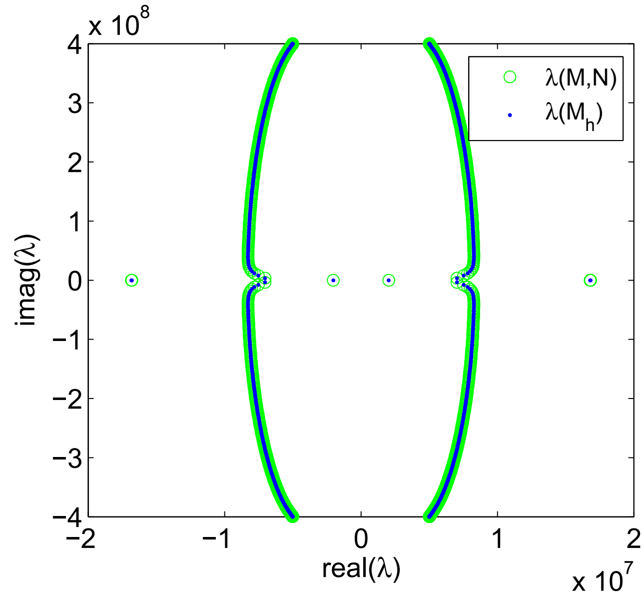


Figure 2: Eigenvalues of the EHP and regular Hamiltonian matrix for a transmission line system. $\lambda(M,N)$ are the eigenvalues of the EHP, and $\lambda(M_h)$ are the eigenvalues of the Hamiltonian matrix.

The comparison of CPU times is shown in Table 2. It is shown that the proposed method possesses a 24X∼45X speed-up over the general-purpose eigensolver from LAPACK, and the speed-up increases as the size of the problems grows. For large problems, e.g. #4, #7, #8, the general-purpose eigensolver failed to return the answer, while the Laguerre's method computes all the eigenvalues in a few minutes.

Table 1: Summary of Test Cases

Problem	#poles	#ports	problem size
#1	600	3	1206
#2	1000	5	2010
#3	2000	10	4020
#4	4000	20	8040
#5	1200	3	2406
#6	2000	5	4010
#7	4000	10	8020
#8	8000	20	16040

We have to emphasize that although the algorithm from [6, 7] is also much faster than direct eigensolver, they do not guarantee finding all the eigenvalues, thus they may potentially give inaccurate results in passivity assessment.

6. CONCLUSIONS

This paper enriches the theory of Extended Hamiltonian Pencil (EHP), in that it applies EHP to S-parameter systems, in contrast to existing literature that deals only with hybrid systems [1]. Also reported are several useful properties of EHP, which allows efficient passivity enforcement that is consistent with methods based on regular Hamiltonian matrix.

Table 2: Comparison of CPU time for LAPACK and this work.

Problem	#Iter	LAPACK	This work	Speed-up
#1	6	20	0.82	24X
#2	6	87	3.14	28X
#3	7	594	23.9	25X
#4	7	–	251	–
#5	6	150	3.28	46X
#6	7	573	13.2	44X
#7	7	–	86	–
#8	8	–	862	–

7. ACKNOWLEDGEMENT

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