Improved Performance and Variation Modelling for Hierarchical-based optimisation of Analogue Integrated Circuits

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Abstract

A new approach in hierarchical optimisation is presented which is capable of optimising both the performance and yield of an analogue design. Performance and yield trade-offs are analysed using a combination of multi-objective evolutionary algorithms and Monte Carlo simulations. A behavioural model that combines the performance and variation for a given circuit topology is developed which can be used to optimise the system level structure. The approach enables top-down system optimisation, not only for performance but also for yield. The model has been developed in Verilog-A and tested extensively with practical designs using the Spectre simulator. A performance and variation model of a 5 stage voltage controlled ring oscillator has been developed and a PLL design is used to demonstrate hierarchical optimisation at the system level. The results have been verified with transistor level simulations and suggest that an accurate performance and yield prediction can be achieved with the proposed algorithm.

1 Introduction

Advances in silicon technology over the last decade have led to increased integration of analogue and digital functional blocks onto the same chip. In such a mixed signal environment, the analogue circuits must use the same transistors as their digital neighbours. The increasing complexity and accuracy of device models has led to wide acceptance of simulation and optimisation based design techniques for the design of analogue blocks rather than hand calculations [1-3]. With reducing transistor sizes, the impact of process variations on analogue design has become very prominent and can lead to circuit performance and yield falling below specification. This issue has led to the consideration of yield in the design process, known as design for yield (DFY) [4]. In optimisation-based design techniques, the performance of the circuit must be evaluated for a large number of different circuit variables, a process known as design space exploration. Running the entire performance evaluation at transistor level is computationally intensive especially for large and complex circuits. Due to this limitation, hierarchical design is commonly used to break down a large system into its constituent building blocks. A typical hierarchical design is shown in figure 1. Not only does this approach simplify the design task but it also speeds up the design flow by encouraging design reuse. Hierarchical based optimisation is one method used to reduce simulation time and involves the use of behavioural models prior to transistor level simulation [5-7]. Behavioural and macro modelling is a useful technique that involves developing models from simulation that relate performance to circuit parameters. Although the initial time investment is high, subsequent design flows are significantly faster [8]. Recently, macromodelling has been used to predict the parametric yield and performance of a design [9].

![Figure 1. A typical system design hierarchy.](image)

In this paper, a novel approach is proposed that develops a combined performance and statistical variation model.
2 Background

Simulation-based optimisation techniques are widely used for analogue circuit design and several synthesis tools have been developed that use spice-like simulators for their evaluation engine. However, these approaches are processor intensive which limits their use to smaller building blocks [11]. To overcome this problem, hierarchical based design has been proposed to divide the large system into sub blocks that can be optimised separately [11].

2.1 Multi-Objective Optimisation

The optimisation formulation for more than one objective function is called multi-objective optimisation (MOO) which can be generally stated as:

\[
\begin{align*}
\text{Minimise} / \text{Maximise } & f_m(x), m = 1, 2, ..., M \\
\text{Subject to } & g_j(x) \geq 0, j = 1, 2, ..., J
\end{align*}
\]  

Where \(f_m(x)\) is the set of \(M\) performance functions and \(g_j(x)\) is the set of \(J\) constraints and the outcome is a set of optimal solutions [12]. Figure 2 shows the relationship between the parameter space and objective space, where each point in the parameter space is a solution that corresponds to a point in the objective space. The black curve shown on the objective space is called the Pareto front and all solution points lying on this curve are called Pareto-optimal solutions. The algorithm used in this work for the MOO is called Non-dominated Sorting Genetic Algorithm -II (NSGA-II). This evolutionary algorithm employs an elite preserving strategy which makes sure that good design solutions found early in the optimisation will be carried to the next generation. The following shows an outline flow of an NSGA-II algorithm:

<table>
<thead>
<tr>
<th>NSGA Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generate initial random population size (N).</td>
</tr>
<tr>
<td>Create offspring population</td>
</tr>
<tr>
<td>- Combine parent and offspring population to form (R = R_P \cup Q_c)</td>
</tr>
<tr>
<td>- Perform non-dominated sorting and identify fronts, (F_i (i=1, 2, ..., etc))</td>
</tr>
<tr>
<td>Set new population, (P_{i+1}), and fill (P_{i+1}) with (F_i (P_{i+1} = F_i))</td>
</tr>
<tr>
<td>Perform crowding sort and place most widely spread solution in (P_{i+1})</td>
</tr>
<tr>
<td>Create offspring population (Q_i) from (P_{i+1}) and repeat until final generation</td>
</tr>
</tbody>
</table>

2.2 Table Model Functions

Behavioural models employing table model functions require the generation of sampled data points from circuit simulation. Interpolation and extrapolation techniques are then used to estimate a new value from the set of known values. Verilog-A supports three types of spline interpolation: linear, quadratic and cubic. The choice of interpolation is a trade off between accuracy and complexity. Cubic spline interpolation has been employed in this work to maximise accuracy. The third degree polynomial used to create the piece-wise interpolation curve is defined by equation (3), where \(a_i, b_i, c_i\), and \(d_i\) are the coefficients for the polynomials.

\[
S_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i, \quad (3)
\]

2.3 Hierarchical-based Design

A hierarchical methodology consists of a top-down design and bottom-up verification process. Important aspects of both processes are circuit decomposition and specification propagation. Once the system architecture has been divided into sub-blocks, automatic optimisation algorithms can be applied to solve for the circuit sizing. The optimisation for hierarchical based design can be divided into two steps: In step one, the behavioural level blocks are optimised using a behavioural simulation and the design parameters that meet the system level specification are determined. In the second step, the design parameters from the previous optimisation are taken as the specifications for the circuit level optimisation which propagates the system level specification to the bottom level. The relationship between transistor level and system level optimisation in hierarchical design is shown in figure 3. The behavioural-level Pareto front determines all the solution points that meet the system level specifications. The design space of this Pareto front is then taken as the design objective for the sub-block circuit level. At the circuit level, the Pareto front is then used to determine the design parameters that best meet the design objective, resulting in the transistor dimensions.
3 Proposed Algorithm

The key steps in the proposed algorithm are shown in Figure 4. These steps are now discussed in more detail.

3.1 Netlist and Objective Function Generation

The starting point for the proposed algorithm is a circuit topology, process models and a set of performance functions. The first step involves generating a transistor level netlist for the chosen circuit topology. From this netlist a set of designable parameters are derived which will be used to change the circuit’s performance. Examples of designable parameters include a transistor’s length and width. Each parameter will have constraints imposed by the designer and once determined, these define the parameter space. The performance functions of the circuit are defined as the objective functions and testbench netlists are defined to simulate the performance for a certain set of design parameters.

3.2 Multi-Objective Optimisation

In this stage, the design space is explored and the objective functions are improved iteratively. The optimisation implementation is based on an evolutionary algorithm known as Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [12]. The genetic algorithm procedure involves generating a number of individuals and optimising these over a number of generations. The individuals are encapsulated in a set of parameters defined as the GA string. During the optimisation, the algorithm determines the quality of the individuals through the fitness score of each individual. The fitness score is measured from the performance evaluation.

3.3 Performance and Variation Modelling

The outcome of the multi objective optimisation is a Pareto-front (a set of optimal solutions). All the solution points on the Pareto-front and their respective design parameters can be extracted and a model can be created that represents these data points. Interpolation is one of the techniques that can be used to model these data points.
3.4 Lookup Table Model Development

This step in the proposed architecture involves developing a behavioural model description for the circuit block so that it can be used in system level optimisation. The performance and variation model developed in the previous stage will be defined in a behavioural language and will be added in the behavioural description of the circuit block. The performance and variation model is defined as a look-up table using the table model function in Verilog-A. This function allows the module to approximate the behaviour of a system by interpolating between the performance and variation data points extracted from the Pareto-front. The syntax of the table model function is shown below:

```
$table_model(f1,f2,"datafile.tbl", control_string);
```

Where \(f1\) and \(f2\) are the performance functions, ‘datafile.tbl’ is the text file that contains the performance functions and design parameters, and ‘control string’ determines the interpolation and extrapolation method. In this algorithm, a cubic spline method is used for the interpolation and no extrapolation method is used, in order to avoid approximation of the data beyond the sampled data points. The algorithm creates a table model function for both performance and variation functions.

4 Experimental Results

This section presents a complete design example using a voltage control oscillator (VCO) as a test case for the performance and variation model development. This model is used to demonstrate a behavioural modelling infrastructure to design a system level PLL using hierarchical-based design. A block diagram of a PLL is shown in figure 5. The PLL is designed to operate with an output frequency range of 500MHz to 1.2GHz and a locking time and current consumption of less than 1\(\mu\)s and 5mA respectively. Jitter will be minimised as part of the process. All the following simulations were performed using the industry standard Cadence SpectreRF™ simulator with foundry level BSIM3v3 transistor models from a standard 0.12\(\mu\)m process.

![Figure 5. PLL system block diagram](image)

4.1 Design Setup

The chosen VCO topology is a 5 stage ring oscillator as shown in figure 6. The first step is to determine the designable parameters for the topology. In this example, these include the transistor lengths and widths making a total of 7 designable parameters. The performance functions for which the Pareto front must be generated are jitter, current consumption, gain, minimum frequency and maximum frequency. A testbench netlist was created to evaluate the performance functions.

![Figure 6. Five stage VCO schematic.](image)

4.2 Multi-Objective Optimisation

The designable parameters must be constrained within a reasonable range which defines the design space of the optimisation. In this case all transistor lengths and widths were specified to be between 0.12\(\mu\)m-1\(\mu\)m and 10\(\mu\)-100\(\mu\)m. A Genetic Algorithm then generates the design parameters according to the specified design parameters and it is these that are used in the Spice netlist for simulation. A total of 30 generations each with a population size of 100 were used in this case, giving 3,000 total samples for the optimisation.

![Figure 7. 3D plot of Pareto-optimal front.](image)

The testbench netlist is used to evaluate the performance for each design parameter set (defined by GA) and the result of the simulations determines the fitness score of the individuals. Non-dominated sorting
and crowding distance sorting are applied to the solution for each generation in order to determine the final set of Pareto-fronts. The result of the optimisation is a full set of designable parameters and their performance functions.

4.3 Performance and Variation Modelling

The outcome of the MOO is a Pareto-front consisting of solution points which defines the performance model of the circuit. Figure 7 shows the 3D plot of the Pareto-front for three competing objectives: jitter, current and gain. To develop the variation model of the Pareto-front, every optimal solution undergoes a Monte Carlo simulation using foundry variation and mismatch models. 100 samples were chosen for the MC simulation and from these the variation for each performance is calculated. This completes the variation model and results are stored in a data file. At this point, a combined performance and variation model for the VCO is developed. A look-up table is defined for the table model function in the Verilog-A model given in listing 1. Table 1 shows a selection of samples points from the table and their performance and variation values.

Listing 1. Performance and Variation Model

```
module vco(out, outmin, outmax, in);

...........

parameter real kvco=500e6;
parameter real ivco=3e-3;
..........

analog begin
kvco = \texttt{Stable-model}(kvco, "kvco.delta.tbl", "3E");
jvco = \texttt{Stable-model}(jvco, "jvco.delta.tbl", "3E");
ivco = \texttt{Stable-model}(ivco, "ivco.delta.tbl", "3E");

...........

p1 = \texttt{Stable-model}(kvco, ivco, jvco, "data.tbl", "3E, 3E");
p2 = \texttt{Stable-model}(kvco, ivco, jvco, "data.tbl", "3E, 3E");
P7 = \texttt{Stable-model}(kvco, ivco, jvco, "data.tbl", "3E, 3E");
fptr=\texttt{fopen}("params.dat");
Sfwrite(fptr, "n Generated Design Parameters\n");
Sfwrite(fptr, "\%e \%e \%e \%e \%e \%e\n", p1,p2,p3,p4,p5,p6,p7);
Sfclose(fptr);
..........
end
```

### Table 1. Performance and variation values.

<table>
<thead>
<tr>
<th>Design</th>
<th>Kvco (MHz/V)</th>
<th>ΔKvco</th>
<th>Jvco (ps)</th>
<th>ΔJvco</th>
<th>Ivco (mA)</th>
<th>ΔIvco</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>997</td>
<td>0.50%</td>
<td>0.13</td>
<td>22%</td>
<td>8.62</td>
<td>2.9%</td>
</tr>
<tr>
<td>21</td>
<td>373</td>
<td>0.45%</td>
<td>0.11</td>
<td>22%</td>
<td>3.58</td>
<td>2.7%</td>
</tr>
<tr>
<td>22</td>
<td>1090</td>
<td>0.32%</td>
<td>0.29</td>
<td>25%</td>
<td>2.79</td>
<td>2.6%</td>
</tr>
<tr>
<td>23</td>
<td>1620</td>
<td>0.30%</td>
<td>0.19</td>
<td>23%</td>
<td>8.46</td>
<td>2.9%</td>
</tr>
<tr>
<td>24</td>
<td>2280</td>
<td>0.28%</td>
<td>0.36</td>
<td>26%</td>
<td>4.98</td>
<td>2.7%</td>
</tr>
<tr>
<td>27</td>
<td>1850</td>
<td>0.29%</td>
<td>0.21</td>
<td>23%</td>
<td>6.74</td>
<td>2.8%</td>
</tr>
<tr>
<td>28</td>
<td>1450</td>
<td>0.29%</td>
<td>0.12</td>
<td>22%</td>
<td>6.16</td>
<td>2.8%</td>
</tr>
<tr>
<td>29</td>
<td>1600</td>
<td>0.35%</td>
<td>0.30</td>
<td>25%</td>
<td>2.68</td>
<td>2.6%</td>
</tr>
</tbody>
</table>

4.4 Behavioural Description

The resulting performance and variation model become a part of the behavioural model of the VCO block. This model is given in listing 2 which includes the lookup-table of the Pareto-front. All the individual blocks in a PLL system including the PFD, CP and VCO were behaviourally modelled based on [13].

Listing 2. VCO behavioural model

```
module vco(out, outmin, outmax, in);

............

............

jvco = \texttt{Stable-model}(kvco, ivco, "data.tbl", "3E, 3E");
jvco_min = \texttt{Stable-model}(kvco_min, ivco_min, "data.tbl", "3E, 3E");
jvco_max = \texttt{Stable-model}(kvco_max, ivco_max, "data.tbl", "3E, 3E");

............

@((cross(phase-0.25,+1,ttl))begin
    dt = \texttt{delta}*\texttt{sdist_normal}(seed, 0, 1);
    dt_min = \texttt{delta_min}*\texttt{sdist_normal}(seed, 0, 1);
    ........
    V(out)<+\texttt{transition}(vout,0,tt);
    V(outmin)<+\texttt{transition}(vout_min,0,tt);
    V(outmax)<+\texttt{transition}(vout_max,0,tt);
end
```

4.5 Hierarchical optimisation

The behavioural models are combined together for PLL system level optimisation using the NSGA-II algorithm. The designable parameters for the optimisation are gain (Kvco) and current (Ivco) for the VCO and C1, C2 and R1 for the loop filter.

During the optimisation, the variation model of VCO is used to interpolate the minimum and maximum VCO gain and VCO current for each of their nominal values generated by GA. Based on these values, the performance model is then used to interpolate the nominal, minimum and maximum of the VCO jitter. Therefore the output of the VCO behavioural block which will include the nominal, minimum and maximum values will be used to determine the PLL performances. At the end, the optimised system level will have their nominal as well as minimum and maximum performances. Table 2 shows some of the optimal solution samples for the PLL system level optimisation. From the optimal solutions, a solution that meets the specifications and variations will be selected as the design solution (shown by the shaded area).
Table 2. PLL system level solution samples

<table>
<thead>
<tr>
<th>Kv</th>
<th>Kvmin</th>
<th>Kvmax</th>
<th>Iv</th>
<th>Ivmin</th>
<th>Ivmax</th>
<th>C1</th>
<th>C2</th>
<th>R1</th>
<th>Lt</th>
<th>Jit</th>
<th>Jit_min</th>
<th>Jit_max</th>
<th>Curr</th>
<th>Curr_min</th>
<th>Curr_max</th>
</tr>
</thead>
<tbody>
<tr>
<td>1540</td>
<td>1536</td>
<td>1545</td>
<td>16.1</td>
<td>15.68</td>
<td>16.52</td>
<td>2.1p</td>
<td>1.8p</td>
<td>2k</td>
<td>0.90</td>
<td>4.30</td>
<td>4.23</td>
<td>4.38</td>
<td>17.1</td>
<td>15.6</td>
<td>18.6</td>
</tr>
<tr>
<td>684</td>
<td>680.9</td>
<td>687.1</td>
<td>3.78</td>
<td>3.68</td>
<td>3.88</td>
<td>5p</td>
<td>2p</td>
<td>3.8k</td>
<td>0.82</td>
<td>4.24</td>
<td>4.18</td>
<td>4.30</td>
<td>3.83</td>
<td>3.78</td>
<td>3.91</td>
</tr>
<tr>
<td>1050</td>
<td>1046</td>
<td>1053</td>
<td>3.96</td>
<td>3.95</td>
<td>3.97</td>
<td>3p</td>
<td>1p</td>
<td>1k</td>
<td>0.85</td>
<td>4.37</td>
<td>4.27</td>
<td>4.66</td>
<td>3.99</td>
<td>3.85</td>
<td>4.07</td>
</tr>
<tr>
<td>1480</td>
<td>1475.7</td>
<td>1484.3</td>
<td>10.4</td>
<td>10.1</td>
<td>10.7</td>
<td>4.8p</td>
<td>3p</td>
<td>2k</td>
<td>0.79</td>
<td>4.17</td>
<td>4.13</td>
<td>4.21</td>
<td>10.50</td>
<td>10.30</td>
<td>10.90</td>
</tr>
</tbody>
</table>

To verify the predicted yield given by the proposed approach, a Monte Carlo analysis with 500 samples was run on the final design. This analysis confirmed a yield of 100%. Figure 8 shows the transistor level result for the PLL locking time simulation.

5 Conclusions

This paper has presented a new approach for hierarchical-based design that combines performance and yield optimisation for a mixed-signal system. Multi-objective optimisation based on an evolutionary algorithm is used to explore tradeoffs between performance and yield, leading to a set of Pareto optimal solutions for the design. Monte Carlo variation analysis is performed on all the Pareto optimal solutions, and a table is constructed for both the performance and variation analysis. A behavioural model developed in Verilog-A is used together with this table to determine the parameters required to achieve the highest yield within a given specification. The behavioural model is used for a system level simulation and the approach demonstrates a successful top-down optimisation. These benefits are enjoyed without a corresponding drop in accuracy. A 5 stage VCO and PLL system design were used to demonstrate the proposed algorithm and the behaviour has been verified with transistor level simulations.

References