

Steady-State Analysis of Nonlinear Circuits Using Discrete Singular Convolution Method

Xin Zhou¹, Dian Zhou¹, Jin Liu¹, Ruiming Li¹, Xuan Zeng² and Charles Chiang³
ECE Dept., University of Texas at Dallas, Richardson, TX75083, USA¹
Microelectronics Dept., Fudan University, Shanghai 200433, P.R.China²
Synopsys Inc., Mountain View, CA 94043, USA³

Abstract*

In this paper, we propose a novel time-domain based method, Discrete Singular Convolution algorithm, for computing steady-state response in nonlinear circuit. Properties and advantages of Discrete Singular Convolution method are discussed, compared with some other approaches. The accuracy and efficiency of this method are tested by the numerical experiments.

1 Introduction

A main difficulty to get the steady-state response in nonlinear circuit simulation is that the transient response may take a very long time before the circuit reach the steady state. The traditional simulation method to get the steady-state response has to take an extraordinary computing time throughout the transient regime.

A lot of time-domain and frequency-domain methods have been proposed to solve the periodic steady-state problem [1]-[7]. Time-domain based method that have been proposed include shooting methods [2], [3] and wavelet-balance method [7]. Frequency-domain based methods that have been proposed include harmonic methods and their modifications [1], [6]. The shooting methods iteratively simulate the circuit over one period intervals in order to find initial conditions, which makes the signals at the end of the period exactly match those at the beginning, i.e., let the circuit start directly in steady state. However, the shooting methods require to integrate the

equations repeatedly, which costs substantial computation time. The harmonic balance methods expand the unknown variables in the circuits by Fourier series. In order to achieve an accurate solution, it needs many harmonic components. During the implementation of harmonic method, it has to execute DFT and IDFT repeatedly. So they also consume a lot computing time. [4], [5] present approaches that based on harmonic method, but formulate the system equations in the time domain, which avoid repeatedly executing DFT and IDFT. The wavelet balance method assumes the solutions in terms of wavelet series instead of Fourier series.

Recently, A discrete singular convolution (DSC) method [9], [10], [11] has been proposed and considered as potential method to solve partial differential equations. It has been applied to many scientific and engineering problems, such as electromagnetic problems [11] and quantum eigenvalue problems [10]. However, using DSC method to find the steady state response in nonlinear circuit has not been explored. In this paper, we propose a time-domain DSC method to solve the steady state response problem in nonlinear circuit. A regularized Shannon's kernel is used during the implementation of DSC method, which dramatically decreases the truncation error. Comparing with other approaches, DSC method has several advantages, such as controllable accuracy and computational bandwidths for numerical computations by choosing the different computational parameters.

The rest of the paper is organized as follows. Section II introduces discrete singular convolution method and how to find the steady state response using DSC. Section III shows the numerical experiment and results. Section IV draws the conclusions.

2 Steady-State Analysis With DSC Method

2.1 Principle of Steady-State Analysis Using DSC Method

* This research is supported by NSF CCR-0098275, CCR-0306298, NSFC 90307017, 60176017, 90207002, Synopsys Inc., Cross-Century Outstanding Scholar's fund of Ministry of Education of China, National 863 plan projects 2002AA1Z1340 and 2002AA1Z1460, doctoral program foundation of Ministry of Education of China 2000024628, Science & Technology key project of Ministry of Education of China 02095, Shanghai Science and Technology committee project 01JC14014 and Shanghai AM R&D fund 0107.

Consider the nonlinear system that is described by an ordinary differential equation as following:

$$\frac{dx(t)}{dt} = f(x(t), t) \quad (1)$$

where $f(x(t), t)$ is the given nonlinear vector function and $x(t) = [x_1(t) \ x_2(t) \ \dots \ x_L(t)]^T$ which are L unknown variables.

According to Shannon's sampling theorem, a L^2 continuous function with maximum frequency $\omega_{\max} \leq \pi/h$ can be written as:

$$x(t) = \sum_{k=-\infty}^{\infty} \delta_h(t-t_k)x(t_k) \quad (2)$$

where $t_k = kh$, $\delta_h(t-t_k)$ is the Shannon's kernel:

$$\delta_h(t-t_k) = \frac{\sin[(\pi/h)(t-t_k)]}{(\pi/h)(t-t_k)} \quad (3)$$

Note that $\delta_h(t_i-t_k) = 0$ when $i \neq k$ and equals 1 when $i = k$. The interpolative property makes the computation simple and accurate.

Truncating the infinite series in (2) to a finite series of M terms, a function and its n th derivatives can be approximated:

$$x^n(t) \approx \sum_{k=-M}^M \delta_h^{(n)}(t-t_k)x(t_k) \quad n=0,1,2.. \quad (4)$$

$\delta_h^{(n)}(t-t_k)$ is the n th derivative of $\delta_h(t-t_k)$, which is given by :

$$\delta_h^{(n)}(t-t_k) = \left(\frac{d}{dt}\right)^n (\delta_h(t-t_k)) \quad (5)$$

In our DSC method, we use a regularized Shannon's kernel instead of normal Shannon's kernel (3):

$$\delta_{h,\sigma}(t-t_k) = \frac{\sin \frac{\pi}{h}(t-t_k)}{\frac{\pi}{h}(t-t_k)} e^{-\frac{(t-t_k)^2}{2\sigma^2}} \quad (6)$$

The regularized kernel is a more compactly supported kernel, where $[h = T/N]$ is the grid spacing, T is the total simulation domain, which is also the steady state response period. N is the total number of collocation sample points, which is another parameter that can be chosen according to accuracy requirement. h should be set

small enough so that $(\pi/h) > B$, where B is the highest frequency that can be reached in the Fourier representation among all the observed variables in computational domain. $2M+1$ is the computational bandwidth, M can be chosen accordingly, which depends on the accuracy requirement. $\sigma = rh$, where r is another parameter in computation, which normally can be chosen by practical experiments. $t_k = kh$. The truncation error can be dramatically reduced by using regularized Shannon's kernel. Estimation of the truncation error is given in [8], which provides a guide of how to choose M , σ and h . For $n=0$ in the equation (4), if L^2 norm error is set to $10^{-\lambda}$, the following equations are deduced:

$$r(\pi - Bh) > \sqrt{4.61\lambda} \quad (7a)$$

$$(M/r) > \sqrt{4.61\lambda} \quad (7b)$$

Note that the regularized Shannon's kernel retains the interpolation property, which also simplifies the computation.

Notice in (4) that in order to find out the value of $x(t)$ near the boundary of the computational domain, we have to fake points $x(t_k)$ which are outside of the domain. The values of $x(t_k)$ are determined according to the boundary conditions. For example, in a computational domain $[a, b]$, if $x(t)$ at boundary point $x(a)$ and $x(b)$ are known, the $x(t_k)$ located on the left of point a can be considered as $x(a)$ and $x(t_k)$ located on the right of the point b can be considered as $x(b)$. For the periodic boundary condition, the $x(t_k)$ can be generated by expanding the correspondent points inside the computational domain.

Now plug the equation (4) into the equation (1) and use the regularized kernel to replace Shannon's kernel and let $n=1$, for each variable $x_l(t)$:

$$\sum_{k=-M}^M \delta'_{h,\sigma}(t-t_k)x_l(t_k) = f(x_l(t), t) \quad (8a)$$

$$l = 1, 2, \dots, L$$

At each point t_i , the equation (8a) should be satisfied, then we have:

$$\sum_{k=-M}^M \delta'_{h,\sigma}(t_i-t_k)x_l(t_k) = f(x_l(t_i), t_i)$$

$$t_i = ih \quad i = 1, 2, \dots, N \quad l = 1, 2, \dots, L \quad (8b)$$

where

$$\begin{aligned} \delta'_{h,\sigma}(t_i - t_k) &= \frac{d}{dt} \left(\frac{\sin \frac{\pi}{h}(t - t_k)}{\frac{\pi}{h}(t - t_k)} e^{-\frac{(t-t_k)^2}{2\sigma^2}} \right) \Big|_{t=t_i} \\ &= \frac{(-1)^{i-k}}{h(i-k)} e^{-\frac{(t_i-t_k)^2}{2\sigma^2}} \quad i \neq k \quad (9) \end{aligned}$$

and equals 0 when $i = k$. So the equation (8b) become

$$\sum_{\substack{k=-M \\ i \neq k}}^M \frac{(-1)^{i-k}}{h(i-k)} e^{-\frac{(t_i-t_k)^2}{2\sigma^2}} x_l(t_k) = f(x_l(t_i), t_i)$$

$$t_i = ih \quad i = 1, 2, \dots, N \quad l = 1, 2, \dots, L \quad (10)$$

In a non-autonomous circuit, the steady state response period T is known, the total unknown variables $x_l(t_k)$ are $N \times L$, the total independent algebraic equations in (10) are also $N \times L$, so (10) can be solved using Newton's method.

For an autonomous circuit, the steady state response period T is an unknown variable, which make the total number of unknown variables in (10) equal $N \times L + 1$. Since the total independent algebraic equations in (10) are $N \times L$, we add the following equation into (10), which also enhance the periodic boundary condition in DSC method.

$$\sum_{l=1}^L (x_l(t_1) - x_l(t_N))^2 = 0 \quad (11)$$

Therefore, the total number of variables and independent algebraic equations are all $N \times L + 1$, and we can also use Newton's method to solve it. After the coefficients $x_l(t_k)$ are found, computation is end because they are exactly the values of variables we want to solve.

2.2 Advantages Of DSC

First, the DSC method works at time domain, it can efficiently handle some problems, such as nonlinearity, which is difficult to deal with by using frequency domain based approaches.

Second, truncation errors in DSC method can be estimated by choosing different values of M , σ , r and h [8], so it has a controllable accuracy and computational

bandwidth. For a high accuracy requirement computation, a bigger M can be chosen while for a fast requirement computation, a smaller M can be chosen.

Third, according to the interpolation property of the kernel used in DSC, the expansion coefficients are the values of solutions we want, therefore, no additional computation is needed after we get the coefficients.

Fourth, for non-autonomous nonlinear circuit steady state response, the periodic boundary condition is automatically included in the computation by expanding the correspondent points inside the simulation domain. Therefore, no additional computation is needed to deal with the boundary condition.

Fifth, A regularized Shannon's kernel is used in DSC method, which dramatically lessen the truncation error, i.e., A smaller M can be used to reach the same accuracy by using regularized Shannon's kernel compare to using normal Shannon's kernel.

3 Numerical Results

In this section, we use two nonlinear circuits as examples to show the effectiveness of DSC method. The examples are simulated on AMD-900M PC with 128M memory.

3.1 Van Der Pol Oscillator

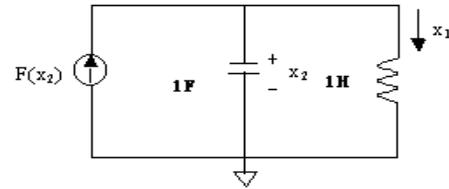


Figure1. Schematic of Van der Pol oscillator, where $F(x_2) = 5(x_2 - x_2^3/3)$.

Figure 1. shows a nonlinear Van der Pol oscillator, which is a simple nonlinear circuit, yet services well to test different algorithms [3], [4], [5], [7] for autonomous circuits. x_2 is the voltage across capacitor and x_1 is the current across the inductor. The Van der Pol equations are:

$$dx_1 / dt = x_2 \quad (12a)$$

$$dx_2 / dt = 5(x_2 - x_2^3/3) - x_1 \quad (12b)$$

Simulations are conducted with different combinations of computation parameters: M , the parameter controlling the computation depth and accuracy; N , total sampling points and r .

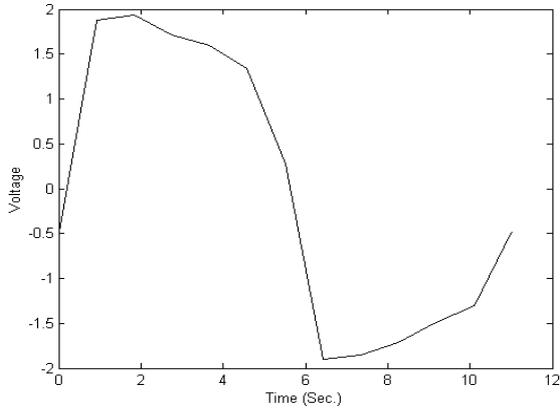


Figure. 2 Waveform of x_2 using 13 points, where $M = 6$, $N = 13$, $r = 1.5$ and $T = 11.222s$

Figure 2 shows the experiment result with $M = 6$, $N = 13$ and $r = 1.5$. The steady state response period $T = 11.222s$. The accuracy of T is much better than that generated with 15 points in [5]. Figure 3 shows the result with $M = 25$, $N = 51$ and $r = 10$. We get the T equals 11.589 seconds. The result is also much better than what was presented in [5] with 60 sample points. Taking advantage of the convenience of DSC method, we can increase M to get more accuracy result without adding more sample points, we get $T = 11.619s$ with $M = 30$, $N = 25$ and $r = 10$. This result is almost same as the result presented with 240 sample points in [5], and is better than the result presented in [7] with 63 basis functions. We can say that, with much fewer sampling points, results produced by using DSC method with regularized Shannon's kernel are the same as or better than the results presented in [7]. With the same parameters M and N , the DSC method reaches a significant better result by using regularized Shannon's kernel compare to by using the normal Shannon's kernel. Table 1. shows the simulation results by using different approaches.

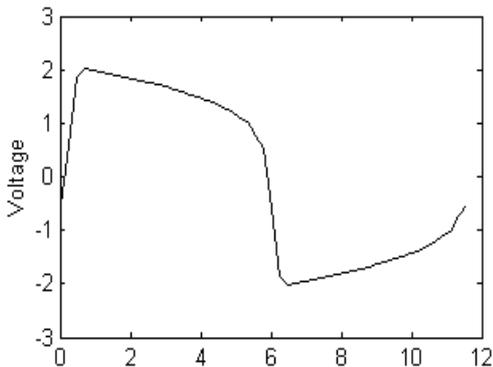


Fig. 3. Waveform of x_2 using 51 points, where $M = 25$, $N = 51$, $r = 10$ and $T = 11.589s$

Table 1. Simulation results comparison

Method	Number of Sample points/Basic functions	T(second)
Relaxation Algorithm (Bilinear Mapping)	15	10.1028
	30	11.067
	240	11.6171
Wavelet Balance Algorithm	23	11.51
	43	11.57
	63	11.61
DSC with Shannon kernel	13	12.110
	51 with $M = 25$	11.364
DSC with regularized kernel	13 with $M = 6$, $r = 1.5$	11.222
	51 with $M = 25$, $r = 10$	11.589
	51 with $M = 30$, $r = 10$	11.619

3.2 Three-inverter-oscillator

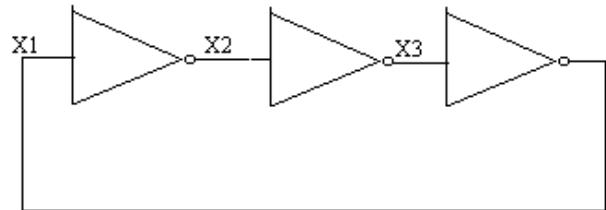


Figure4. Schematic of three-inverter-oscillator

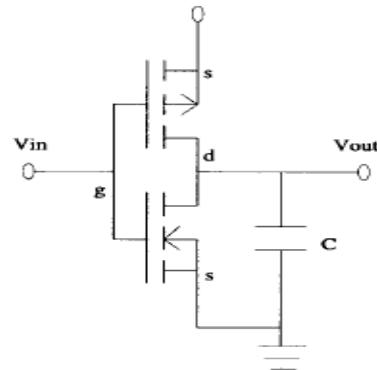


Figure. 5 Schematic of inverter

Currently simulation of PLL (Phase Lock Loop) is a challenging work in Analog Circuit design. VCO (Voltage Controlled Oscillator) is part of PLL circuit and three-inverter-oscillator is one of the simplest circuits to implement VCO. Figure 4. shows a three-inverter-oscillator circuit. x_2 , x_3 and x_1 are the output voltages of three

inverters, respectively. Figure 5. shows the schematic of one inverter. Following equation describes the circuit behavior for one inverter:

$$\frac{dv_{out}}{dt} = \frac{I_{ds,pmos}(t) - I_{ds,nmos}(t)}{C} \quad (13)$$

Where I_{ds} is determined by following equations:

$$I_{ds} = \begin{cases} K \frac{W}{L} (v_{gs} - v_{th} - \frac{v_{ds}}{2}) v_{ds} (1 + \lambda v_{ds}), & v_{gs} > v_{th} \quad \text{and} \quad v_{ds} < v_{gs} - v_{th} \\ K \frac{W}{L} (v_{gs} - v_{th})^2 (1 + \lambda v_{ds}), & v_{gs} > v_{th} \quad \text{and} \quad v_{ds} > v_{gs} - v_{th} \\ 0 & \text{else} \end{cases} \quad (14)$$

where $K, W, L, v_{gs}, v_{ds}, v_{th}, \lambda$ are the parameters of NMOS and PMOS. Combination of equation (13) and (14) is the nonlinear equations for one inverter.

Figure 6. shows the simulation result of x_1 by using DSC method, where $M = 25, N = 31, r = 3$. The period of x_1 is about 3.8ns. A same result is got by implementing the fourth-order Runge-Kutta algorithm. In order to keep the waveform not to drift away quickly, we have to dramatically increase the number of points one period in Runge-Kutta algorithm, which is, in our case, at least 15,000 points per period. DSC algorithm only needs 31 points one period to get the same result and doesn't have the error accumulation problem.

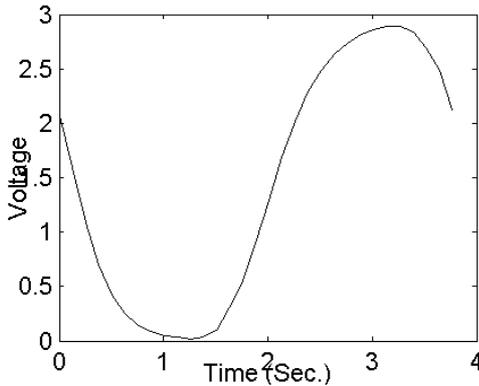


Figure6. Waveform of X_1

4 Conclusion

A novel time-domain based approach, discrete singular convolution method, is presented in this paper for getting the steady state response in nonlinear circuit. By expanding variables into DSC regularized Shannon's kernel series in time domain, the differential equations become a set of algebraic equations. Exploiting the superior computational properties of DSC method and its δ type kernel, the proposed method shows a great efficiency

and accuracy. Some comparisons to other approaches have been made through the numerical results.

5 Acknowledgments

Li,Xin of Carnegie Mellon University gave us a lot of help during the course of this work. Prof. Bao,Gang of Michigan State University is also gratefully acknowledged.

6 References

- [1] K. S. Kundert and A. Sangiovanni-Vincentelli, "Simulation of nonlinear circuits in the frequency domain", *IEEE Trans. Computer-Aided Design*, vol. 5, pp. 521-535, Oct. 1986
- [2] T. J. Aprille and T. N. Trick, "steady state analysis of nonlinear circuits with periodic inputs", *Proc. IEEE*, vol. 60, no. 1, pp. 108-114, Jan. 1972
- [3] J. R. Parkhurst and L. L. Ogborn, "Determining the steady state output of nonlinear oscillatory circuits using multiple shooting", *IEEE Trans. Computer-Aided Design*, vol. 14, pp. 882-889, July 1995
- [4] D. Frey and O. Norman, "An integral equation approach to the steady state problem in nonlinear circuits", *IEEE Trans. Circuits Syst. I*, vol.39, pp. 744-755, Sept. 1992
- [5] D. R. Frey, "A class of relaxation algorithms for finding the periodic steady state solution in nonlinear system", *IEEE Trans. CASI*, vol. 45, pp. 659-663, Jun. 1998
- [6] A. Ushida and L. O. Chua, "Frequency-domain analysis of nonlinear circuits driven by multi-tone signals", *IEEE Trans. CAS*, vol. 31, pp. 766-779. Sept. 1984
- [7] Xin Lin, Bo Hu, Xieting Ling and Xuan Zeng, "A wavelet-balance approach for steady-state analysis of nonlinear circuits", *IEEE Trans. Circuits Syst. I*, vol. 49, pp. 689-694. May 2002
- [8] L. W. Qian and G.W. Wei, "A note on regularized Shannon's sampling formulae", *Preprint arXiv:math. SC/0005003*, 2000
- [9] G. W. Wei, Y .B. Zhao and Y. Xiang, "Discrete singular convolution and its application to the analysis of plates with internal supports", *Int. J. Numer. Meth. Engng*, 55:913-971, 2002
- [10] G.W.Weii, "Solving quantum eigenvalue problems by discrete singular convolution kernels", *J. Phys. B* 33:343-352, 2000
- [11] Gang Bao, G. W. Wei and Shan Zhao, "Local spectral time-domain method for electromagnetic wave propagation", *Optics Letters*, vol. 28, 7: 513-515, April 2003