

Extended subspace identification of improper linear systems

Gerd Vandersteen
IMEC vzw.
Kapeldreef 75
B-3001 Heverlee, Belgium

Rik Pintelon
Vrije Universiteit Brussel
Dept. ELEC
Pleinlaan 2
B-1050 Brussel, Belgium

Dimitri Linten, Stéphane Donnay
IMEC vzw.
Kapeldreef 75
B-3001 Heverlee, Belgium

Abstract

The modeling of linear transfer functions is often required prior to the simulation of electronic systems. An example is the modeling of on-chip inductors starting from 2-port measurements. The modeling is often done using state-space models that can only represent proper systems. This leads to modeling problems in the case of improper systems such as in the case of 2-port modeling of the admittance matrix of an on-chip inductor. This paper first describes an extended state-space model to represent improper systems. Afterwards, the paper introduces an extension to classical frequency-domain subspace identification methods. The usefulness of both the extended state-space model and the extended subspace modeling technique are illustrated by comparing them with commercially available solutions. This includes a comparison on measurements of an on-chip inductor and on simulations of a coplanar waveguide.

1. Introduction

Accurate and efficient models are required for the design and simulation of high-performance electronic systems. Models for linear systems can be extracted starting from either complex models generated by (partial) differential equation solvers or by table-based frequency-dependent transfer functions obtained by either simulations or measurements.

Passivity is a major issue when identifying a continuous-time model since non-passive models can lead to simulation problems. Unwanted oscillations can be observed during transient simulations when using non-passive models.

It has been observed that specifying incorrect model orders often leads to non-passive models. This comes from the fact that the poles / zeros of the model are used such that they compensate for the presence / absence of the poles / zeros in the real system.

The modeling of an inductor based on 2-port S-parameter measurements clearly illustrates the problem: If the simulator uses a modified nodal analysis, then a state-space representation for the admittance matrix is a natural choice. The admittance matrix representation of a 2-port inductor is improper as will be demonstrated below. This leads to modeling errors when using state-space models. In practice, this often leads to the generation of non-passive models to approximate the measurements.

State-space models are often used to represent continuous-time linear systems using

$$\begin{cases} sX = AX + BU \\ Y = CX + DU \end{cases} \quad (1)$$

where s denotes the Laplace variable; A, B, C and D are the system matrices and U, Y and X represent respectively the vector of n_u inputs, n_y outputs and n_a state variables. Hence, the order of the system is n_a .

State-space models are unable to represent improper systems, i.e. systems where the order of the numerator is higher than the order of the denominator. The transfer function of (1) equals

$$H(s) = C(sI - A)^{-1}B + D \quad (2)$$

which clearly demonstrates that the order of the numerator is at most the order of the denominator. This implies that not all passive networks can be represented using a state-space model. Consider as example the simplified model of the inductor in figure 1 which only takes the parasitic capacities at both ports of the inductor into account. The system's admittance matrix equals

$$H(s) = \frac{1}{sL} \begin{bmatrix} s^2 LC_{par} + 1 & -1 \\ -1 & s^2 LC_{par} + 1 \end{bmatrix} \quad (3)$$

Matrix (3) clearly demonstrates that the model is improper.

Subspace identification techniques can be used to extract a state-space model out of table-based frequency response data and – optionally – a frequency dependent weighting [6].

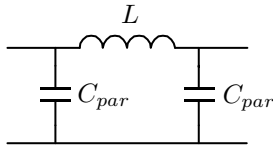


Figure 1. Simplified on-chip inductor model with nominal value L and parasitic capacitors C_{par} .

The state-space matrices are estimated in a non-iterative way using oblique projections and least-squares estimators.

This work presents a subspace-based modeling technique for modeling improper continuous-time systems satisfying

$$\begin{cases} sX = AX + BU \\ Y = CX + DU + \sum_{i=1}^N s^i E_i U \end{cases} \quad (4)$$

where N equals the number of zeros minus the number of poles. Note that N is small in practical applications: it can be shown that $N \leq 1$ for passive circuits composed out of lumped elements [2].

The proposed modeling technique – described in section 2 – is inspired on a state-space modeling technique. Both the necessary transformations and the determination of the poles and zeros are adapted to allow the modeling of improper models. The applicability of the method is demonstrated in section 3 where it is used to model an on-chip inductor and a coplanar waveguide starting from respectively measurement and simulation data. The proposed method is compared with commercially available solutions, namely Spectre simulations. Based on the observations on these different types of experiments, it is concluded in section 3 that the proposed model and modeling method outperforms for modeling improper systems.

2. The Extended subspace modeling technique

The extended state-space model and the developed subspace-based modeling technique are described in this section. After a short review of subspace identification (section 2.1), the model properties for the extended state-space model is given in section 2.2. Extending the classical subspace identification methods towards the extended state-space models is discussed in section 2.3. This requires the modification of both the oblique projection and the least squares estimators used. Section 2.4 finally deals with several practical issues such as the ability of stabilizing the poles of A .

2.1. Reviewing subspace identification

A wide spectrum of subspace-based identification methods are described in the literature [5, 6, 7, 8, 9] covering

- both time-domain and frequency-domain identification;
- both discrete-time and continuous-time models;
- the use of frequency dependent weighting;
- the use of orthogonal polynomials to improve the numerical conditioning;
- the stabilization of the estimated state-space model.

The proposed method described in this paper is based on frequency-domain subspace methods for discrete-time and continuous-time systems. The frequency-domain system representation uses the z -transform for discrete-time representation, while the Laplace transform is used to represent continuous-time systems in the frequency domain. Hence, the state-space models can be written as

$$\xi_k X(k) = AX(k) + BU(k) \quad (5a)$$

$$Y(k) = CX(k) + DU(k) \quad (5b)$$

where $\xi_k = z_k = e^{j\omega_k}$ for discrete-time systems and $\xi_k = s_k = j\omega_k$, for continuous time systems (z and s are respectively the Z-transform and the Laplace transform variables). The repetitive application of (5a) onto (5b) gives

$$W_r(k)Y(k) = O_r X(k) + S_r W_r(k)U(k) \quad (6)$$

with

$$W_r(k) = \begin{bmatrix} 1 \\ \xi_k \\ \vdots \\ \xi_k^{r-1} \end{bmatrix}, O_r = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{r-1} \end{bmatrix} \quad (7)$$

and

$$S_r = \begin{bmatrix} D & 0 & \dots & 0 & 0 \\ CB & D & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{r-2}B & CA^{r-3}B & \dots & CB & D \end{bmatrix} \quad (8)$$

Collecting equations (6) for F frequencies $k = 1, \dots, F$ gives

$$\mathbf{Y} = O_r \mathbf{X} + S_r \mathbf{U} \quad (9)$$

with

$$\begin{aligned} \mathbf{Y} &= [W_r(1)Y(1) \quad W_r(2)Y(2) \quad \dots \quad W_r(F)Y(F)], \\ \mathbf{U} &= [W_r(1)U(1) \quad W_r(2)U(2) \quad \dots \quad W_r(F)U(F)], \\ \mathbf{X} &= [X(1) \quad X(2) \quad \dots \quad X(F)] \end{aligned}$$

where $\mathbf{Y} \in \mathbb{C}^{r \times F}$, $\mathbf{U} \in \mathbb{C}^{r \times F}$, $\mathbf{X} \in \mathbb{C}^{n_a \times F}$, $O_r \in \mathbb{R}^{r \times n_a}$ and $S_r \in \mathbb{R}^{r \times r}$. Equation (9) is converted as a real set of equations as

$$\mathbf{Y}^{re} = O_r \mathbf{X}^{re} + S_r \mathbf{U}^{re} \quad (10)$$

where $(\cdot)^{re}$ locates the real and imaginary parts beside each other, for example

$$\mathbf{Y}^{re} = [\text{Re}(\mathbf{Y}) \quad \text{Im}(\mathbf{Y})].$$

Subspace identification algorithms are basically three-step algorithms:

1. An estimate of the extended observability matrix O_r is obtained by applying a projection

$$\Pi^\perp = I - \mathbf{U}^{reT} (\mathbf{U}^{re} \mathbf{U}^{reT})^{-1} \mathbf{U}^{re} \quad (11)$$

onto (10) such that

$$\mathbf{Y}^{re} \Pi^\perp = O_r \mathbf{X}^{re} \Pi^\perp. \quad (12)$$

The range space of $\mathbf{Y}^{re} \Pi^\perp$ equals the range space of O_r if r is larger than the order n_a of the system [6]. An efficient and numerically stable implementation does not compute the matrix products of (11) directly but performs the projections through a QR-decomposition [10].

2. An estimate of the matrix A is found as the linear least-squares solution of the over-determined set of equations given by the extended observability matrix O_r :

$$O_{r[1:r-1,:]} A = O_{r[2:r,:]} \quad (13)$$

An estimate of C is given by $O_{r[1:n_y,:]}.$

3. Estimates of the matrices B and D are found using a linear least-squares solution of

$$\sum_{k=1}^F \|Y(k) - [C(\xi_k I - A)^{-1} B + D]U(k)\|_{W(k)}^2$$

with

$$\|X\|_W^2 = X^H W X.$$

Note that step 2 and 3 are independent of r which makes that a single (time consuming) first step can be used to determine models for various model orders [6].

It is known that continuous-time systems – with $\xi = s$ – can only be modeled for small values of n_a . Numerical conditioning problems occur for larger values of n_a . The conditioning can be improved by applying a change in variables such that the estimates are performed using another parameterization. A bilinear transformation can be used for this purpose [6, 11]

$$s \mapsto \omega_{scale} \frac{z-1}{z+1}$$

with ω_{scale} a frequency scaling which the user is free to specify under the constraint that $\omega_{scale}/2$ is not a pole of the continuous-time system. This scaling can be seen as a kind of sampling frequency.

2.2. Properties of the extended subspace model

The computation of poles and zeros of the transfer function is important to verify the passivity of the network. A dedicated method is required to compute the zeros of the extended state-space model. This method is described after reviewing the pole/zero calculation of ordinary state-space models.

The computation of the poles for both the ordinary and the extended state-space model can be formulated as an eigenvalue problem $\lambda X = AX$.

The computation of the zeros of (2) start from the original state-space model. A zero of the transfer function generates a zero output for an arbitrary input. This leads to a generalized eigenvalue problem

$$\lambda \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ U \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ U \end{bmatrix}.$$

Similarly, the zeros of the extended state-space model can be formulated as a generalized eigenvalue problem:

$$\lambda \begin{bmatrix} I & 0 & \cdots & 0 & 0 \\ 0 & -E_1 & \cdots & -E_{N-1} & -E_N \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \begin{bmatrix} X \\ U \\ U_2 \\ \vdots \\ U_N \end{bmatrix} = \begin{bmatrix} A & B & 0 & \cdots & 0 \\ C & D & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & I \end{bmatrix} \begin{bmatrix} X \\ U \\ U_2 \\ \vdots \\ U_N \end{bmatrix}.$$

It can be concluded that both the zeros and the poles of the extended state-space model can be computed in a numerically stable way using (generalized) eigenvalue decompositions.

2.3. Subspace identification of the extended state-space model

The extended subspace method is based on the continuous-time modeling using a bilinear transformation [6]. This approach is followed since (i) it allows different types of stabilization techniques [5, 8], and (ii) it is known that the parameter estimation problem is well conditioned for discrete-time transfer functions because the powers of $\xi = e^{j\omega}$ form a natural orthogonal basis [3].

The subspace modeling technique can not be applied straightaway since the projection Π^\perp needs to be modified such that not only the input \mathbf{U} is projected away, but the terms $s^i\mathbf{U}$ for $i = 1, \dots, N$ as well. All three subspace identification steps are therefore re-examined in the context of the extended state-space model.

The first step is to get an estimate of O_r using an oblique projection. Similar to the original state-space case, it is possible to write

$$\mathbf{Y} = O_r\mathbf{X} + S_r\mathbf{U} + \sum_{i=1}^N s^i T_r^i \mathbf{U} \quad (14)$$

with

$$T_r^i = \begin{bmatrix} E_i & 0 & \dots & 0 & 0 \\ CB & E_i & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ CA^{r-2}B & CA^{r-3}B & \dots & CB & E_i \end{bmatrix}. \quad (15)$$

The extended subspace method must now estimate O_r by projecting the output \mathbf{Y} onto the null space of both \mathbf{U} and all contributions of $s^i\mathbf{U}$ for $i = 1, \dots, N$. Hence, it requires the projection

$$\bar{\Pi}^\perp = I - Z^T(ZZ^T)^{-1}Z \quad (16)$$

with

$$Z = [\mathbf{U}^T, s\mathbf{U}^T, \dots, s^N\mathbf{U}^T]^T.$$

It is important – from numerical point of view – to apply the projection using a QR-decomposition [10] instead of calculating the matrix products of (16) directly.

The second step is identical for both the ordinary and the extended subspace method. Both estimate the matrices A and C out of O_r . Representations in both the z -domain and the s -domain are possible. If the model is extracted in the z -domain, then a bilinear state-space transformation is required as described in [6, 11].

The third step of the subspace modeling requires a linear least squares to estimate the matrices B and D . This step must be extended towards the linear least-squares estimation of B , D , and E_i for $i = 1, \dots, N$ using

$$\sum_{k=1}^F \|Y(k) - Y_{model}(k)U(k)\|_{W(k)}^2$$

with

$$Y_{model}(k) = C(s_k I - A)^{-1}B + D + \sum_{i=1}^N s_k^i E_i.$$

Note that the projection in the first step is done in the Laplace variable s . This implies that only small values of N are allowed to avoid numerical conditioning problems. However, this is not an issue since $N \leq 1$ for passive systems composed out of lumped elements [2]. Numerical problems can therefore be avoided by a proper scaling of the Laplace variable s .

2.4. Subspace identification in practice

Imposing stability or passivity on the estimated model is an active research topic. Imposing the stability of a state-space model requires that all poles are in the left half plain in the s -domain or – alternatively – that the poles are in-sight the unity circle in the z -domain. Imposing passivity of a system is far more difficult. The passivity condition depends on the actual representation of the circuit [1]. For an admittance and an impedance matrix representation, it is a necessary (but not sufficient) condition that both admittance and impedance matrices are stable. For S-parameter representation, passivity requires that $H(s)$ is bounded-real [1].

Imposing the stability of the estimated admittance models is used when identifying models for the passive circuits. Although it does not guarantee the passivity of the final model, it increases the probability of having it.

Imposing stability during a subspace identification can be done using two different techniques. Both stabilization methods are applied in step two of the subspace identification, i.e. the estimation of A and C . The least-squares estimation of B and D in a later stage can correct introduced modeling errors. A first method moves the instable poles into the stable region using a projection from the right to the left half plain in the s -domain and using a projection into the unit circle in the z -domain [6]. A second method is only applicable to the z -domain and uses a regularization of the linear least-squares problem to compute A such that all poles are within the unit circle [8].

In practice, a scan with respect to r is performed and models are extracted for different model orders. A stable – and inversely stable – model was found in practice out of a large set of estimated models if the original data represented a passive system.

3. Experimental verification

The modeling of on-chip inductors and transmission lines are often encountered in practice. Therefore, the proposed method is verified on 2 different sets of data, namely measurement data of an on-chip inductor and simulation data of a low-loss coplanar waveguide. All verifications are performed under the following conditions:

- All simulations are performed using the Spectre simulator of Cadence. The Nport model of Spectre – implementing the method described in [4] – is used to compare the extended state-space model with an ordinary state-space model.
- The extended state-space model is automatically generated for Spectre using an ordinary state-space model (cktrom-model in Spectre), combined with additional

SpectreHDL components to model a symmetric E_1 matrix.

- The extended state-space model is extracted using a weighting which is proportional to the given admittance matrix.

3.1. Modeling a measured inductor

The modeling a measured on-chip $0.18\mu\text{m}$ CMOS UMC inductor is used to demonstrate both the necessity of the extended state-space model and the ability of estimating this extended state-space model using an extended subspace method.

The original data was obtained using 2-port S-parameter measurements from 500MHz till 26GHz. All measured Y -parameters and the residual modeling errors are shown in figure 2. These residual modeling errors are computed as the complex difference between the data and the model.

The inductor is modeled using both the extended subspace and the Nport model available in Spectre. The comparison of the different models is done on the bases of the order of the model, i.e. the number of internal states. Figure 3 shows the maximum and the mean relative error as function of the model order. The filled symbols in the plot indicate that a non-passive model was obtained. The Nport model of the second order is not shown due to AC-simulation problems.

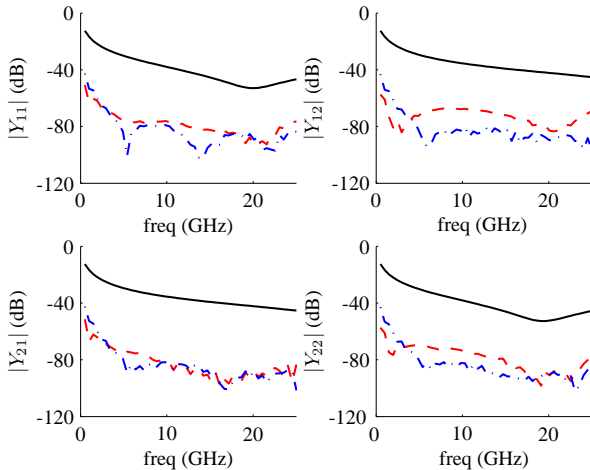


Figure 2. The Y -parameter of a $0.18\mu\text{m}$ CMOS UMC inductor and the modeling errors $|Y - Y_{model}|$ for an 8^{th} order model. Measured data: black —; Extended subspace model: red —; Nport model: blue ·—.

It can be concluded from figure 3 that

- extending the state-space model is important to handle improper models;
- modeling improper models using an ordinary state-space models can result into non-passive models due to the introduced modeling errors;

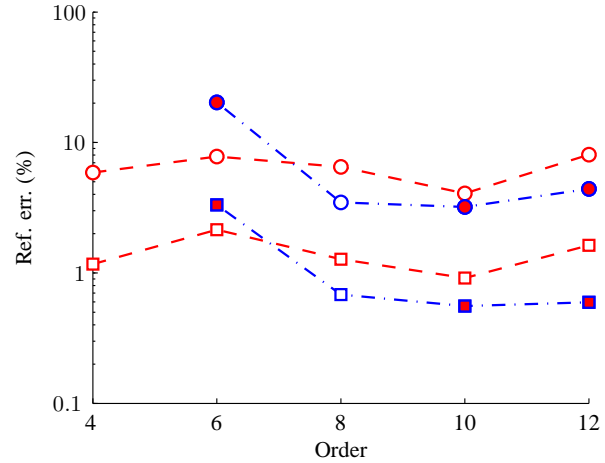


Figure 3. Maximum (○) and mean (□) relative modeling error $|Y - Y_{model}|$ for the on-chip inductor as function of the model order for both the extended subspace model (red —) and the Nport model (blue ·—). Filled symbols represent non-passive models. The 4^{th} order Nport model is not available due to Spectre AC-simulation problems.

3.2. Modeling a Coplanar waveguide

An experiment is set up using simulation data of a coplanar waveguide to analyze the impact of modeling errors. The coplanar waveguide used is a 50Ω transmission line of 10 mm long in a low-loss – Rogers RO3003 – PCB.

All Y -parameters are obtained from DC till 26GHz and are shown in figure 4. This figure clearly shows the high dynamics in the admittance matrices.

The comparison of the modeling error for different models as function of the model order is shown in figure 5. Contrary to the inductor case, it has been found that all Nport models are passive. Using figure 5, it can be concluded that the extended state-space model is also able to model transmission lines accurately.

4. Conclusions

State-space modeling of linear systems is often used to include table-based frequency-dependent transfer functions

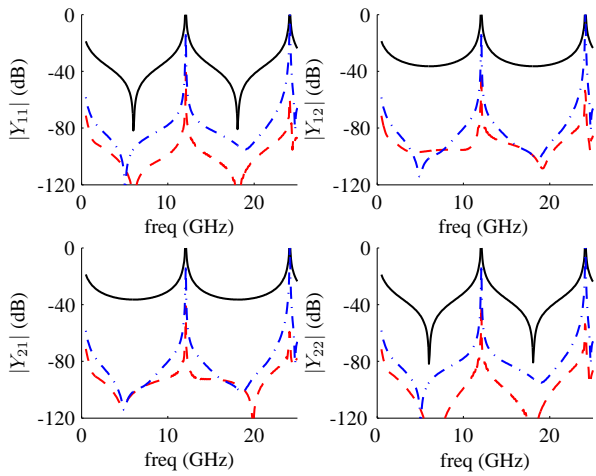


Figure 4. The Y -parameter of a coplanar waveguide and the modeling errors for a 14th order model. Measured data: black —; Extended subspace model: red ---; Nport model: blue -.-.

into transient-like simulators. State-space models are not able to represent all linear systems. They can't represent improper systems such as the admittance matrix of an on-chip inductor.

This paper introduces an extended state-space model which enables the modeling of improper systems. This includes the extraction of system properties (such as poles and zeros) out of the extended state-space model. Afterwards a subspace-based identification algorithm is described to estimate the extended state-space model.

Both the extended state-space model and the extended subspace modeling technique are applied on measurements of an on-chip inductor and on simulations of a coplanar waveguide. They lead to the conclusion that

- the model extension towards improper models leads to lower order models;
- the lack of modeling errors in the extended state-space model influences the passivity of the model: modeling errors introduced in e.g. the inductor can lead to non-passive models for the ordinary state-space model.

Given the above observations it can be concluded that the extended state-space model is a necessity when modeling e.g. inductors when using admittance matrices. It is also observed that the extended subspace method works fine for moderate modeling orders.

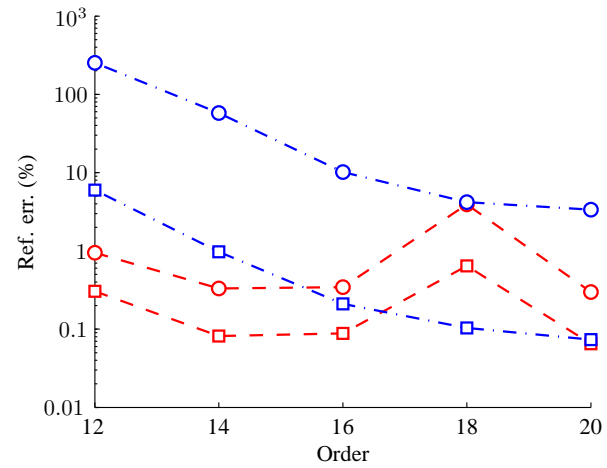


Figure 5. Maximum (○) and mean (□) relative modeling error for the on-chip inductor as function of the model order for both the extended subspace model (red ---) and the Nport model (blue -.-).

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