# Efficient Static Compaction of Test Sequence Sets through the Application of Set Covering Techniques 

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#### Abstract

The test sequence compaction problem is modeled here, first, as a set covering problem. This formulation enables the straightforward application of set covering methods for compaction. Because of the complexity inherent in the first model, a second more efficient, formulation is proposed where the test sequences are modeled as matrix columns with variable costs (number of vectors). Further, matrix reduction rules appropriate to the new formulation, which do not affect the optimality of the solution, are introduced. Finally, the reduced problem is minimized with a Branch \& Bound algorithm. Experiments on a large number of test sets show significant reductions to the original problem by simply using the presented reduction rules. Experimental results comparing our method with others from the literature and also with the absolute minima of the examples, computed separately with the MINCOV algorithm, support the potential of the proposed approach.


## 1. Introduction

The cost of testing a digital system is greatly affected by the length of the set of test sequences applied. Automatic Test Pattern Generation (ATPG) methods for sequential circuits try to find sequences of input vectors (test sequences) that detect all single stuck-at faults [1] in the circuit. Since ATPG is a highly complex task usually very long test sequences are produced and therefore shorter (compact) test sequences are always desirable.

Many compaction procedures, static or dynamic, have been proposed $[1,2,3,4,5,6,7,8,9,12,19]$. Dynamic compaction is applied during test generation. Static compaction is applied after test generation, independently of the particular test generator. Here we shall concentrate on static compaction methods.

Static compaction methods may be divided into those that iteratively fault simulate a single (produced usually
after concatenation) test sequence and into those that try to exploit the existing redundancy in a given set of test sequences (without fault simulation). The Vector Omission [5] method operates on a single sequence and achieves very good compaction ratios, at the expense of long execution times (fault simulation is required at every step). The Vector Restoration [8, 9] method also achieves very high levels of compaction and operates on a single sequence but relies heavily on fault simulation.

In this work we consider static compaction methods that try to exploit the existing redundancy in a given set T of test sequences (test set). This situation is usual in fault oriented [1, 24] ATPG methods which produce each test sequence by targeting a specific fault.

There exist many static compaction methods which operate on a set of test sequences [2, 3, 4, 6, 7]. In [2] the test sequences are statically compacted, without fault simulation, by using a Genetic Algorithm (GA) to find and remove redundant vectors from the sequences. In [5] a procedure named Vector Selection is proposed in which test subsequences for every fault are extracted from a single test sequence. After collecting all subsequences, a set covering method is applied with the purpose of selecting a minimal subset of sequences to detect all faults. The method, however, relies on fault simulation. In [6] the test compaction problem is formulated as a minimization problem and a static compaction method is developed. However, results are presented only for combinational circuits and no details are given for the algorithm developed for sequential circuits. In [7] an exact method based on a Branch \& Bound algorithm is presented.

In this paper we propose an algorithm for the static compaction of test sets for sequential circuits, exploiting the principles of a set-covering model $[10,22]$ without employing fault simulation. Since, however, the transformation of the compaction problem into a setcovering formulation [10, 11, 22] with single costs results in a matrix expansion, this formulation was modified here to account for variable column costs, necessary for the present problem (partial columns may be selected). A set
of reduction rules is then applied aiming at simplifying the initial covering matrix. Finally, a B\&B procedure is proposed to solve the remaining reduced problem.

The paper is organized as follows: In section 2 the compaction problem is formulated as a set-covering problem. In section 3 a modified formulation of the problem and matrix reduction rules are proposed. In section 4 the final compaction algorithm is presented. In section 5 the experimental results concerning the efficiency of the proposed method are presented.

## 2. The compaction problem

Let us consider a set of test sequences $T=\left\{S_{1}, S_{2}, \ldots, S_{n}\right\}$ detecting (covering) the set of faults $F=\left\{f_{1}, f_{2}, \ldots, f_{m}\right\}$ of a sequential circuit. Every test sequence $S_{i}=\left(v_{1}, \ldots, v_{L i}\right)$, $i=1 \ldots n$, is an ordered set of the $L_{i}$ test vectors $v_{1}, \ldots, v_{L i}$, where $L_{i}$ is the length of $S_{i}$. For example, the test set $\mathrm{T}=\left\{\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{3}, \mathrm{~S}_{4}\right\}$ of fig. 1 detects the set of faults $\mathrm{F}=\left\{\mathrm{f}_{1}, \mathrm{f}_{2}, \mathrm{f}_{3}, \mathrm{f}_{4}, \mathrm{f}_{5}, \mathrm{f}_{6}\right\}$ and its sequences have lengths $\mathrm{L}_{1}=7$, $L_{2}=6, L_{3}=7$ and $L_{4}=7$. A fault $f_{i}$ within a sequence $S_{j}$ has a detection $\operatorname{cost} \mathrm{d}_{\mathrm{ij}}$ equal to the number of vectors from the beginning of the sequence until $f_{i}$ becomes detected in $S_{j}$. In fig. 1 , for fault $\mathrm{f}_{1}$ it is $\mathrm{d}_{11}=7, \mathrm{~d}_{12}=2$, and $\mathrm{d}_{14}=3$.

The compaction problem is to find a collection of subsequences, i.e. subsets of vector sequences, so that all faults in F are covered and the test length of the collection is a minimum.


Figure 1. Set of Test Sequences
We may formulate the compaction problem as a set covering problem, as follows: Every test sequence $\mathrm{S}_{\mathrm{i}}=\left(\mathrm{v}_{1}, \ldots, \mathrm{v}_{\mathrm{Li}}\right)$ is expanded into all possible subsequences $S_{i 1}=\left(v_{1}\right), S_{i 2}=\left(v_{1}, v_{2}\right), \ldots S_{i L i}=\left(v_{1}, \ldots, v_{L i}\right)$, i.e. every sequence $S_{i}$ generates $L_{i}(i=1 \ldots n)$ subsequences with lengths $L_{i 1} \ldots$ $\mathrm{L}_{\mathrm{ii}}$. Then, matrix $\mathrm{C}_{\mathrm{mp}}$ is built, with rows the m faults, and $p=\sum_{i} L_{i}$ columns, a column for each subsequence. The matrix element $\mathrm{c}_{\mathrm{ij}}$ takes the value $\mathrm{c}_{\mathrm{ij}}=1$ when subsequence (column) j detects fault (row) i and $\mathrm{c}_{\mathrm{ij}}=0$ otherwise.

It is noted that it is not necessary to expand a sequence to all its possible subsequences. It is sufficient to regard only the detection subsequences, i.e. those subsequences the tails of which detect (cover) at least a fault.

The compaction problem now becomes a problem of
selecting from $\mathrm{C}_{\mathrm{mp}}$ columns (subsequences) of length $\mathrm{L}_{\mathrm{j}}$, covering (detecting) rows $\mathrm{F}_{\mathrm{j}}$ (set of faults), so that:

$$
\bigcup_{j} F_{j}=F \quad \text { and } \quad \sum_{j} L_{j} \text { is minimum }
$$

Many methods have been proposed for solving set covering problems and a long experience exists in that field [10, 11, 23, 25]. For example, many problems in logic synthesis may be formulated as set covering problems, i.e. the minimization of logic functions [10, 11, $20,21,22,23]$, the state minimization of finite state machines [10, 22], etc.

The advantage of the above problem formulation is that it enables the straightforward application of algorithms, readily available from the field of logic synthesis, to solve such problems, for example ESPRESSO [23].

Here, we have used the MINCOV algorithm from ESPRESSO, which is available in source code, to solve the compaction problem as formulated by Matrix $\mathrm{C}_{\mathrm{mp}}$. The set covering problems, however, are inherently NPcomplete, and, with the expansion of the test sequences into subsequences, the size of the covering matrix suffers from exponential blow-up. This is shown in the experiments we performed (section 5) to test the potential of this model.

For the above reasons, we propose, in this work, a more compact formulation of the (modified) covering matrix, as well as certain matrix reduction rules, appropriate to the nature of the present compaction problem (section 3). Following the reductions, a B\&B algorithm is applied to solve the remaining, smaller, problem (section 4).

## 3. Modified formulation and reduction

We propose to model our compaction problem with a modified Covering Matrix $D_{m n}$, the elements $d_{i j}$ of which are extended to deal with variable costs, as follows: The $m$ faults $f_{i}(i=1, \ldots, m)$ form the rows of $D_{m n}$ and the $n$ sequences $S_{j}(j=1, . ., n)$ form its columns. Matrix element $\mathrm{d}_{\mathrm{ij}}$ is a positive integer that represents the detection cost for fault $f_{i}$ in sequence $S_{j}$. The convention $d_{i j}=d_{\infty}$ (a very large integer value) denotes that fault $f_{i}$ is not detected by sequence $S_{j}$. The Matrix $D_{m n}$ for the example of fig. 1 is given in fig. 2 a . When the detection costs $\mathrm{d}_{\mathrm{ij}}$ are not given explicitly they may be computed separately by an initial fault simulation of the given test sequences.

A matrix formulation called Detection Matrix, similar to Matrix $\mathrm{D}_{\mathrm{mn}}$, is proposed in [2], though the compaction approach followed in [2] is different.

The compaction problem now becomes a problem of selecting from each column $j$ of $D_{m n}$ (sequence $j$ ) a subset $F_{j}$ of rows (subsequence detecting faults $F_{j}$ ) with cost $w_{j}$ $\left(\mathrm{w}_{\mathrm{j}}=\max \left(\mathrm{d}_{\mathrm{ij}}\right.\right.$ of $\left.\left.\mathrm{F}_{\mathrm{j}}\right)\right)$ so that:

|  |  |  |  |  | $\sum_{j} w_{j}$ is minimum. |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{S}_{1}$ | $\mathrm{S}_{2}$ | $\mathrm{S}_{3}$ | $\mathrm{S}_{4}$ | $\mathrm{S}_{1}$ | $\mathrm{S}_{2}$ | $\mathrm{S}_{3}$ | $\mathrm{S}_{4}$ |
| $\mathrm{f}_{1}$ | 7 | 2 | - | 3 | 7 | 2 | - | - |
| $\mathrm{f}_{2}$ | 4 | 6 | - | 7 | 4 | 6 | - | - |
| $\mathrm{f}_{3}$ | 3 | - | 7 | - | 3 | - | 3 | - |
| $\mathrm{f}_{4}$ | 3 | - | 2 | - | - | - | - | - |
| $\mathrm{f}_{5}$ | 3 | 5 | - | 6 | - | - | - | - |
| $\mathrm{f}_{6}$ | - | - | 4 | - | - | - | - | - |
| (a) Initial Matrix |  |  |  |  | (b) | med |  |  |
|  | gur | 2 |  |  | vering | Matri |  |  |

Since the size of matrix $\mathrm{D}_{\mathrm{mn}}$ is large, though much smaller than that of $\mathrm{C}_{\mathrm{mp}}$, we borrow techniques from logic synthesis (essentiality [20], row and column dominance [10, 21], partitioning [10, 11] or Gimpel's reduction [10] method) to first try to simplify the problem and afterwards solve the reduced problem. However, in our case, the elements of matrix $\mathrm{D}_{\mathrm{mn}}$ are free to take any positive integer value, so, it is necessary to extend the definitions and the techniques of essentiality and dominance before they can be applied to our problem.

Next, we propose certain rules which, iteratively applied, try to reduce the size of $\mathrm{D}_{\mathrm{mn}}$ while preserving the optimality of the solution.

## Rule 1 (essentiality)

A column j is an essential column, if it is the only column that covers a row $i$ (row $i$ is called essential). If column j is an essential column, then a part of it (subsequence of $\mathrm{S}_{\mathrm{j}}$ ) must be selected in every solution of the problem, with a $\operatorname{cost} \mathrm{w}_{\mathrm{j}}$ at least equal to the $\operatorname{cost} \mathrm{d}_{\mathrm{ij}}$ of that essential fault i.e. $w_{\mathrm{j}} \geq \mathrm{d}_{\mathrm{ij}}$. If more essential faults are covered by column j then $\mathrm{w}_{\mathrm{j}} \geq \mathrm{Z}_{\mathrm{j}}$ where $\mathrm{Z}_{\mathrm{j}}=\max \left(\mathrm{d}_{\mathrm{ij}}\right.$, row i is essential). The rule operates as follows:

For every column j that is identified as essential, we remove every row i with $\mathrm{d}_{\mathrm{ij}} \leq \mathrm{Z}_{\mathrm{j}}$ and change the cost of the remaining rows $\mathrm{p}\left(\mathrm{d}_{\mathrm{pj}}>\mathrm{Z}_{\mathrm{j}}\right)$ to $\mathrm{d}_{\mathrm{pj}}=\mathrm{d}_{\mathrm{pj}}-\mathrm{Z}_{\mathrm{j}}$.

## Rule 2 (row elimination)

Given rows i and p , row p may be removed, without affecting the optimality of the solution, if and only if:

- For all columns j it is: $\mathrm{d}_{\mathrm{ij}} \geq \mathrm{d}_{\mathrm{pj}}$,
- For at least one column k it is: $\mathrm{d}_{\mathrm{ik}}<\mathrm{d}_{\infty}$.


## Rule 3 (column set dominance)

Let column j is covering the set of faults $\mathrm{F}_{\mathrm{j}}=\left\{\mathrm{f}_{0}, \mathrm{f}_{\mathrm{l}}\right.$, $\left.\mathrm{f}_{2}, \ldots, \mathrm{f}_{\mathrm{n}}\right\}$, having costs: $\mathrm{d}_{0, \mathrm{j}} \leq \mathrm{d}_{1, \mathrm{j}} \leq \mathrm{d}_{2, \mathrm{j}} \leq \ldots \mathrm{d}_{\mathrm{n}, \mathrm{j}}$.

Let the set of columns $\mathrm{C}=\left\{\mathrm{k}_{0}, \mathrm{k}_{1}, \ldots, \mathrm{k}_{\mathrm{q}}\right\} \quad(\mathrm{j} \notin \mathrm{C})$ is covering at least $F_{j}$ and let $c_{i}$ be the minimum detection cost for fault $f_{i}$ within $C$.

Then we say that set C dominates j if and only if:

$$
\text { - } \quad \mathrm{d}_{\mathrm{r}, \mathrm{j}} \geq \sum_{\mathrm{i}=0}^{\mathrm{r}} \mathrm{c}_{\mathrm{i}} \text {, for } \mathrm{r}=0,1, \ldots, \mathrm{n}
$$

In this rule, if column set C dominates column j then column j may be removed without affecting the optimality of the solution.
Proof: From the above relations, we have that every fault and every subset of faults covered by column j may be also covered by a proper combination of subsequences from C with an equal or smaller collective cost. Therefore, by removing column j solution optimality is retained.

Rule 3 is applied as follows:
For $\mathrm{j}=1$ to n

1) Let C consist of the $\mathrm{q}=\mathrm{n}-1$ columns ( $\mathrm{j} \notin \mathrm{C}$ ): $\mathrm{k}_{1}$, $\mathrm{k}_{2}, \ldots, \mathrm{k}_{\mathrm{n}-1}$
2) The $\mathrm{q}=\mathrm{n}-1$ columns are replaced with a temporary column c with elements $\mathrm{c}_{\mathrm{i}}$ such that for every fault i covered by the column set C it is $\mathrm{c}_{\mathrm{i}}=\min \left\{\mathrm{d}_{\mathrm{i}, 1}, \mathrm{~d}_{\mathrm{i}, 2}, \ldots, \mathrm{~d}_{\mathrm{i}, \mathrm{n}-1}\right\}$.
3) Column $j$ is checked against $c$ for possible dominance.
The cost of applying the three Rules 1,2 , and 3 on $\mathrm{D}_{\mathrm{mn}}$ is $\mathrm{O}\left(\mathrm{m}^{2} \mathrm{n}\right)$ and becomes smaller as the matrix is reduced.

The order by which the three rules may be applied on the matrix is not important, provided they are cycled until no further reduction is made. The systematic application of the reduction rules will not only reduce the size of the problem but sometimes may lead directly to the optimum solution.

## 4. The compaction algorithm

Our test compaction algorithm, hereafter referred to as DSeqComp (fig. 3), consists of a reduction phase, followed by an exact $B \& B$ algorithm applied to the reduced problem (Reduced Matrix $\mathrm{D}_{\mathrm{mn}}$ ).

```
Input: A matrix \(\mathrm{D}_{\mathrm{mn}} / *\) Covering Matrix */
    Do \{ /** Do Reductions **/
        Find essential columns and add the essential subseq.
        to the solution with all the covered faults; /* Rule 1 */
        Apply row elimination; /*Rule 2 */
        Apply column set dominance; /* Rule 3 */
        If (no reductions are applicable)
            For (all columns \(\neq \mathrm{j}\) ) Apply Rule 3 with \(\mathrm{q}=1\);
    \} While (reductions are applicable);
    If \(\left(\mathrm{D}_{\mathrm{mn}}=\varnothing\right)\) return solution;
    Else enter Branch\&Bound algorithm;
```

Figure 3. Compaction algorithm
In the reduction phase of the algorithm (fig. 3) the rules of section 3 are repeatedly applied on the Covering Matrix $\mathrm{D}_{\mathrm{m} n}$, until the cyclic core is reached. If the reduced matrix becomes empty, then the solution obtained thus far
(reduction rules do not affect solution optimality) is the optimal one.

The $\mathrm{B} \& \mathrm{~B}$ algorithm tries to solve the reduced matrix by exploiting certain bounds to prune the search space.

Although, for the examples presented in section 5, DSeqComp found optimal solutions, in some cases the produced results may not be optimal, due to user imposed constraints (e.g. CPU time, memory).

## 5. Experimental results

The proposed compaction algorithm DSeqComp has been implemented in C. The efficiency of the algorithm was measured by running the ISCAS'89 (and Addendum'93), ITC'99 benchmark circuits [13, 17] on a Pentium III/933 MHz machine with 256 MB . We experimented with the test sequences from [2]. For comparison we have used the results obtained by the compaction method of [2] which is a GA-based method. Also, we have computed, whenever possible, the minimum solution of the given example circuits, using the $\mathrm{B} \& \mathrm{~B}$ algorithm MINCOV of ESPRESSO [10, 23]. Therefore, we have, for most circuits an independent base for comparison on how close the results of our compaction method are to the optimal solution.

The results in Table 1 refer to the GATTO [15] test sequences. Under the heading 'Original problem' columns \#faults and \#seq determine the initial size of the covering matrix and \#vec is the collective length of the \#seq sequences. Next, under 'MINCOV' (wherever feasible) the minimum results (column 'Compacted Set') obtained by applying the MINCOV algorithm to the expanded set of subsequences (column 'Initial Size') are presented. Under 'method [2]' are presented the compaction results obtained [16] from [2] and under 'DSeqComp' are our compaction results where 'Reduced Problem' is the problem resulting after the iterative application of the reduction rules (section 3).

From Table 1 it is seen that:
a) Significant reductions are obtained on all circuits, by only applying our reduction rules. For most of the circuits (e.g. s641 etc.) the reduced matrix is empty, giving directly the optimal solution. For the few remaining circuits, the reduced covering matrix is very small. DSeqComp, compared with method [2], obtains comparable or better results.
b) The sizes of the expanded Covering Matrices (column 'Initial Size'), that MINCOV has to deal with, are very large and contain 9 to 31 times more columns than the original problem. This size increase results in higher memory demands and may cause MINCOV to fail to build the expanded Covering Matrix, as in the case of examples s35932 and s38584 where the memory
demands were higher than 560MB.
Table 2 refers to test sequences produced by HITEC [14]. For 17 circuits out of 22 the reduced matrix is empty, the optimal solution being obtained with the reduction rules only. For four of the remaining circuits the results of DSeqComp are optimal. Here also, MINCOV failed in building the Covering Matrix for s35932.

Results for the application of DSeqComp on larger circuits from the ITC'99 benchmark suite [17] (ARTIST [17] examples b15, b17, b21) and RAGE [18] examples b14, b20) are presented in Table 3. From Table 3 we see that by simply applying the reduction rules the Covering Matrix either becomes empty (column 'Reduced Problem') or is very simplified. The actual running time of DSeqComp (column 'net Time') is very small. MINCOV succeeded in solving only two of the examples.
As we see from Tables 1, 2 and 3, DSeqComp reduces the initial test sets about $50 \%$ on the average.

The speed of DSeqComp is compared in Table 4 with that of the GA compaction algorithm of [2] and with MINCOV, for the examples where MINCOV succeeded in solving the Covering Matrix. Since method [2] was run on a different machine (SUN SPARCstation 5/110) the results from [2] are multiplied by the factor 110/933 (row 'Norm. CPU (sec)'). From Table 4 we see that DSeqComp is orders of magnitude faster (including disk I/O time) as compared to [2] and MINCOV, while it attains comparable or better compaction results.

In our examples we observed, experimentally, that not only the complexity of applying the reduction rules is $\mathrm{O}\left(\mathrm{m}^{2} \mathrm{n}\right)$ but also the execution time of our combined method (reductions + Branch\&Bound) shows polynomial behavior.

## 6. Conclusions

The test set compaction problem without fault simulation is formulated here, first, as a Set Covering Problem. However, because of its complexity, a Modified Covering Matrix is proposed, where the matrix elements indicate the variable costs (number of vectors) of selecting partial columns (subsequences) to cover specific rows (faults). Further, three simplification rules are proposed, which iteratively applied, lead to a new matrix of smaller size, without sacrificing the optimality of the solution of the original problem. To the smaller problem a $B \& B$ minimization algorithm is applied.

Experimental results indicate that the reduction rules alone effect significant reductions on the size of the problem and in many cases they produce directly the optimum solution. The results of our method are compared with results (a) from the literature and (b) with the absolute minima of these test sets as computed by

ESPRESSO, whenever possible, to get a better measure of efficiency. The obtained compaction results achieve, for most examples for which the minima are known, the
optimal solution. Considerable is also the speed of the algorithm.

Table 1. Problem size and results for GATTO Test Sets

| circuit | Original Problem |  |  | MINCOV |  |  |  | method [2] |  | DSeqComp |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Initial Size |  | Compacted Set |  |  |  | Reduced Problem |  | $\begin{aligned} & \text { acted } \\ & \text { et } \end{aligned}$ |
|  | \#faults | \#seq | \#vec. | \#seq. | \#vec. | \#seq | \#vec. | \#seq | \#vec. | \#faultsx\#seq | \#seq | \#vec. |
| s510 | 551 | 37 | 989 | 926 | 18042 | 7 | 239 | 7 | 239 | $5 \times 6$ | 7 | 239 |
| s641 | 407 | 48 | 395 | 432 | 2567 | 24 | 223 | 24 | 223 | $0 \times 0$ | 24 | 223 |
| s713 | 481 | 55 | 557 | 594 | 4051 | 23 | 252 | 23 | 252 | $0 \times 0$ | 23 | 252 |
| s820 | 435 | 38 | 669 | 492 | 7730 | 14 | 349 | 14 | 349 | $0 \times 0$ | 14 | 349 |
| s838 | 389 | 37 | 1323 | 502 | 9315 | 11 | 475 | 11 | 475 | $3 \times 4$ | 11 | 475 |
| s938 | 389 | 37 | 1323 | 502 | 9315 | 11 | 475 | 11 | 475 | $3 \times 4$ | 11 | 475 |
| s953 | 1044 | 75 | 1099 | 1153 | 10720 | 32 | 541 | 32 | 541 | $0 \times 0$ | 32 | 541 |
| s967 | 1019 | 72 | 1223 | 1217 | 13474 | 31 | 671 | 31 | 671 | $5 \times 6$ | 31 | 671 |
| s991 | 857 | 20 | 448 | 334 | 5214 | 9 | 367 | 9 | 367 | $0 \times 0$ | 9 | 367 |
| s1196 | 1200 | 133 | 1805 | 1747 | 19340 | 74 | 1126 | 73 | 1133 | $0 \times 0$ | 74 | 1126 |
| s1238 | 1227 | 133 | 1554 | 1513 | 15671 | 74 | 1006 | 72 | 1009 | $0 \times 0$ | 74 | 1006 |
| s1269 | 1306 | 52 | 450 | 500 | 2868 | 29 | 247 | 29 | 247 | $11 \times 11$ | 29 | 247 |
| s1423 | 1418 | 107 | 2691 | 1892 | 39653 | 28 | 1281 | 28 | 1286 | $0 \times 0$ | 28 | 1281 |
| s1488 | 1422 | 65 | 1824 | 1575 | 38500 | 19 | 948 | 19 | 948 | $0 \times 0$ | 19 | 948 |
| s1494 | 1412 | 62 | 1244 | 1060 | 16816 | 19 | 654 | 19 | 654 | $0 \times 0$ | 19 | 654 |
| s1512 | 774 | 52 | 772 | 718 | 6659 | 14 | 291 | 14 | 291 | $3 \times 4$ | 14 | 291 |
| s3271 | 3188 | 132 | 2529 | 2210 | 23891 | 50 | 1180 | 50 | 1534 | $8 \times 11$ | 50 | 1180 |
| s3330 | 2336 | 108 | 2028 | 1916 | 28824 | 43 | 1069 | 44 | 1069 | $4 \times 5$ | 43 | 1069 |
| s3384 | 3096 | 58 | 888 | 872 | 11503 | 22 | 412 | 22 | 412 | $5 \times 6$ | 22 | 412 |
| s4863 | 4482 | 112 | 1533 | 1475 | 16252 | 41 | 747 | 42 | 748 | $5 \times 7$ | 41 | 747 |
| s5378 | 3271 | 71 | 919 | 920 | 8789 | 42 | 495 | 41 | 495 | $0 \times 0$ | 42 | 495 |
| s6669 | 6507 | 64 | 592 | 656 | 3840 | 36 | 303 | 36 | 305 | $6 \times 7$ | 36 | 303 |
| s13207 | 1994 | 34 | 544 | 518 | 5035 | 9 | 189 | 9 | 189 | $0 \times 0$ | 9 | 189 |
| s35932 | 34302 | 59 | 903 | 880 | 12733 |  |  | 8 | 310 | $0 \times 0$ | 8 | 310 |
| s38417 | 6516 | 95 | 1617 | 1614 | 20376 | 30 | 686 | 31 | 699 | $7 \times 9$ | 30 | 686 |
| s38584 | 18390 | 271 | 8065 | 8334 | 178918 |  |  | 108 | 3891 | $7 \times 9$ | 105 | 3808 |

Table 2. Problem size and results for HITEC Test Sets

| circuit | Original Problem |  |  | MINCOV |  |  |  | method [2] |  | DSeqComp |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Initial Size |  | Compacted Set |  |  |  | Reduced Problem | Com | $\begin{aligned} & \text { acted } \\ & \text { et } \end{aligned}$ |
|  | \#faults | \#seq | \#vec. | \#seq. | \#vec. | \#seq | \#vec. | \#seq | \#vec. | \#faultsx\#seq | \#seq | \#vec. |
| s832 | 750 | 112 | 1057 | 1170 | 8123 | 60 | 619 | 60 | 620 | $0 \times 0$ | 60 | 619 |
| s838 | 382 | 52 | 675 | 335 | 4365 | 12 | 312 | 12 | 312 | $6 \times 6$ | 12 | 312 |
| s938 | 382 | 52 | 675 | 335 | 4365 | 12 | 312 | 12 | 312 | $6 \times 6$ | 12 | 312 |
| s953 | 974 | 111 | 825 | 935 | 5661 | 38 | 406 | 38 | 406 | $0 \times 0$ | 38 | 406 |
| s967 | 939 | 120 | 831 | 950 | 5681 | 38 | 409 | 38 | 409 | $0 \times 0$ | 38 | 409 |
| s991 | 825 | 50 | 83 | 133 | 312 | 25 | 48 | 25 | 48 | $0 \times 0$ | 25 | 48 |
| s1196 | 1200 | 189 | 509 | 698 | 2259 | 110 | 339 | 109 | 341 | $0 \times 0$ | 110 | 339 |
| s1238 | 1241 | 191 | 513 | 704 | 2291 | 109 | 334 | 108 | 334 | $0 \times 0$ | 109 | 334 |
| s1269 | 1148 | 67 | 254 | 322 | 1192 | 26 | 138 | 25 | 157 | $0 \times 0$ | 26 | 138 |
| s1423 | 987 | 50 | 282 | 304 | 2119 | 17 | 189 | 16 | 189 | $0 \times 0$ | 17 | 189 |
| s1488 | 740 | 24 | 69 | 93 | 288 | 16 | 58 | 16 | 58 | $0 \times 0$ | 16 | 58 |
| s1494 | 1217 | 60 | 523 | 573 | 3866 | 43 | 420 | 43 | 426 | $0 \times 0$ | 43 | 420 |
| s3271 | 3054 | 61 | 984 | 889 | 10142 | 19 | 489 | 22 | 491 | $3 \times 4$ | 19 | 489 |
| s3330 | 2274 | 133 | 763 | 890 | 4148 | 86 | 550 | 85 | 550 | $0 \times 0$ | 86 | 550 |
| s3384 | 3042 | 18 | 211 | 212 | 2795 | 9 | 166 | 8 | 166 | $0 \times 0$ | 9 | 166 |
| s4863 | 4404 | 106 | 375 | 463 | 1733 | 58 | 258 | 57 | 258 | $0 \times 0$ | 58 | 258 |
| s5378 | 3061 | 95 | 250 | 345 | 993 | 49 | 154 | 49 | 154 | $0 \times 0$ | 49 | 154 |
| s6669 | 6493 | 68 | 466 | 534 | 3379 | 23 | 261 | 22 | 261 | $0 \times 0$ | 23 | 261 |
| s13207 | 1712 | 15 | 96 | 112 | 621 | 6 | 59 | 6 | 59 | $0 \times 0$ | 6 | 59 |
| s35932 | 34136 | 376 | 1712 | 2084 | 13835 |  |  | 13 | 246 | $0 \times 0$ | 11 | 244 |
| s38417 | 4210 | 281 | 805 | 1087 | 3782 | 15 | 133 | 14 | 133 | $0 \times 0$ | 15 | 133 |
| s38584 | 11448 | 48 | 509 | 557 | 6059 | 30 | 437 | 30 | 437 | $0 \times 0$ | 30 | 437 |

Table 3. Problem size and results for ARTIST-RAGE Test Sets

| circuit | Original Problem |  |  | MINCOV |  |  |  | DSeqComp |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Initial Size |  | Compacted Set |  | Reduced <br> Problem\#faultsx\#seq | Compacted Set |  |  |
|  | \#faults | \#seq | \#vec. | \#seq | \#vec. | \#vec. | Time (s)/net Time(s) |  | \#seq | \#vec. | Time (s)/net Time(s) |
| b14 | 10503 | 105 | 7715 | 5752 | 269174 | - | - | $0 \times 0$ | 39 | 2996 | 1.20 / 0.03 |
| b15 | 7146 | 59 | 2733 | 1323 | 26111 | 1359 | 24.5 / 4.4 | $0 \times 0$ | 36 | 1359 | 1.25 / 0.05 |
| b17 | 9591 | 69 | 1197 | 980 | 14829 | 694 | 52.9 / 7.24 | $0 \times 0$ | 24 | 694 | 4.06 / 0 |
| b20 | 20421 | 142 | 10932 | 9754 | 415166 | - | - | $3 \times 4$ | 78 | 5517 | 3.46 / 0.06 |
| b21 | 22010 | 81 | 7015 | 5569 | 406820 | - | - | $0 \times 0$ | 40 | 5265 | 2.08 / 0 |

Table 4. CPU time results

|  | GATTO Test Set |  | HITEC Test Set |  | GATTO Test Set |  | HITEC Test Set |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | method [2] | DSeqComp | method [2] | DSeqComp | MINCOV | DSeqComp | MINCOV | DSeqComp |
| Total Time ( $\mathrm{CPU} \mathrm{sec)}$ | 8495 | 12 | 11325 | 19.4 | 409 | 3.25 | 102.75 | 3.6 |
| Norm. CPU (sec) | 1002 | 12 | 1335 | 19.4 | 409 | 3.25 | 102.75 | 3.6 |
| Norm. to DSeqComp | 83 | 1 | 69 | 1 | 126 | 1 | 29 | 1 |

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