Analysis and White-Box Modeling of Weakly Nonlinear Time-Varying Circuits

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Abstract

The architectural study of wireless communication systems typically requires simulations with high-level models for different analog and RF blocks. Among these blocks, frequency-translating devices such as mixers pose problems in RF circuit simulation since their response typically covers a mix of long- and short-time scales. This paper proposes a technique to analyze and model nonlinear frequency-translating RF circuits such as up- and downconversion mixers. The proposed method is based on a generalized Volterra series approach for periodically time-varying systems. It enables a multi-tone distortion analysis starting from a circuit description and derives simplified high-level models based on the most important nonlinear contributions. These models give both insight in the nonlinear behavior and enable an efficient high-level simulation during architectural design of front-ends of RF transceivers.

1 Introduction

To improve the performance of wireless transceivers a detailed architectural study of the analog front-end of such transceivers is very important. Indeed, architectural improvements often have a more considerable impact on the overall system performance than optimizations on the individual blocks of the front-end. To enable an accurate architectural analysis, several simulation tools are available, ranging from general-purpose mathematical programs such as MATLAB [1] to extensions of analog or digital simulators [2] or very dedicated approaches [3][4]. The nonlinear behavior of the analog circuits complicates the modeling and the simulation of analog architectures at a level that is higher than the transistor level. Moreover, nonlinear behavior is much more difficult to interpret and to understand than linear behavior.

Signal paths in wireless transceiver front-ends are usually weakly nonlinear: ideally, they are linear, but due to inevitable curvatures of device characteristics, a slight deviation from linear behavior results, giving rise to crosstalk, intermodulation distortion, desensitization, … For high-level modeling of weakly nonlinear behavior of circuits such as amplifiers and active filters, a modeling approach has been presented in [5]. This approach is based on Volterra series, which goes further than the classically used Taylor series, since Volterra series can take into account frequency dependence of the nonlinear behavior in an elegant way. Moreover, the approach of [5] decomposes the nonlinear behavior observed at the output into different contributions. By keeping only the contributions of the nonlinearities that dominate the overall nonlinear circuit behavior, a compact high-level model can be generated that both yields insight and can be evaluated efficiently during simulations.

This paper describes an extension of the high-level modeling approach mentioned above to weakly nonlinear periodically time-varying (WNPTV) systems such as downconversion and upconversion mixers. Again, the result is a high-level model that is suitable for interpretation and efficient simulation. To this end, a mixer is considered as a circuit that behaves in a weakly nonlinear way with respect to the input signal, but it varies in a periodic way with the cycles of the local oscillator (LO). In this way a mixer is a periodically time-varying system and the input signal is considered as a small excursion around a periodically varying operating point. With this way of looking at a mixer, it is not problematic that the mixer behaves in a strongly nonlinear way with respect to the LO (which is usually the case in integrated active RF mixers). On the other hand, if one would try to model a mixer as a two-input system with the LO as one of the inputs, then a (multiple-input) Volterra series [6] would diverge if one wants to describe the strongly nonlinear behavior due to a large LO amplitude.

To describe the behavior of linear periodically time-varying (LPTV) systems, that are linear with respect to the signal input (RF input for a downconverter, low-frequency input for an upconverter), the so-called large-signal small-signal analysis or conversion matrix method [7] has been used for many years. This method can be extended to time-varying Volterra series to describe the weakly nonlinear behavior with respect to the input signal. Such series describe small excursions (due to the signal input) around the steady state that is due to the periodic excitation (here the LO) only. Just as with time-invariant Volterra series, the frequency dependence of the nonlinear behavior can be taken into account. The first term of a time-varying Volterra series corresponds to the solution of large-signal small-signal analysis, just as the first term of a time-invariant Volterra series corresponds to the response computed with an AC analysis.

Time-varying Volterra series have been used already for the analysis of mixers [8], sampled-data systems [9] and even for black-box modeling [10] and for efficient circuit-level simulations [11]. However, the combination of generation of high-level models that can be evaluated efficiently, with insight in the nonlinear behavior, as described in this paper, is a new contribution in the field of time-varying Volterra series.

The modeling method described in this paper starts from a SPICE-like netlist of a circuit and generates a high-level simulation model of the circuit. This model consists of static nonlinearities (x^2, x^3 and ideal multipliers) and linear time-varying transfer functions. The latter can in turn be represented as a combination of ideal multipliers and linear time-invariant transfer functions.

As will be shown with the example of a simple balanced mixer, our method also identifies the dominant nonlinear contributions, which gives more insight in the nonlinear circuit behavior and enables to construct simplified high-level models based only on the most important contributions with a prescribed error. These models can be used for efficient high-level simulation of the RF front-end using a high-level simulator.

The outline of the paper is as follows. In section 2 time-varying nonlinearities are discussed. Section 3 describes the concept of a time-varying transfer function and deals with its high-level modeling in terms of linear transfer functions and static nonlinearities. The extension to weakly nonlinear behavior is made in section 4, while section 5 contains its application to a single-balanced downconversion mixer.

2 Description of time-varying nonlinearities

A circuit contains many nonlinear elements that are each described by a model equation. This model equation typically expresses the current through the nonlinear element as a function of the controlling voltage(s). For our analysis, the nonlinear model equations of each component are expanded in a power series around the periodic large-signal voltages. For the collector current of a bipolar transistor this yields

$$i_c(t)=\sum_{m=1}^{\infty}\sum_{n=0}^{m}\left[\left.\frac{\partial^m I_c(V_{be},V_{ce})}{\partial V_{be}^n\partial V_{ce}^{m-n}}\right|_{\substack{V_{be}=V_{beLO}(t)\\V_{ce}=V_{ceLO}(t)}}\cdot\frac{v_{be}^n(t)}{n!}\cdot\frac{v_{ce}^{m-n}(t)}{(m-n)!}\right]. \quad (1)$$

where the periodic large-signal voltages $V_{beLO}(t)$ and $V_{ceLO}(t)$ define the time-varying operating point of a transistor. The small-signal voltages $v_{be}(t)$ and $v_{ce}(t)$ represent the RF components of the controlling voltages. For convenience we rewrite the above expression as

$$\begin{aligned}i_c(t)&=g_m(t)v_{be}(t)+g_o(t)v_{ce}(t)+K2_{g_m}(t)v_{be}^2(t)+K2_{g_o}(t)v_{ce}^2(t)\\&+K3_{g_m}(t)v_{be}^3(t)+K3_{g_o}(t)v_{ce}^3(t)+K2_{g_m\&g_o}(t)v_{be}(t)v_{ce}(t)+\ldots,\end{aligned} \quad (2)$$

where the time-varying first-order derivative terms with respect to v_{be} and v_{ce} are the small-signal parameters $g_m(t)$, and $g_o(t)$ of a bipolar transistor. The coefficients *K2*.. and *K3*.. in equation (2) are referred to as second- and third-order nonlinearity coefficients since they are proportional to the second- and third-order derivative of the collector current i_C. The coefficients can be also classified as a one-, two- or higher dimensional, depending on the number of controlling voltages. The same process can be followed for a nonlinear capacitor with a nonlinear characteristic $Q = f_q(V(t))$. Here the current is found by differentiating with respect to time. For all nonlinear elements we limit the expansion to the order three, which is a reasonable assumption in most practical cases. However, the described approach remains valid for nonlinear behavior of order higher than three.

The determination of the higher-order time-varying nonlinearity coefficients, *K2*… and *K3*…, represents the first step in the analysis. Commercial simulators usually provide only the first-order time-invariant coefficients, which are in fact the well-known small-signal parameters. To obtain the time-varying coefficients up to order three, we derived the higher-order derivatives in a symbolic algebra program [12] starting from the model equations. The resulting expressions for the small-signal parameters and *K2*… and *K3*… are then evaluated for the steady-state response of their controlling voltages that is found using the shooting method or harmonic balance method depending on the circuit simulator used.

Every nonlinearity coefficient *K2*… gives rise to a contribution to the second- and third-order time-varying response and the coefficients *K3*… each yield a contribution to the third-order response. The second-order contributions are given by the product of the coefficient *K2*… with the (time-varying) transfer function from the input to the controlling voltage(s) of each nonlinear element, and with the (time-varying) transfer function from a fictitious current source between the terminals of the nonlinear element to the output of the circuit. For the third order the transfer function from the input to the controlling voltage(s) of each nonlinear element should be replaced by the second-order time-varying response at the controlling voltage(s).

3 Time-varying transfer functions

As mentioned in the end of the previous section, the determination of responses of order one, two and three with respect to the input signal, it is necessary to determine time-varying linear transfer functions in the circuit under consideration. The large-signal-small-signal analysis [7] covers the analysis of linear time-varying circuits. It introduces the concept of the linear time-varying operator called *conversion matrix*. The conversion matrix performs a frequency translation in addition to the normal frequency dependency of a transfer function. In an LPTV circuit a small-signal input at a frequency ω_{RF} yields a response at frequencies that are offset by multiples of the fundamental frequency.

Let us denote $\omega_0 = |\omega_{RF}-\omega_{LO}|$ and introduce the indexing with respect to an operational frequency with the following meaning. A symbol $\boldsymbol{S}_n$ stands for complex amplitude of a signal $\tilde{\mathbf{s}}$ at the frequency $\omega_n = \omega_0 + n\omega_{LO}$. The symbol tilde above the name of a signal stresses the periodically time-varying nature of the signal. We use this notation also for the naming of time-varying transfer functions to distinguish them from simple (time-invariant) transfer functions.

Suppose we are interested in a time-varying transfer function between a small-signal input $\tilde{\mathbf{x}}$ (an input voltage or current) and small-signal response $\tilde{\mathbf{y}}$ (an output voltage or current). Typically it is a transfer function between an input excitation of a circuit and controlling nodes of a nonlinear element, or a transfer function between an output current of a nonlinear element and a circuit output of interest. Limiting the highest multiple of ω_{LO} to K allows us to represent the wanted relation between $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}$ by the following matrix equation

$$\tilde{\mathbf{y}}(\omega) = \tilde{\mathbf{H}}(\omega)\tilde{\mathbf{x}}(\omega), \qquad (3)$$

where

$$\begin{aligned} \tilde{\mathbf{x}}(\omega) &= [\boldsymbol{X}_{-K}, \boldsymbol{X}_{-K+1}, \ldots, \boldsymbol{X}_0, \ldots, \boldsymbol{X}_{K-1}, \boldsymbol{X}_K]^T \\ \tilde{\mathbf{y}}(\omega) &= [\boldsymbol{Y}_{-K}, \boldsymbol{Y}_{-K+1}, \ldots, \boldsymbol{Y}_0, \ldots, \boldsymbol{Y}_{K-1}, \boldsymbol{Y}_K]^T \end{aligned} \qquad (4)$$

are column vectors of complex amplitudes at the various harmonic offsets. $\tilde{\mathbf{H}}(\omega)$ is the time-varying transfer function (conversion matrix) with the following structure

$$\tilde{\mathbf{H}}(\omega) = \begin{bmatrix} \boldsymbol{H}_{-K,-K} & \boldsymbol{H}_{-K,-K+1} & \cdots & \boldsymbol{H}_{-K,K} \\ \boldsymbol{H}_{-K+1,-K} & \boldsymbol{H}_{-K+1,-K+1} & & \boldsymbol{H}_{-K+1,K} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{H}_{K,-K} & \boldsymbol{H}_{K,-K+1} & \cdots & \boldsymbol{H}_{K,K} \end{bmatrix}, \qquad (5)$$

where each element $\boldsymbol{H}_{kl}$ represents a transfer function that converts an input signal at frequency $\omega_l=\omega_0+l\omega_{LO}$ to an output signal at frequency $\omega_k=\omega_0+k\omega_{LO}$. Computing the conversion matrices is conceptually straightforward for a given time-varying operating point and a linearization [13].

As stated earlier, the main goal of our modeling procedure is to derive a high-level model that can be efficiently used in high-level simulations of front-end architectures. It will be shown later that this model can be constructed using a limited set of basic buildings blocks, namely linear transfer functions and static nonlinearities. We will now prove that a time-varying transfer function can be realized as a combination of linear time-invariant transfer functions and ideal multipliers.

At first, suppose the simple case when the vector of the input signal has only a nonzero value at the input frequency $\omega_{RF}=\omega_1=\omega_0+1\cdot\omega_{LO}$.

$$\tilde{\mathbf{x}} = [0,\ldots,0, \boldsymbol{X}_1, 0,\ldots,0]^T \qquad (6)$$

Substituting (6) into (3) gives

$$\tilde{\mathbf{y}} = \tilde{\mathbf{H}}_1 \cdot \boldsymbol{X}_1 = [\boldsymbol{H}_{-K,1}, \boldsymbol{H}_{-K-1,1}, \ldots, \boldsymbol{H}_{K,1}]^T \cdot \boldsymbol{X}_1 \qquad (7)$$

where the transfer function $\tilde{\mathbf{H}}_1$ is the column vector with dimension $(2K+1)\times 1$ that is selected from the complete conversion matrix (5) by vector (6). This can be represented by the block diagram of Fig. 1.

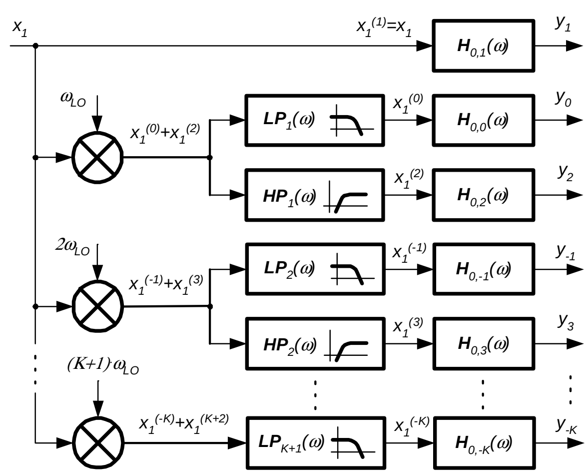

Figure 1. Internal block diagram structure of a time-varying transfer function $\tilde{\mathbf{H}}_1$.

The meaning of the signal notations used in the block diagram of Fig. 1 is the following

$$\begin{aligned} x_1 &= 0.5\cdot\left(\boldsymbol{X}_1 \exp(j\omega_1 t) + \boldsymbol{X}_1^* \exp(-j\omega_1 t)\right) \\ x_1^{(n)} &= 0.5\cdot\left(\boldsymbol{X}_1 \exp(j\omega_n t) + \boldsymbol{X}_1^* \exp(-j\omega_n t)\right) \\ y_n &= 0.5\cdot\left(\boldsymbol{Y}_n \exp(j\omega_n t) + \boldsymbol{Y}_n^* \exp(-j\omega_n t)\right) \end{aligned} \qquad (8)$$

for $n = -K,-K+1,\ldots,0,\ldots,K-1,K$. The frequency translation is realized by mixing the input signal with the auxiliary unit signal (at frequencies equal to the appropriate multiple of ω_{LO}) and applying the set of auxiliary filter blocks - lowpass $\boldsymbol{LP}_n$ and highpass $\boldsymbol{HP}_n$ with edge frequency ω_n - to separate the required mixing products. This block diagram can be simplified when used in high-level simulation environments that are able to treat the signal in frequency band around each carrier of interest separately [4], so-called *multi-carrier* representation of a signal. Then the auxiliary lowpass and highpass filters can be omitted.

A more complex situation occurs when the input excitation vector $\tilde{\mathbf{x}}$ has nonzero elements at all frequency positions. It is characteristic for a transfer function from an output of a nonlinear element (usually represented by a nonlinear current) to a circuit output node of interest. The block structure of the model is displayed in Fig. 2. It exploits repetitively the model structure of Fig. 1 to carry out the transformation represented by the complete time-varying transfer function $\tilde{\mathbf{H}}(\omega)$.

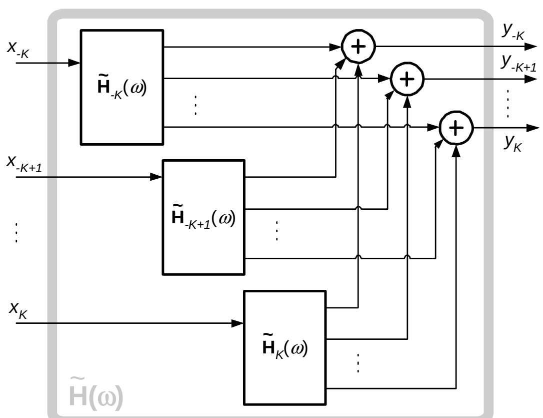

Figure 2. Internal block diagram structure of time-varying transfer function $\tilde{\mathbf{H}}(\omega)$.

4 Nonlinear modeling

The modeling approach proposed in this paper is based on Volterra series theory [6]. Extending these fundamentals for WNPTV systems leads to the time-varying Volterra functional series that establishes a relationship between the input *x(t)* and the output *y(t)*:

$$y(t) = \mathrm{H}_1[x(t)] + \mathrm{H}_2[x(t)] + \ldots + \mathrm{H}_n[x(t)] + \ldots, \tag{9}$$

where the time-domain operator H_n is equal to

$$\mathrm{H}_n[x(t)] = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} h_n(t, \tau_1, \ldots, \tau_n) x(\tau_1) \ldots x(\tau_n) d\tau_1 \ldots d\tau_n \tag{10}$$

The multidimensional function $h_n(t, \tau_1, \tau_2, \ldots, \tau_n)$ is called the n^{th}-order (time-varying) Volterra kernel [11]. According to equation (9) a WNPTV system can be represented by a block diagram (see Fig. 3).

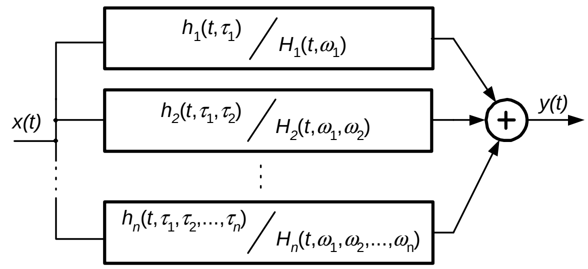

Figure 3. Schematic block representation of a WNPTV system: each path corresponds to a term of the time-varying Volterra series.

This scheme is formed by the parallel combination of blocks (operators) that can be described either in the time-domain (see equation (10)) or in the frequency-domain by means of the so-called *multifrequency network function* $H_n(t, \omega_1, \omega_2, \ldots, \omega_n)$

$$H_n(t, \omega_1, \ldots, \omega_n) = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} (h_n(t, \tau_1, \ldots, \tau_n) \cdot \\ \cdot e^{-j\omega_1(t-\tau_1)} \ldots e^{-j\omega_n(t-\tau_n)} d\tau_1 \ldots d\tau_n \tag{11}$$

We adopted the Volterra theory described in [6] to model the n^{th}-order time-varying Volterra kernel. Consider a second-order time-varying nonlinearity that contributes to the overall second-order harmonic and intermodulation distortion. The first-order response of the voltage or current that controls the nonlinearity is computed first. This first-order response is computed using the conversion matrix method on the linearized time-varying circuit. This response is then squared by the second order nonlinearity and multiplied by the time-varying nonlinearity coefficient to produce a second-order time-varying "nonlinear" current. The resulting signal propagates further to the output of the linearized time-varying circuit. This reasoning holds for all second-order time-varying nonlinearity coefficients in the circuit.

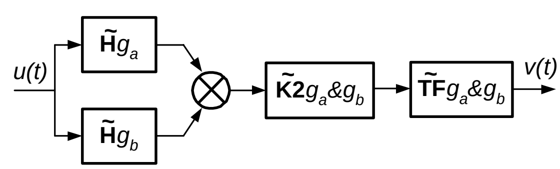

Figure 4. Block diagram of the $K2g_a\&g_b(t)$ contribution to the 2nd order nonlinear behavior.

The block diagram in Fig. 4 shows the signal path through which the second-order two-dimensional (time-varying) nonlinearity coefficient $K2g_a\&g_b(t)$ contributes to the overall second-order nonlinear behavior (in terms of the frequency $\omega_0 = |\omega_{RF}-\omega_{LO}|$). The transfer function from the input to the controlling voltage of the nonlinearity is denoted by $\tilde{\mathbf{H}}_{\ldots}$. Transfer functions from a second-order current to the output of the circuit are denoted by $\widetilde{\mathbf{TF}}_{\ldots}$. They are computed along the RF paths in the time-varying linearized circuit. They are modeled by blocks with the inner structure discussed in section 3 and depicted in Fig. 1 or Fig. 2, depending on the form of their input excitation.

The kernel model comprises many parallel paths, one for each time-varying nonlinearity coefficient. The structure of each path can be different depending on the type of nonlinearity. Generally each path consists of time-varying transfer functions, time-varying nonlinearity coefficients and static nonlinearities (e.g. x^2, x^3 or an ideal multiplier).

The large number of nonlinearities in a practical circuit results in a complex time-varying high-level model. Fortunately, many nonlinearities give a negligible contribution in practice. This is exploited in a pruning algorithm that eliminates all negligible contributions up to a user-definable error on the magnitude and phase of the kernel transforms in a given frequency band of interest.

A translation of the dominant contributions into a block diagram yields the final model in the form of relatively simple block diagram with a limited number of static nonlinearities and linear time-invariant transfer functions.

During simulation we often deal with carrier-modulated signals that have to be sampled at a rate that is at least twice the highest frequency in the signal spectrum. Requirements for efficient high-level simulation RF systems in conjunction with digitally modulated signals are the main motivation for transformation building blocks of a high-level model described in s-domain to their *lowpass equivalents* [14] and their description in *z-domain*. The complex envelope method uses complex lowpass equivalent signals. If the signal spectrum around a carrier at frequency f_c is contained in the band $<f_c-B/2,\ f_c+B/2>$, then with a complex lowpass representation the sampling frequency only needs to be B or higher. The high-level models of time-varying systems discussed above can also be represented in a complex lowpass format.

To summarize, the proposed modeling approach can be summarized as follows:

1. Determine the time-varying nonlinearity coefficients and the linear time-varying transfer functions from the input to the voltages that control nonlinear elements and from a fictitious current source in parallel with each nonlinearity to the output and the controlling voltages.
2. Determine the most important nonlinear contributions.
3. Construct the simplified high-level model in the analog s-domain according to the user-definable error on the magnitude and phase of each kernel transform.
4. Transform the s-domain representation of the linear transfer function into a complex lowpass representation and with digital filters if a dataflow simulation model is required.

5 Example: single-balanced BiCMOS mixer

The technique described in this paper, that has been prototyped in MATLAB, is illustrated with the analysis of a single-balanced 0.35µm BiCMOS downconversion mixer (see Fig. 5).

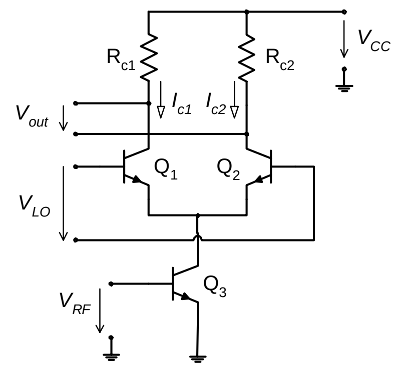

Figure 5. A single-balanced downconversion mixer.

In this circuit the 0 dBm LO signal at 1 GHz downconverts the RF signal (5 carriers in a 50 MHz frequency band, total power of –25dBm) to the frequency band <100 MHz, 150 MHz>. The mixing operation is actually performed by the transistors Q1 and Q2. If their base would be excited by a small LO signal, then the nonlinear distortion in this mixing operation would be mainly caused by the nonlinearity of the collector current of Q1 and Q2. However, the large amplitude of the LO signal makes Q1 and Q2 switch from cutoff to the forward active region and further into saturation. As a result, additional contributions play a role (see Fig. 7): since the transistor beta, averaged over an LO cycle is fairly small, the nonlinearity of the base current, modeled by a nonlinear r_π (= $1/g_\pi$) and C_π, also plays a role. Further, the nonlinear dependence of C_π on V_{bc}, which becomes significant when the transistor saturates, also has a significant influence.

If no mismatches between Q_1 and Q_2 are present, then the nonlinear even-order contributions arising from the two transistors are identical, apart from the sign, such that they cancel. With mismatches this cancellation is not perfect anymore. As a result, the wanted downconverted signal between 50 MHz and 150 MHz is corrupted by the second- and third-order nonlinear responses between DC and 300 MHz. This observation is also evident with Spectre RF simulations. The accuracy of our approach is validated by the comparison of the 1st- and 2nd-order signal response with Spectre RF results in the frequency band of interest (see Figure 6.).

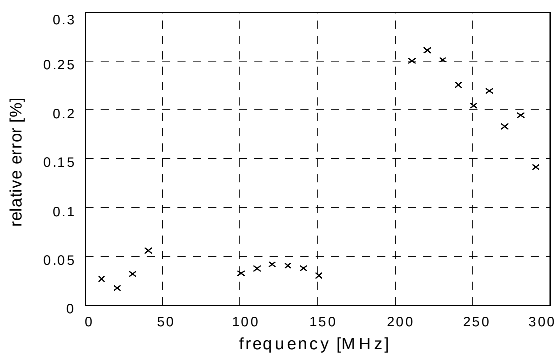

Figure 6. Relative error of the1st-and the 2nd-order signal magnitudes

The above simulation takes into account all nonlinearities in the circuit. Next an approximated model is derived that only contains the dominant contributions to the nonlinear behavior. The dominant contribution of Q1 and Q2 can be combined, leading to a simpler model (see Fig. 7). The time-varying block denoted $j\mathbf{\Omega}$ performs a time-derivative operation in the frequency domain (the block models a diagonal matrix $(2K+1)\times(2K+1)$ with diagonal elements – transfer functions – equal to $\boldsymbol{H}_{nn}(\omega) = = j\cdot\omega_n = j\cdot(|\omega\text{-}\omega_{\mathrm{LO}}|+n\cdot\omega_{\mathrm{LO}})$ for n=-K,-K+1,…,0,..,K-1,K).

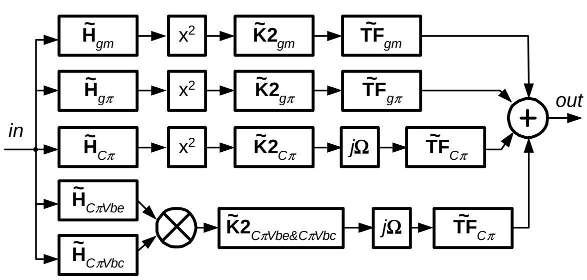

Figure 7. High-level model for the 2nd order nonlinear behavior contributed by transistor Q_1 or Q_2 (the time-varying blocks are translated into a combination of time-invariant blocks).

A complete mixer model for the second-order nonlinear behavior (to model e.g. signals at $f_{RF1}+f_{RF2}$-$2*f_{LO}$) that has an accuracy of 3% compared to the exact Volterra kernel, has been derived with our method and it also contains a contribution from Q_3, which is dominant in the absence of mismatches.

Finally, the high-level model of Fig. 7 can be transformed to a complex lowpass equivalent model and used e.g. in efficient dataflow simulations [4] with an arbitrary *RF* signal excitation, such as a wideband digitally modulated RF signal

6 Conclusions

Due to their nonlinear and time-varying nature, it is difficult to understand and to model the behavior of mixers, that are used in almost any wireless transceivers. An analysis and modeling technique is presented here to describe the weakly nonlinear behavior of such circuits. The procedure starts from a netlist circuit description and determines the dominant nonlinear contributions to the overall nonlinear behavior of the appropriate order. This information can be used by the designers to obtain insight into the nonlinear behavior of the circuit. As evidenced by an analysis of a simple 0.35µm BiCMOS downconversion mixer, this approach enables to construct simplified high-level models for efficient high-level simulations of RF communication systems excited with digitally modulated signals.

7 References

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