Approximation Approach for Timing Jitter Characterization in Circuit Simulators

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Abstract

A new computational concept of timing jitter is proposed that is suitable for exploitation in circuit simulators. It is based on the approximation of computed noise characteristics. To define jitter value the parameter representation is used. The desired parameters are obtained after noise simulation process in time domain by minimization of integral residual L_2-norm. The approach is illustrated by examples of jitter computation using Spice-like simulator.

1. Introduction

Computation of timing jitter is an important problem of practical design for wide variety of circuits. Estimation of jitter allows to predict the timing instability [1-5].

The development of numerical procedures for noise simulation is an open problem. Numerical procedures of timing jitter computation at the transistor level have been investigated in detail for autonomous circuits [8]. Some general theories to estimate timing jitter or phase noise have been developed for oscillators (see for instance [3-10]). The decomposition of the full noise into amplitude and phase noise (jitter) contributions is ambiguous even for autonomous circuits [9,14,15]. But in this case the definition of jitter is in a good agreement as a rule with physics of circuit behavior. The extension of these results to the case of arbitrary circuit is not clear. In particular this is related with the absence of the generalized jitter definition as a probabilistic characteristic that will be invariant for circuit applications. As a rule the individual intuitive definition for each class of circuits is used. The different definitions are applied even to oscillators circuits (see for instance [2,4]).

To implement jitter analysis in conventional simulation packages it is desired to use the universal computational definition that is well suited to exploited standard numerical procedures.

In this paper the formalized approach to define jitter metric is presented that is based on the approximation representation of phase functions. The obtained expressions correspond to the transient noise simulation technique using spectral decomposition of stochastic process [12,16].

In section 2 the limitations of known jitter definitions are discussed. In section 3 the suggested jitter definition is considered. The numerical procedures based on introduced definition are presented in 4. The examples of application are given in section 5.

2. Motivation

Time instability can be characterized by timing jitter *j(t)* or phase noise in frequency domain [1,8,11]. Timing jitter is defined at qualitative level as a random variation in the sampling phase [1-5,11].

Let us to consider the natural definition [1,3,5] of timing jitter and its metric J that is widely used in analytical and numerical techniques for jitter characterization. This definition leads to the computation of J at time points of threshold crossing using the following formulae [5]:

$$J^2 = var(j(t_q)) = \frac{var[y(t_q)]}{S_q^2} \qquad (1)$$

where y is the total noise response and S_q is the slewrate or the value of large signal time derivative at the output node and t_q is the expected time of switching.

This definition is successfully used in applications but it has some essential disadvantages:

- the definition (1) leads to the dependency of the local jitter estimate on choosing of the time point t_q from the considered interval T;

- it is based on the variance of the total noise and does not take into account the dominant role of low frequency noise in timing jitter.

For this reason a new jitter definition is desired that provides the universality for wide class of circuits and is suitable for jitter computation by conventional Spice like simulators.

3. Interval Jitter Metric

In comparison with other works on separation of the phase noise component from the total noise (see, for instance [13,15,17]) the proposed below approach for the characterization of timing jitter is based on the concept of approximation of the noise perturbed response.

Let the nonlinear circuit be described by the following system of equations:

$$\dot{q}(v(t)) + i(v(t)) + b(t) + Au(t) = 0 \tag{2}$$

where *q(v), i(v)* are vectors of node charges or fluxes, *b(t), u(t)* are vectors of large signals and noise sources respectively (the *N by K* matrix *A* reflects the connections of noise sources). The vector *v(t)* includes node voltages and some branch currents. The solution vector can be presented in the form *v(t)=x(t)+y(t),* where *x(t)* is the deterministic solution of (2) with *u(t)=0* and *y(t)* is the pure noise.

The following jitter computational definition is suggested for this type of circuit model. Let the timing jitter metric *J* be determined by the value τ_ξ that minimizes the following L_2-norm on the specified time interval *T*:

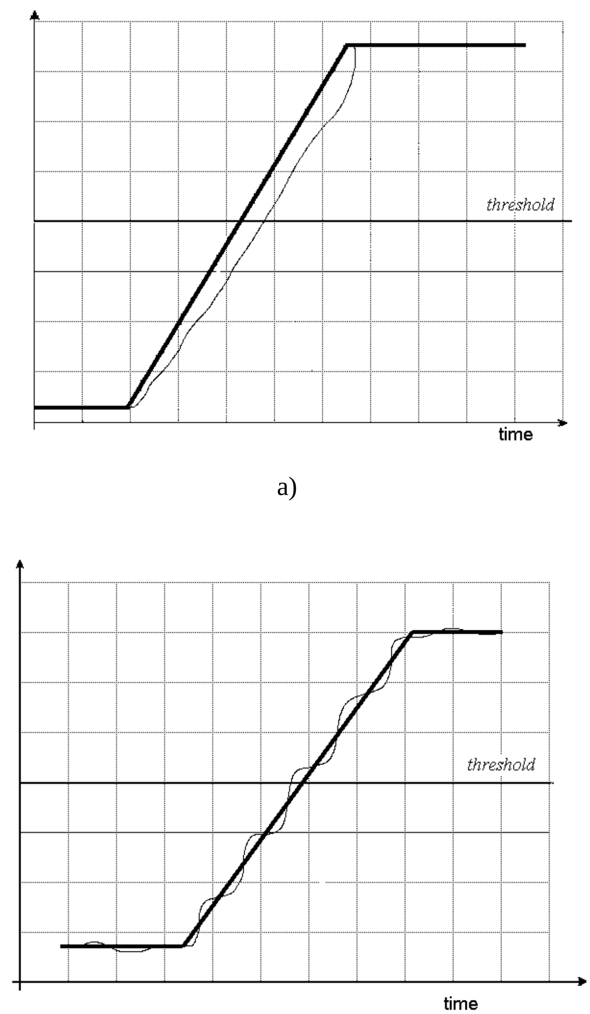
$$w(\theta) = \int_0^T (y_\xi(t) - \theta\dot{x}(t))^2 dt$$
$$w(\tau_\xi) = \min_{\theta} w(\theta) \tag{3}$$

Here $y_\xi(t)$ is a realization of the noise stochastic process *y(t)*, $\dot{x}(t)$ is the time derivative of large signal noiseless solution and θ is a parameter. Thus, τ_ξ is a value of random variable τ and its variance yields the metric of jitter $J^2 = E[\tau^2]$.

This definition satisfies to above mentioned requirements and provides relatively simple jitter computation during the conventional noise simulation as it will be shown below.

Note that the definition (3) expects the specifying of the interval of switching event and does not require to know the time point of switching t_q or threshold level. In contrast the local definition (1) requires quit exact numerical determining the time point t_q for real computed waveforms.

The effect of the dominant role of low frequency noise in timing jitter is illustrated in Fig.1. The computed realization of the output signal in the presence of low frequency noise is shown in Fig.1a. The time shift can be clearly detected and the correct contribution to the resulting jitter value can be computed. Fig.1b shows the realization of the output signal in the presence of high frequency noise. The high frequency means the oscillations with the period that is much less than rise/fall time. It can be seen from this illustrative example that taking into account the high frequency noise contributions is undesired for jitter estimation. However, the definition (1) converts the total noise directly into jitter value without frequency limitations. As it will be shown below the introduced definition (3) eliminates these high frequency noise contributions.



b)

Figure 1. The effect of the dominant role of low frequency noise in timing jitter: noiseless signal and low frequency noise (a); noiseless signal and high frequency noise (b).

4. Basic Computational Procedure

4.1 Transient Noise Analysis

The considered below type of transient noise analysis is based on the representation of noise sources by the set of sinusoidal sources with random uncorrelated amplitudes [12, 16].

Assuming the contribution of the noise sources is small the equation (2) can be linearized about the large-signal noise-free solution *x(t)* giving the following equation with respect to the vector of noise response *y*:

$$\frac{d}{dt}(C(t)y(t)) + G(t)y(t) + Au(t) = 0 \tag{4}$$

where *C(t)* and *G(t)* are the time varying matrices of capacitances and conductances.

This equation is linear and can be solved for every *k*-th noise source independently. Thus, the overall noise response is the superposition of noise responses:

$$y(t) = \sum_{k=1}^{K} y^{(k)}(t) \tag{5}$$

Using the spectral decomposition for *k*-th noise source in the complex form

$$u(t) = \sum_{l=-L}^{L} \xi_l s_u(\omega_l, t)e^{j\omega_l t} \tag{6}$$

where ξ_l are uncorrelated values with variance $\Delta\omega_l$ and $s_u(\omega_l, t)$ is a modulated spectral density, we obtain [16]

$$\begin{gathered}\frac{d}{dt}(C(t)y^{(k)}(t)) + G(t)y^{(k)}(t) + \\ a\sum_{l=-L}^{L} \xi_l s_u(\omega_l, t)e^{j\omega_l t} = 0\end{gathered} \tag{7}$$

where *a* is *k*-th column of matrix *A*.

Because the equation (7) is linear the solution at any time point can be expressed as the linear combination of responses with the coefficients ξ_l:

$$y^{(k)}(t) = \sum_{l=-L}^{L} \xi_l \cdot y^{(k)}(\omega_l, t) \tag{8}$$

where $y^{(k)}(\omega_l, t)$ is obtained from the equation

$$\begin{gathered}\frac{d}{dt}(C(t)y^{(k)}(\omega_l, t)) + G(t)y^{(k)}(\omega_l, t) + \\ as_u(\omega_l, t)e^{j\omega_l t} = 0\end{gathered} \tag{9}$$

This equation yields the noise response corresponding to one noise source k and frequency ω_l. The method of solving (9) is discussed in [16]. In particular, (9) is rewritten and solved with respect to envelope $z^{(k)}(\omega_l, t)$ ($y^{(k)}(\omega_l, t) = z^{(k)}(\omega_l, t)e^{j\omega_l t}$).

Thus, taking into account (5), (8) a realization of noise stochastic process can be presented in the form

$$y_\xi(t) = \sum_{k,l} \xi_{kl} y^{(k)}(\omega_l, t) \tag{10}$$

4.2 Approximation Approach for Timing Jitter

To construct the numerical procedure of jitter estimation based on the definition (3) we start from the characterization of timing displacement in a realization of a noisy signal. We assume that a noisy signal contains a timing displacement of a large signal $x(t+\theta)$ and a deviation. Then the following expression approximates this deviation

$$x(t) + y_\xi(t) - x(t+\theta) \cong y_\xi(t) - \dot{x}(t)\theta \tag{11}$$

Therefore the cost function (3) minimizes the deviation. The expression (3) can be written in the form

$$w_\xi(\theta) = \int_0^T (y_\xi(t) - \dot{x}(t)\theta)^2 dt = a\theta^2 - 2b\theta + c \tag{12}$$

where

$$a = \int_0^T (\dot{x}(t))^2 dt \tag{13}$$

$$b = \int_0^T \dot{x}(t) y_\xi(t) dt \tag{14}$$

The minimum value of (12) is attained at

$$\tau_\xi = \theta_{min} = \frac{b}{a} \tag{15}$$

Now taking into account (10) the expression (15) yields for τ_ξ

$$\tau_\xi = \sum_{k,l} \xi_{kl} \tau^{(k)}(\omega_l) \tag{16}$$

where

$$\tau^{(k)}(\omega_l) = \frac{1}{a}\left(\int_0^T \dot{x}(t) y^{(k)}(\omega_l, t) dt\right) \tag{17}$$

Thus the jitter metric J or the variance of τ can be calculated by standard expression for linear combination of uncorrelated random variables:

$$var(\tau) = \sum_{k,l} (\tau^{(k)}(\omega_l))^2 \Delta\omega_l \tag{18}$$

where ω_l are noise frequencies for which noise equations are solved. Here we used that $var(\xi_{kl}) = \Delta\omega_l$.

For practical implementation of the proposed jitter characterization we need to compute integrals (13), (17). The numerical computation of (13), (17) can be performed simultaneously with the integration (9). Thus, this jitter characterization is quite efficient.

The introduced approach suppresses high oscillating noise due to averaging high frequency components in integrals (12) or (17) that is important feature for jitter characterization.

The proposed definition reduces the impact of the stationary noise on the jitter value. The jitter phenomena is related with nonstationary noise. Noise response of time-invariant circuits corresponds to a stationary noise that does not impact on jitter. To amplify the suppression of the stationary noise contribution, functions $y^{(k)}(\omega, t)$ in integral (17) are replaced by the following functions

$$\widehat{y}^{(k)}(\omega, t) = y^{(k)}(\omega, t) - \widehat{z}(\omega) e^{j\omega t} \tag{19}$$

where $\widehat{z}$ is an average on the time interval envelope of $y(\omega, t)$:

$$\widehat{z}(\omega) = \frac{1}{T}\int_0^T z(\omega, t) dt \tag{20}$$

The elimination of the average envelope $\widehat{z}$, responsible for the stationary noise components, improves the separation of jitter from the total output noise.

5. Application

The first example is given to show that the introduced interval jitter definition (3) yields an estimate of jitter that is close to the result obtained by local definition (1). The correctness of the introduced definition is also confirmed by the computation of timing jitter for relaxation oscillator [3]. The typical dependences of jitter value on time are presented in Fig. 2. Note the growth of jitter with increasing of time. The obtained by definitions (1), (3) curves are in good agreement at the qualitative level. To apply definition (1) switching time points t_q are specified by time points with maximal time derivatives of large signal solution. Similar results were obtained for the CMOS ring oscillator [8,9].

The noise simulation of comparator (Fig.3a) is a typical example of jitter computation in nonautonomous circuits. The input and noiseless output waveforms are shown in Fig.3b. The computed RMS noise characteristic versus time is given in Fig.3c. The estimate of jitter value equals to 2.1 nsec and 0.57 nsec obtained by definition (1) and (3) respectively. The last estimate is more realistic due to the elimination of high frequency noise contribution from the total noise. Note that the presented approach avoids specifying the switching time point. Note also that the jitter values for internal nodes differ from one at the output node. So the jitter value at internal node N7 obtained by (3) equals to 0.067 nsec. The definition (1) yields 5 nsec at node N7 that is more than 2 times larger in comparison with the jitter value at the output node. This impractical result can be explained by formal converting the total noise to jitter with using the definition (1). This example demonstrates the usefulness of the suppression of the high frequency noise contribution.

This effect is also illustrated by the following test examples. The noiseless comparator is based on opamp ua741. The transistor models contain no capacitors. The input excitation includes a linear signal and white noise. If the upper bound of the frequency range of the input noise is extended then the RMS value of the amplitude noise is increased. Respectively the jitter value computed with using the standard definition (1) is unlimited (see Fig. 4a). In contrast the definition (3) based on interval averaging reduces the high frequency contribution into jitter value and provides more realistic results. The corresponding computed lower curve, that is flat, is shown in Fig. 4a.

The similar results can be observed in Fig. 4b for the relaxation oscillator [3]. It can be seen that the computed by new definition jitter value is stabilized after 1MHz while the standard definition (1) gives the growth of jitter value with increasing the upper bound of noise frequency range.

The effect of the elimination of the stationary noise is examined by the simulation of the circuit containing the comparator (Fig.3a) and the amplifier. Jitter is measured at the input and output of the amplifier. The noisy linear amplifier causes only stationary noise and can not be an additional jitter source. According to the definition (1) jitter value is 2.1 nsec and 15.8 nsec at the input and output respectively. The definition (3) yields the equal values (0.57 nsec) for both input and output that corresponds to the natural understanding of jitter phenomena.

6. Conclusion

The concept of timing jitter suited to numerical analysis by circuit simulators has been proposed. This concept is in agreement with the notion of phase deviation that is widely used in practice.

The advantages of the proposed approximation approach for jitter estimation are desired universality and invariance. Moreover it provides the adaptation to the noise frequency range.

7. References

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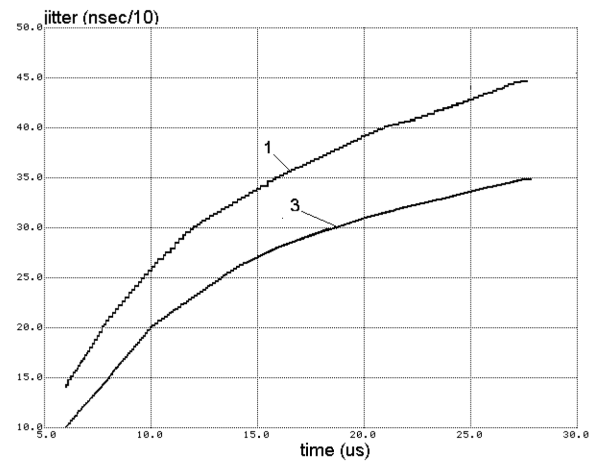
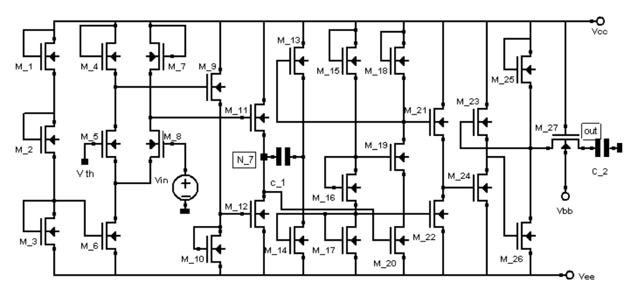


Figure 2. Timing jitter vs. time in relaxation oscillator obtained by using definitions (1) and (3).



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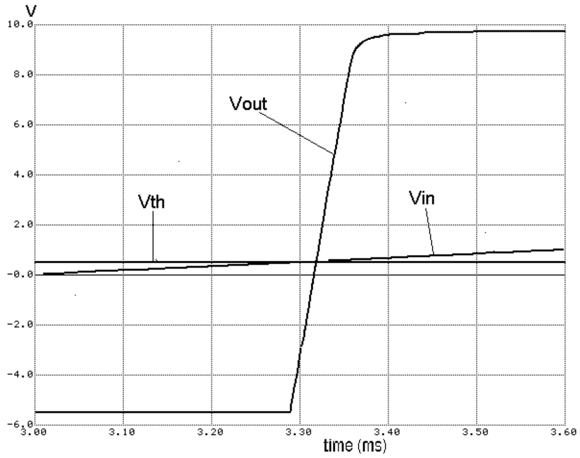

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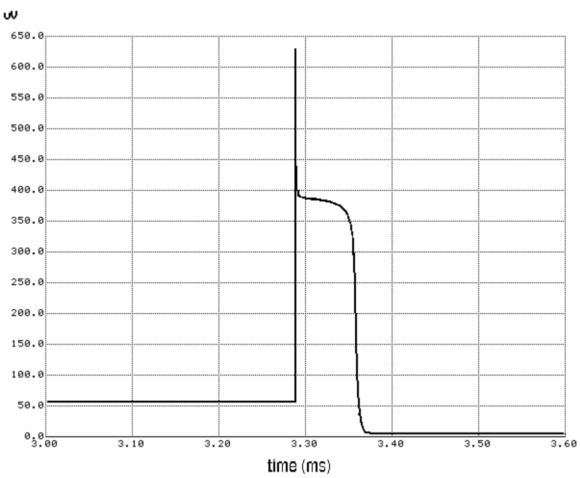
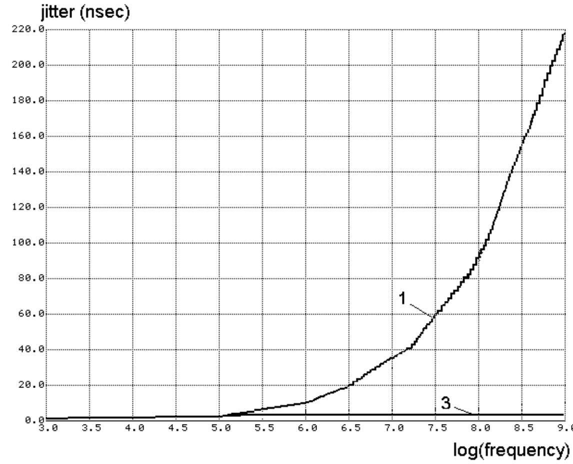


c)

Figure 3. Comparator circuit (a), input and output waveforms (b) and RMS noise vs. time (c).



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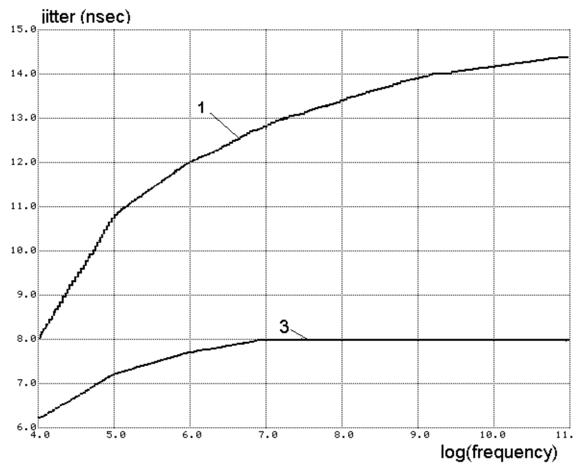

b)

Figure 4. The dependence of jitter on noise frequency range for the comparator circuit (a) and for the relaxation oscillator (b) obtained by using definitions (1) and (3).