

Composition Trees in Finding Best Variable Orderings for ROBDDs

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The algorithms for static reordering of Reduced Ordered Binary Decision Diagrams (ROBDDs) rely on dependable properties for grouping of variables. Two such properties have been studied so far: keeping symmetric variables adjacent [1] and minimizing the ROBDD's width [2]. However, counterexamples have been found for the both cases [1], [3]. In this paper, we introduce a new condition for grouping of variables, suggesting to keep adjacent the variables from all bound sets of the function which are explicitly given by its composition tree. *Bound set* is a proper subset Y of the variables X of a function $f : \{0, 1\}^{|X|} \rightarrow \{0, 1\}$ resulting in the decomposition of type $f(X) = g(h(Y), Z)$, where $Z = X - Y$. *Composition tree* $T(f)$ of $f(X)$ is a structure reflecting all its non-overlapping bound sets [4]-[6]. *Bound-set-preserving ordering* $\langle X \rangle$ of the variables of a ROBDD for $f(X)$ is a vector, describing the variables of X in order from top to bottom of the ROBDD, in which the variables of any node of $T(f)$ are adjacent in $\langle X \rangle$. For example, if a function $f(x_1, x_2, x_3)$ has a single non-trivial bound set $\{x_1, x_2\}$, then the orderings $\langle x_1, x_2, x_3 \rangle, \langle x_2, x_1, x_3 \rangle, \langle x_3, x_1, x_2 \rangle, \langle x_3, x_2, x_1 \rangle$ are bound-set-preserving ones, while the orderings $\langle x_1, x_3, x_2 \rangle$ and $\langle x_2, x_3, x_1 \rangle$ are not. A composition tree $T(f)$ is unique for f (up to isotopy) and therefore any Boolean function has a unique bound-set-preserving ordering. We prove that the intersection of the set of bound-set-preserving orderings and the set of best orderings is non-empty for any Boolean function:

Theorem 1 *For any Boolean function $f(X)$, there exists a bound-set-preserving ordering which is a best.*

Our condition reveals an interesting link between the symmetry and minimal-width criteria, showing that the reasons why they contribute to improvement of the variable ordering originate from the same source. An ordering, keeping the variables from the bound sets adjacent, often (but not always) keeps the symmetric variables adjacent and, at the same time, minimize the width of a ROBDD.

Some extension of Theorem 1 seems desirable. Not all Boolean functions are disjunctively decomposable. Defined

as it is, the bound-set-preserving condition is not capable to improve the ordering of variables of a function which does not have non-trivial bound sets. An open problem is how to extend Theorem 1 to the more general case of Roth-Karp decomposition $f(X) = g(h_1(Y), h_2(Y), \dots, h_k(Y), Z)$ with functions $h_i : \{0, 1\}^{|Y|} \rightarrow \{0, 1\}$ ($1 \leq i \leq k$) and $g : \{0, 1\}^k \times \{0, 1\}^{|Z|} \rightarrow \{0, 1\}$. Roth-Karp decomposition includes as a subclass simple disjoint decompositions ($k = 1$) as well as nondisjoint decompositions. As long as $f(X)$ is a function of more than three variables, such a decomposition can always be found with h_1, h_2, \dots, h_k and g each having fewer variables than f . Thus, Roth-Karp decomposition is applicable to any Boolean function.

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