Efficient Model Reduction of Linear Time-Varying Systems via Compressed Transient System Function

Emad Gad and Michel Nakhla
Department of Electronics, Carleton University
Ottawa, ON, Canada, K1S-5B6
Ph: (613)520-5780, Fax: (613) 520-5708, e-mail:msn@doe.carleton.ca

Abstract

This paper presents a new approach for model-order reduction of linear time varying system based on expanding the time-varying system in the right half plane of the s-domain. The proposed algorithm is developed through introducing Krylov Subspace-based reduction to time-varying transfer functions. The proposed algorithm does not require solution of large system of equations to construct a basis for the time-varying moments. Instead, it computes such a basis through time-domain integration of the corresponding linear time-varying differential algebraic equations. Numerical experiments show that expanding in the right-half plane compresses the transient phase of the response of these equations by several orders of magnitude.

1. Introduction

Abstraction of large communication blocks with small macromodels that preserve their basic features is essential for efficient hierarchical system-level simulation. Generating small accurate reduced-order models that capture the behavior of much bigger system has proven to be a key to developing successful simulation algorithms.

In recent years, model-reduction of Linear Time-Invariant (LTI) systems has been well-established and successful for efficient simulation of large linear circuits [1–4]. However, there are many other systems that can not be modeled as LTI systems. For example, RF components are designed to have an almost linear response for the small signal path but behave nonlinearly in response to some other large excitation signal. Such systems can be accurately modelled as rather Linear Time-Varying (LTV) systems, where the nonlinear system is linearized around the strong excitation signal resulting in a system that is linear time-varying for the small signal path.

The problem of generating reduced-order LTV macromodels has been addressed recently in [5] and [6]. The main idea in this approach is to separate the time scale of the system from the time scale of the input. This enables the construction of a time-varying transfer function, $\mathbf{h}(s,t)$, that models the linear time-dependent path of the small signal. In particular, if the LTV system is periodical, as it is always the case in circuit applications, then $\mathbf{h}(s,t)$ is also periodic in t, and hence classical steady-state approaches, (e.g. Harmonic Balance (HB) or Finite Difference (FD) method [7]) can be invoked to compute a compact representation. Generating macromodels is then carried out through obtaining a reduced-order approximations of $\mathbf{h}(s,t)$ via applying Krylov-subspace techniques to the large equations that arise in HB methods.

This approach has been successful in being able to model the time-varying small signal path using a reduced-order time-varying Padé (TVP) approximation. However, it requires a solution of a large system of equations that arise in the inner loop of the HB or FD methods to construct the reduced-order model.

In this paper, we present a new approach that does not require a solution of large system of equations. The proposed algorithm is motivated by the fact that $\mathbf{h}(s,t)$ has very short transient response for values of s that are in the right-half plane of the s-domain. This means that the time-varying moments of $\mathbf{h}(s,t)$ w.r.t. s take a very short time before settling into steady state, and hence can be obtained efficiently through direct time-domain integration [8].

In the proposed algorithm, a generalized framework of Krylov subspace is developed in order to avoid computing the time-varying moments of $\mathbf{h}(s,t)$ explicitly. In fact, we show that implicit moment matching methods using Krylov-subspace techniques can be applied to approximating transfer functions whose moments are related through a differential operator.

In addition, we present an approach to eliminate numerical noise resulting from the truncation error generated by the time-domain integration method.

The paper is organized as follows. In Section II we present a brief background on LTV systems and the concept of time-varying transfer function. Section III presents a quick review of the recent approaches to model-order reduction of LTV systems. Section IV presents outlines the proposed algorithm. Section V and VI present some numerical results and conclusion.

2. Analysis of Time-Varying Systems

A nonlinear system driven by a large signal $b_L(t)$ and a small signal $u(t)$ can be described by a set of nonlinear differential algebraic equations,

$$\begin{aligned} \frac{d\mathbf{q}(\mathbf{x}(t))}{dt} + \mathbf{f}(\mathbf{x}(t)) &= \mathbf{b}_L(t) + \mathbf{b}u(t) \\ \mathbf{z}(t) &= \mathbf{d}^T\mathbf{x}(t) \end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the vector of node voltages; $\mathbf{q}(.)$ and $\mathbf{f}(.)$are nonlinear functions describing the charge/flux and resistive elements, respectively; $\mathbf{z}(t)$ is the set of output nodes; $\mathbf{b}$ and $\mathbf{d}$ are selector vectors that map the input and output ports to the space of the networks, and n is the number of nodes in the network. Splitting the response into two parts:

$$\mathbf{x}(t) = \mathbf{x}_L(t) + \mathbf{x}_s(t) \tag{2}$$

where $\mathbf{x}_L(t)$ and $\mathbf{x}_s(t)$ are the responses due to the large signal $\mathbf{b}_L(t)$ and small signal $u(t)$, respectively. Substituting from (2) into (1) and linearizing around $\mathbf{x}_L(t)$ yields the LTV system,

$$\frac{d(\mathbf{C}(t)\mathbf{x}_s(t))}{dt} + \mathbf{G}(t)\mathbf{x}_s(t) = \mathbf{b}u(t) \tag{3}$$

where

$$\mathbf{G}(t) = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_L(t)} \qquad \mathbf{C}(t) = \left.\frac{\partial \mathbf{q}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_L(t)} \tag{4}$$

Zadeh [9] has introduced the concept of time-varying system function to describe the response of LTV systems. According to Zadeh's formalism, an LTV system can be characterized by using a time-varying transfer function, $\mathbf{h}(s,t)$, where the response of the system due to an input of the form e^{st} is given by $e^{st}\mathbf{h}(s,t)$ [10]. Thus, by substituting in (3) we get,

$$\frac{d(\mathbf{C}(t)\mathbf{h}(s,t))}{dt} + s\mathbf{C}(t)\mathbf{h}(s,t) + \mathbf{G}(t)\mathbf{h}(s,t) = \mathbf{b} \tag{5}$$

The transfer path from the input to the set of the output nodes can be represented by a time-varying transfer function $\mathbf{\Phi}(s,t)$ where,

$$\mathbf{\Phi}(s,t) = \mathbf{d}^T\mathbf{h}(s,t) \tag{6}$$

It is clear from (6) that a full characterization of the system transfer function requires a solution of LTV system for many values of s.

3. Model Reduction of LTV Systems

Computing the time-varying path of signals thorugh solving LTV system for various valus of frequency s can be very expensive. Model-reduction of LTV systems, [5] and [6], was proposed to characterize transfer characteristics of such systems efficiently. A quick review of this approach is presented here to facilitate understanding the basic ideas in the proposed algorithm. The task of generating small LTV macromodels is essentially constructing reduced-order approximation for $\mathbf{\Phi}(s,t)$. In particular, if the matrices $\mathbf{C}(t)$ and $\mathbf{G}(t)$ are periodic, as it is always the case in circuit applications, then $\mathbf{h}(s,t)$ (and hence $\mathbf{\Phi}(s,t)$) becomes periodic in t, and classical steady-state approaches can be invoked to obtain compact representations for it. This is discussed in the next subsections.

3.1. Frequency-Domain Matrix Form

A frequency-domain representation is obtained by representing all time varying terms in (5) in Fourier series. Expanding $\mathbf{C}(t)$, $\mathbf{G}(t)$, and $\mathbf{h}(s,t)$ as Fourier series,

$$\begin{aligned} \mathbf{C}(t) &= \sum_{i=-\infty}^{\infty} \mathbf{C}_i e^{\jmath i\omega_0 t} \\ \mathbf{G}(t) &= \sum_{i=-\infty}^{\infty} \mathbf{G}_i e^{\jmath i\omega_0 t} \\ \mathbf{h}(s,t) &= \sum_{i=-\infty}^{\infty} \mathbf{H}_i(s) e^{\jmath i\omega_0 t} \end{aligned} \tag{7}$$

Substituting in (5) we get the following the following system of equations

$$[s\mathcal{C}_{FD} + (\mathcal{G}_{FD} + \Omega\mathcal{C}_{FD})]\,\vec{\mathbf{H}}_{FD}(s) = \vec{\mathbf{B}}_{FD} \tag{8}$$

where

$$\begin{aligned} \mathcal{C}_{FD} &= \text{toeplitz}\,(\cdots, \mathbf{C}_{-1}, \mathbf{C}_0, \mathbf{C}_1, \cdots) \\ \mathcal{G}_{FD} &= \text{toeplitz}\,(\cdots, \mathbf{G}_{-1}, \mathbf{G}_0, \mathbf{G}_1, \cdots) \\ \vec{\mathbf{H}}_{FD}(s) &= [\cdots, \mathbf{H}_{-1}(s), \mathbf{H}_0(s), \mathbf{H}_1(s), \cdots] \\ \Omega &= \jmath\omega_0 \text{diag}[\cdots, -2I, -I, 0, I, 2I, \cdots)] \\ \vec{\mathbf{B}}_{FD} &= [\cdots, 0, \mathbf{b}^T, 0, \cdots]^T \end{aligned} \tag{9}$$

3.2. Time-Domain Matrix Form

A time-domain version of the equations in (8) can be constructed by collocating $\mathbf{h}(s,t)$ over time samples $t \in [0,T]$ at $N+1$ sample points $t_0, \ldots, t_N$. Using a linear multistep formula (e.g. Backward Euler) and imposing the periodicity constraint on $\mathbf{h}(s,t)$, i.e., $\mathbf{h}(s,t_0) = \mathbf{h}(s,t_N)$, we get a discretized version of the differential operator in (5) in terms of samples,

$$[s\mathcal{C}_{TD} + (\mathcal{G}_{TD} + \Delta\mathcal{C}_{TD})]\,\vec{\mathbf{H}}_{TD}(s) = \vec{\mathbf{B}}_{TD} \tag{10}$$

where

$$\begin{aligned}
\mathcal{C}_{TD} &= \text{BlockDiag}\left[\mathbf{C}(t_0), \cdots, \mathbf{C}(t_N)\right] \\
\mathcal{G}_{TD} &= \text{BlockDiag}\left[\mathbf{G}(t_0), \cdots, \mathbf{G}(t_N)\right] \\
\vec{\mathbf{H}}_{TD}(s) &= \left[\mathbf{H}(s, t_0), \cdots, \mathbf{H}(s, t_N)\right] \\
\Delta &= \begin{pmatrix} \frac{1}{\delta_1}I & & & -\frac{1}{\delta_1}I \\ -\frac{1}{\delta_2}I & \frac{1}{\delta_2}I & & \\ & & \ddots & \\ & & -\frac{1}{\delta_N}I & \frac{1}{\delta_N}I \end{pmatrix} \\
\delta_i &= t_i - t_{i-1} \\
\vec{\mathbf{B}}_{TD} &= \left[\cdots, \mathbf{b}^T, \mathbf{b}^T, \mathbf{b}^T, \cdots\right]^T
\end{aligned} \tag{11}$$

3.3. Model-Reduction Using Block-Krylov Methods

From the analysis in the previous subsections, it can be seen that both (8) and (10) can be put in the form

$$\vec{\mathbf{H}}(s) = [s\mathcal{C} + \mathcal{J}]^{-1}\, B = [I - s\mathcal{A}]^{-1}\mathcal{R} \tag{12}$$

where

$$\mathcal{A} = -\mathcal{J}^{-1}\mathcal{C} \qquad \mathcal{R} = \mathcal{J}^{-1}B. \tag{13}$$

Similarly $\mathbf{\Phi}(s,t)$ can be represented by $\vec{\mathbf{\Phi}}(s)$ where

$$\vec{\mathbf{\Phi}}(s) = \mathcal{D}^T\vec{\mathbf{H}} = \mathcal{D}^T\left[I - s\mathcal{A}\right]^{-1}\mathcal{R} \tag{14}$$

Block-Krylov subspace methods are then used to obtain reduced order approximation of $\vec{\mathbf{\Phi}}(s)$. A typical block-Krylov subspace method is run for q steps with the matrices $\mathcal{A}$, $\mathcal{D}$ and $\mathcal{R}$ as input to produce an $q \times N$ orthonormal matrix $\mathbf{V}$ where

$$\mathbf{V} = \text{colspan}\left[\mathcal{R}, \mathcal{A}\mathcal{R}, \mathcal{A}^2\mathcal{R}, \cdots, \mathcal{A}^{(q-1)}\mathcal{R}\right] \tag{15}$$

A q-th order approximant, $\vec{\mathbf{\Phi}}_q(s)$, for $\vec{\mathbf{\Phi}}(s)$ is then obtained by

$$\vec{\mathbf{\Phi}}_q(s) = D_q^T\left[I_{q\times q} - sT_{q\times q}\right]^{-1} R_q \tag{16}$$

where $T_{q\times q} \in \mathbb{C}^{q\times q}$, $D_q, R_q \in \mathbb{C}^{N\times q}$, and N is the size of the original system in (14).

As seen from the above analysis, constructing the orthonormal basis $\mathbf{V}$ requires a matrix-vector product with $\mathcal{C}$ and linear system solutions with $\mathcal{J}$. These matrices are in general large and difficult to factorize. Iterative techniques [11] have been proposed to solve equations with large sparse systems efficiently. However, fast convergence of these methods require the use of a preconditioner matrix that approximates the inverse of the coefficients matrix, $\mathcal{J}$. Efficient preconditioning can be accomplished easily for diagonally-dominant matrices. In particular, the time-domain matrix form of $\mathcal{J}$ is more diagonally-dominant than the frequency-domain one, especially for systems arising from linearizing strongly nonlinear circuits. However, using a collocation technique in the time-domain presents a practical difficulty due to the need to find a suitable grid that best approximates a “yet-to-be-determined” waveform. This problem gets even more difficult with higher-order time-varying moments, as these moments usually exhibit faster variations and definitley require finer grids.

4. Outlines of the Proposed Algorithm

An important advantage in the proposed algorithm is that it can compute the basis $\mathbf{V}$ in (15) without having to solve for a large dense system of equations. The time-varying transfer function, $\mathbf{h}(s,t)$ is first expanded as a Taylor series around some frequency point s_0,

$$\mathbf{h}(s,t) = \sum_{i=0}^{\infty} \mathbf{m}_i(t)(s - s_0)^i \tag{17}$$

Since $\mathbf{h}(s,t)$ is periodic then $\mathbf{m}_i(t)$ is also periodic with the same period. Substituting from (17) into (5) and equating equal powers of s we get,

$$\frac{d(\mathbf{C}(t)\mathbf{m}_0(t))}{dt} + s_0\mathbf{C}(t)\mathbf{m}_0(t) + \mathbf{G}(t)\mathbf{m}_0(t) = \mathbf{b} \tag{18}$$

$$\frac{d(\mathbf{C}(t)\mathbf{m}_i(t))}{dt} + s_0\mathbf{C}(t)\mathbf{m}_i(t) + \mathbf{G}(t)\mathbf{m}_i(t) = -\,\mathbf{C}(t)\mathbf{m}_{i-1}(t) \quad (i \geq 1) \tag{19}$$

It can be seen from (18) and (19) that the moments $\mathbf{m}_i(t)$ are related through a differential operator. This is in contrast to the LTI case where the moments are related through a matrix algebraic operator.

The objective of the proposed algorithm is to construct the matrix $\mathbf{V}$ through integrating (18) and (19) until the steady-state point is reached.

However, using brute force integration to find the steady-state of $\mathbf{m}_i(t)$ can lead to numerical and computational problems:

1. The first problem is that computing the moments explicitly suffers an ill-conditioning problem. This usually makes moments of order higher than 10 of almost no value [12].

2. Finding the steady-state response through time-domain integration can become extremely computationally expensive for low damped circuits with long transient responses.

These problems are addressed in the following subsections.

4.1. Construction of the basis $\mathbf{V}$

Denote the columns of $\mathbf{V}$ by $v_0, v_1, \cdots, v_{q-1}$. To illustrate the basic steps involved in generating $\mathbf{V}$, we assume that i columns, $v_0, \cdots, v_{i-1}$, have already been computed and it is required to compute v_i. Firstly, a time-domain waveform, $\boldsymbol{\xi}_{i-1}(t)$, is constructed from the harmonics contained in v_{i-1} by

$$\boldsymbol{\xi}_{i-1}(t) = \sum_{k=-K}^{K} \boldsymbol{\Psi}_{k+K+1}\; v_{i-1}\; e^{\jmath k\omega_0 t} \tag{20}$$

where $\boldsymbol{\Psi}_j$ is a selector matrix defined by

$$\boldsymbol{\Psi}_j = [0, \cdots, \underbrace{I}_{j\text{-th position}}, \cdots, 0] \tag{21}$$

This $\boldsymbol{\xi}_{i-1}(t)$ is then used as an input to the system in (19),

$$\frac{d(\mathbf{C}(t)\tilde{\boldsymbol{\xi}}_i(t))}{dt} + s_0\mathbf{C}(t)\tilde{\boldsymbol{\xi}}_i(t) + \mathbf{G}(t)\tilde{\boldsymbol{\xi}}_i(t) = -\mathbf{C}(t)\boldsymbol{\xi}_{i-1}(t) \tag{22}$$

where the response $\tilde{\boldsymbol{\xi}}_i(t)$ is obtained through time-domain integration of (22) until all transients die out. Using FFT, $\tilde{\boldsymbol{\xi}}_i(t)$ can be represented as a truncated Fourier series with coefficients $\tilde{\boldsymbol{\Xi}}_i(k)$

$$\tilde{\boldsymbol{\xi}}_i(t) = \sum_{k=-K}^{K} \tilde{\boldsymbol{\Xi}}_i(k) e^{\jmath k\omega_0 t} \tag{23}$$

To find v_i, a vector $\tilde{v}_i$ is first constructed through

$$\tilde{v}_i = \sum_{k=1}^{2K+1} \mathbf{e}_k \otimes \tilde{\boldsymbol{\Xi}}_i(k-K-1) \tag{24}$$

and then v_i is obtained by

$$v_i = \text{normalize}\left(\tilde{v}_i - \sum_{j=0}^{i-1} \langle \tilde{v}_i, v_j\rangle\, v_j\right) \tag{25}$$

4.2. Transient Compression

It is clear from the previous subsection that the main computational cost involved in constructing the basis $\mathbf{V}$ is that of performing a time-domain integration on the system in (22) until the steady-state response is reached. This can, however, be very costly especially for circuits that have very long time constants.

To overcome this problem, we employ the idea of expanding the transfer function $\mathbf{h}(s,t)$ on the positive real axis of the s-domain. This is done by choosing an expansion point $s_0 = \sigma_0 + \jmath\omega_0$ with $\sigma_0 > 0$. This in effect, shifts all the Floquet exponents of the system far in the left-half plane and leaves the system with very short time constants. This is stated formally by the next two theorems.

Theorem 1 *Let $\boldsymbol{\psi}(t)$ be a fundamental matrix of the periodical LTV system defined by,*

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) \tag{26}$$

and let T be the period of the system. Then there exists a periodic nonsingular matrix $\mathbf{R}(t)$ with period T and a constant matrix $\boldsymbol{\Gamma}$ such that

$$\boldsymbol{\psi}(t) = \mathbf{R}(t)\exp(\boldsymbol{\Gamma}t) \tag{27}$$

Proof: see [10]. ■

Theorem 2 *Let $\boldsymbol{\psi}(t)$ be a fundamental matrix for the system in (26), where $\boldsymbol{\psi}(t) = \mathbf{R}(t)\exp(\boldsymbol{\Gamma}t)$ as described in Theorem 1. Then the system,*

$$\dot{\mathbf{x}}(t) = (\mathbf{A}(t) + \sigma I)\,\mathbf{x}(t) \tag{28}$$

has a fundamental matrix $\boldsymbol{\psi}_s(t)$ given by

$$\boldsymbol{\psi}_s(t) = \mathbf{R}(t)\exp[(\boldsymbol{\Gamma} + \sigma I)t] \tag{29}$$

Proof: By substituting $\boldsymbol{\psi}_s(t)$ for $\mathbf{x}(t)$ in (28), we can see that it satisfies the system equations. Hence, $\boldsymbol{\psi}_s(t)$ is a fundamental matrix. ■

Corollary 1 *Let the set of Floquet exponents of the LTV system $\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}(t)$ be given by $\lambda_1, \cdots, \lambda_N$. Then the Floquet exponents of the system $\dot{\mathbf{x}} = (\mathbf{A}(t) + \sigma I)\mathbf{x}(t)$, are given by $\sigma + \lambda_1, \cdots, \sigma + \lambda_N$.*

4.3. Eliminating Numerical Noise

The process of finding the steady-state response for the system in (22) through time-domain integration and FFT can become a source of significant numerical error. This is because FFT is inefficient in resolving harmonics that are 60-80 dB below the fundamental frequency [13]. This can error can degrade the accuracy of the matrix V and render the reduced model inaccurate. We present a simple correction step to eliminate this source of error.

Denote the steady-state solution of (22), obtained through direct time-domain inegration as described above by $\mathcal{X}$. Then (22) can be written in the frequency domain as

$$\left(s_0\mathcal{C} + (\mathcal{G} + \Omega\mathcal{C})\right)\mathcal{X} = \mathcal{U} + \Delta \tag{30}$$

where $\mathcal{U}$ represents the harmonics in the right-hand side in (22) and Δ is an error term resulting from numerical errors present in the harmonics solution vector $\mathcal{X}$. The goal of the correction step now is to compute a correction $\Delta\mathcal{X}$ that can be applied to $\mathcal{X}$ to eliminate the error term Δ. This problem may be formulated as,

$$\left(s_0\mathcal{C} + (\mathcal{G} + \Omega\mathcal{C})\right)\Delta\mathcal{X} = \Delta \tag{31}$$

To find $\Delta\mathcal{X}$, the system in (31) is converted to the time-domain, and the response is obtained by performing a time-domain integration to find the steady-state point and then using FFT. This process is typically run for 3-5 times to obtain an extremely accurate result.

5 Numerical Results

5.1. Diode Circuit

A rectifier circuit consisting of a diode and capacitors was used to test the validity of the proposed algorithm. First the steady-state response was computed using HB technique. The circuit was then linearized around the steady-state solution, and the resulting LTV was considered for reduction. Figures 1 and 2 show a comparison for the frequency response of the base-band frequency response of $H_1(s)$ as obtained through brute-force calculation of (12) and through the proposed algorithm.

5.2. Mixer Circuit

In this example, we apply the proposed algorithm on a multi-tone circuit. Consider the mixer circuit shown in figure 3. Steady-state solution was first obtained using HB by setting $e_1(t) = 0.15\sin(50\times 10^6\ t)$ and $e_2(t) = 0$. Figure 4 shows the steady-state waveform at one of the output nodes. The circuit was then linearized around the steady-state solution and a LTV system w.r.t. $e_2(t)$ was then constructed. The proposed algorithm was then applied to reducing the transfer function from the source $e_2(t)$ to the output node.

In order to avoid long transient response resulting from the coupling capacitors, the LTV transfer function was expanded on the positive real axis. Figures 5 and 6 demonstrate the effect of expanding on the real axis. It is clear that steady-state point could be reached in only 3 cycles using expansion in the right-half plane, while expanding at the origin took several thousands of cycles.

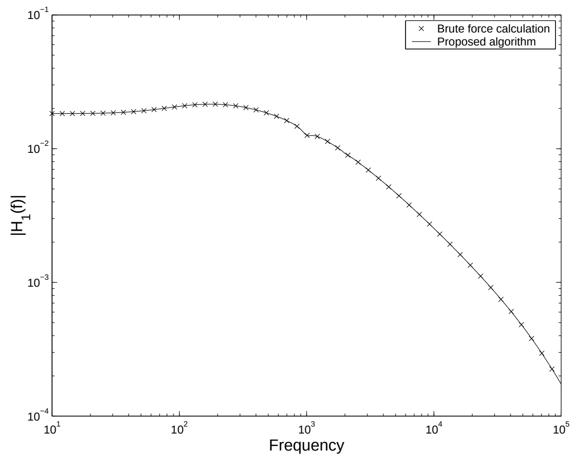


Figure 1. Positive frequency response of diode circuit.

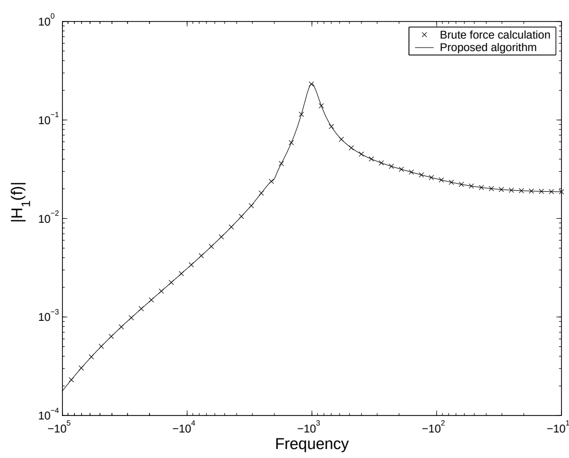


Figure 2. Negative frequency response of diode circuit

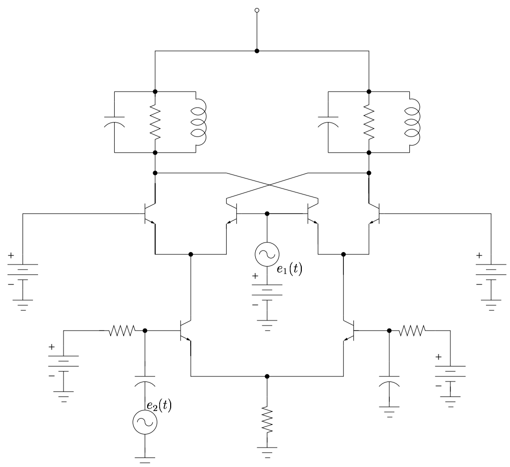


Figure 3. A mixer Circuit

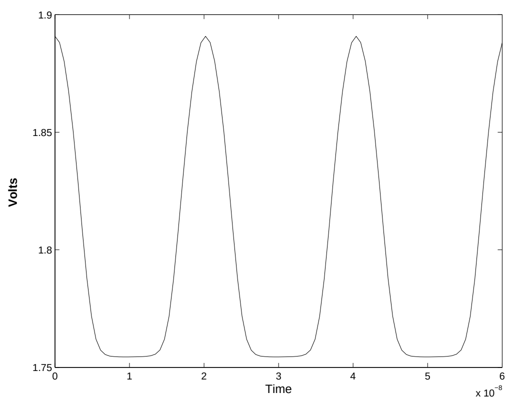


Figure 4. Steady-state response for the mixer circuit.

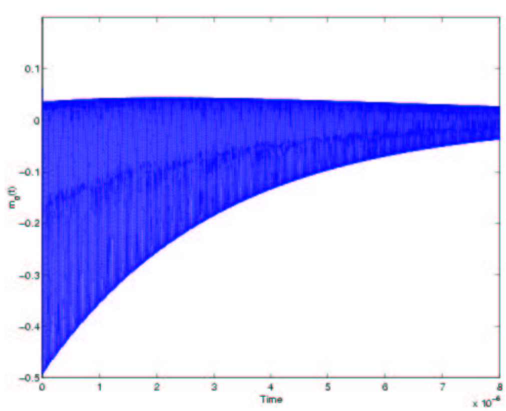


Figure 5. Transient response for the system in (18) by expanding at $s_0 = 0$

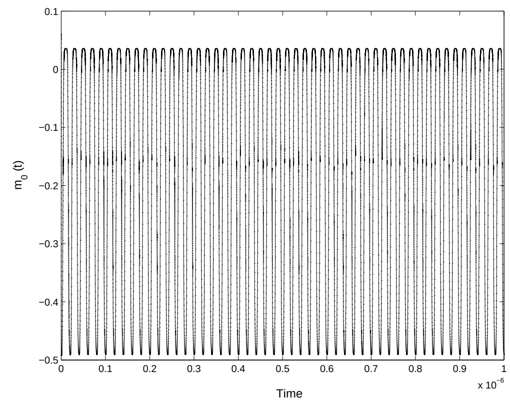


Figure 6. Transient response for the system in (18) by expanding at $s_0 = 100^6$

The proposed algorithm was then applied using an expansion point on the positive real axis at $s_0 = 100^6$. Figure 7 shows a comparison for the frequency response of the base-band referred transfer function $H_1(s)$ as obtained through direct computation and the proposed algorithm, where in the direct computation, the HB equations in (12) were solved directly for different frequency values.

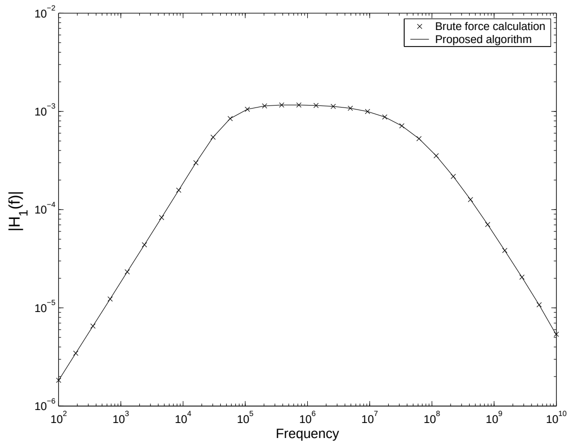
6. Conclusion

The paper presented a new approach for model-reduction of LTV system. The proposed algorithm takes advantage of the fact that for LTV systems, the transfer function has a short transient response in the right-half plane of the s-domain, and hence can be obtained efficiently through time-domain integration of the system DAE equations. To avoid using explicit moment matching techniques, a variant of Krylov subspace methods have been extended to approximating transfer functions whose moments are related through a differential operator. Finally, an approach has been presented to eliminate numerical truncation errors resulting from the time-domain integration process.

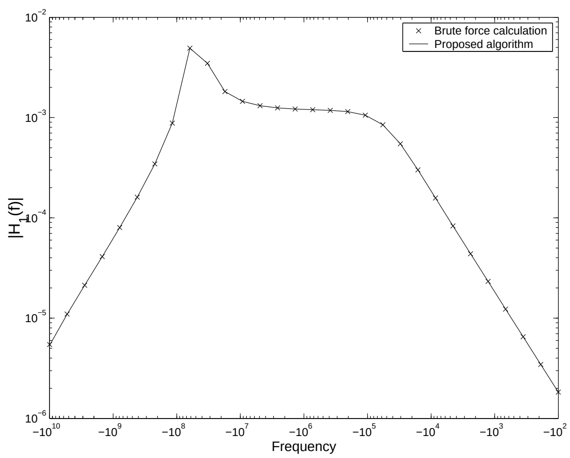
References

[1] L. T. Pileggi and R. A. Roher, “Asymptotic waveform evaluation for timing analysis”, *IEEE Transactions on Computer-Aided design*, vol. 9, pp. 352–366, Apr. 1990.

[2] E. Chiprout and M. S. Nakhla, “Analysis of interconnect networks using complex frequency hopping (CFH)”, *IEEE Transactions on Computer-Aided design*, vol. 14, n. 2, pp. 186–200, Feb. 1995.



(a)



(b)

Figure 7. Frequency response of mixer circuit

[3] A. Odabasioglu, M. Celik and L. T. Pileggi, "PRIMA: passive reduced-order interconnect macromodelling algorithm", *IEEE Transactions on Computer-Aided design*, vol. 17, n. 8, pp. 645–653, Aug. 1998.

[4] Q. Yu, J. M. L. Wang and E. S. Kuh, "Passive multipoint moment matching model order reduction algorithm of multiport distributed interconnect networks", *IEEE Transactions on Circuits and Systems*, vol. 46, n. 1, pp. 140–160, Jan. 1999.

[5] J. Roychowhury, "Reduced-Order Mdelling of Time-Varying Systems", *IEEE Transactions on Circuits and Systems II*, vol. 46, n. 10, pp. 1273–1288, Oct. 1999.

[6] J. R. Phillips, "Model Reduction of Time-Varying Linear Systems Usig Approximate Multipoint Krylov-Subspace Projectors", in *Proc. IEEE ICCAD*, Nov. 1998.

[7] K. S. Kundert, J. K. White and Alberto Sanjiovanni-Vincentelli, *Steady-State Methods for Simulating Analog and Microwave Circuits*, Kluwer Academic Publishers, MA, USA, 1990.

[8] J. Vlach and K. Singhal, *Computer methods for circuits analysis and design*, Van Nostrand, NewYork, NY, 1983.

[9] L. Zadeh, "Frequency Analysis of Variable Networks", *IRE Proc.*, vol. 38, pp. 291–299, 1950.

[10] H. D'angelo, *Linear Time-Varying Systems: Analsysi and Synthesis*, Allyn and Bacon, Boston, MA, 1970.

[11] Y. Saad, *Iterative Methods for Sparse Linear Systems*, PWS, Boston, MA, 1996.

[12] P. Feldmann and R. W. Freund, "Efficient linear cut analysis by Padé via Lanczos process", *IEEE Transactions on Computer-Aided design*, vol. 14, pp. 639–649, May. 1995.

[13] K. S. Kundert, "Accurate Fourier Analyis for Circuit Simulators", in *Proc. IEEE CICC*, May 1994.