Global Optimization Applied to the Oscillator Problem

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Abstract

The oscillator problem consists of determining good initial values for the node voltages and the frequency of oscillation and the avoidance of the DC solution. Standard approaches for limit cycle calculations of autonomous circuits exhibit poor convergence behavior in practice. By introducing an additional periodic probe voltage source to the oscillator circuit, the system of autonomous differential algebraic equations (DAEs) can be reformulated as a system of non-autonomous DAEs with the constraint, that the current through the source has to be zero for the limit cycle. Using a two stage approach leads to a greater range of convergence as the standard approach, but the success of the algorithm is heavily dependent on the initial amplitude of the probe source and the frequency of oscillation. This paper presents a fast and reliable optimization based initialization procedure which overcomes the initialization problem of the two stage algorithm.

1 Introduction

Using modified nodal analysis, electronic circuits can be described mathematically by a system of N (nonlinear) differential algebraic equations (DAEs)

$$f(x,t) = i(x,t) + \frac{\mathrm{d}q(x)}{\mathrm{d}t} = 0, \tag{1}$$

where $x \in \mathbb{R}^N$ is the vector of unknown node voltages and branch currents of inductors, $i : \mathbb{R}^{N\times 1} \to \mathbb{R}^N$ is the vector of conductive contributions to the circuit equations and $q : \mathbb{R}^N \to \mathbb{R}^N$ is the vector of charges and fluxes. It is required that i, q are sufficiently smooth functions and that the partial derivatives with respect to x are available. It is further assumed that i_x is regular for all t, otherwise there is no unique solution of the DAE. In the autonomous case, equation (1) reads as follows

$$f(x) = i(x) + \frac{\mathrm{d}q(x)}{\mathrm{d}t} = 0. \tag{2}$$

It is also assumed that (2) has at least one non-trivial periodic solution $x(t) = x(t+T)$ with an a priori unknown period T and that the DAEs do not exhibit any chaotic behavior. Additionally, it is assumed that (2) is uniquely solvable for any consistent set of initial conditions x_0 at t_0. Under these assumptions the calculation of limit cycles of (2) can be written as the solution of the boundary value problem

$$\begin{aligned} f(x) &= i(x) + \frac{\mathrm{d}q(x)}{\mathrm{d}t} = 0 \\ x(0) &= x(T). \end{aligned} \tag{3}$$

Standard techniques for solving periodic boundary value problems of type (3) are shooting techniques, finite-difference methods and Ritz-Galerkin techniques. Harmonic Balance is a Ritz-Galerkin method employing trigonometric basis functions. In the following we restrict ourselves to the Harmonic Balance technique, which is a reliable technique for simulating steady state responses of electronic systems.

The following section deals with the standard and the two stage approach for limit cycle calculation of oscillators. Some usual initialization techniques are presented in section 3. Section 4 presents the optimization based initialization procedure for the two stage algorithm. Some examples for demonstrating the effectiveness of the new initialization technique are presented in section 5. A short conclusion will close this paper.

2 Limit cycle calculation

In the following, the complex form of the discrete Fourier transform is used for ease of presentation, resulting into $2K + 1$ harmonics of the signal from $-K\omega, ..., K\omega$. The system size can be reduced roughly by a factor of two,

when employing a sine/cosine representation of the waveforms.

In the sections below, lower case characters represent quantities in the time domain (waveforms), upper case letters quantities in the frequency domain (spectra).

2.1 The standard approach

Application of Harmonic Balance to the boundary value problem (3) leads to a system of $N(2K+1)$ nonlinear algebraic equations

$$F(X,\omega) = I(X) + j\,\Omega(\omega)\,Q(X) = 0, \tag{4}$$

where $\omega = \frac{2\pi}{T}$, $j = \sqrt{-1}$, $F : \mathbb{C}^{N(2K+1)} \times \mathbb{R} \rightarrow \mathbb{C}^{N(2K+1)}$. Ω represents the time derivative in the frequency domain

$$\Omega = [\Omega_{nm}]\,,\ \ \Omega_{nm} = \begin{cases} \Omega_{nn} & \text{if} \quad n = m \\ 0 & \text{if} \quad n \neq m \end{cases} \tag{5}$$
$$\Omega_{nn} = \omega\,\mathrm{diag}\,\{-K, ..., -1, 0, 1, ..., K\}\,,\ n = 1, 2, ..., N.$$

Nonlinear equations of type (4) are generally solved by (damped) Newton type techniques, which require the partial derivatives of the constitutive element relations. Let ν be the counter of the Newton procedure. With

$$\begin{aligned} J &= \left[\, J_F(X^{(\nu)}, \omega^{(\nu)})\ \tfrac{\partial F(X^{(\nu)}, \omega^{(\nu)})}{\partial \omega} \,\right] \\ \text{where } J_F &= \partial I(X)\,/\,\partial X + j\,\Omega(\omega)\,\partial Q(X)\,/\,\partial X \\ &= G(X) + j\,\Omega(\omega)\,C(X) \\ \text{and } \frac{\partial F(X,\omega)}{\partial \omega} &= \frac{j}{\omega}\,\Omega(\omega)\,Q(X), \end{aligned}$$

we get

$$J \begin{bmatrix} \Delta X^{(\nu+1)} \\ \Delta \omega^{(\nu+1)} \end{bmatrix} = -\left[\, F(X^{(\nu)}, \omega^{(\nu)}) \,\right]. \tag{6}$$

(4) is an under-determined system, because the additional angular frequency ω is an unknown. This reflects the property of autonomous systems having a continuum of phase-shifted solutions. To overcome this problem, the phase of one harmonic can be fixed, i.e. $\mathrm{Re}\{(e_l^m)^T X\} = 0$ where e_l^m is the l-th canonical vector of dimension $m := N\,(2K+1)$. The standard approach for limit cycle calculations of oscillators can be found e.g. in [6].

2.2 The two stage approach

By introducing an additional periodic probe voltage source $b(t) = b(t+T)$ to the node n of the oscillator circuit, $1 \leq n \leq N$, the autonomous DAE (3) with unknown periodic steady state can be reformulated as a non-autonomous DAE (1) with additional constraints [11, 13]. The probe voltage source injects a current added to the list of unknowns, $x_{N+1}(t)$. From (2), the following system of equations can be obtained

$$\begin{aligned} f(x) = i(x) + \frac{\mathrm{d}q(x)}{\mathrm{d}t} + e_n^N\, x_{N+1} = 0 \\ x_n - b(t) = 0, \end{aligned} \tag{7}$$

where e_n^N is the n-th canonical vector of dimension N. Equation (7) is now a non-autonomous DAE of the form (1). To calculate (7), existing software for solving driven Harmonic Balance problems can be re-used [10, 12]. The autonomous steady state solution of (3) is obtained if and only if $x_{N+1}(t) \equiv 0$. In the frequency domain the following equations are obtained, using Kronecker's $\otimes$ product

$$\begin{aligned} F(X) = I(X) + j\,\Omega\,Q(X) + e_n^N \otimes X_{N+1} \\ X_n - B = 0. \end{aligned} \tag{8}$$

The steady state condition is obtained if and only if the constraint

$$X_{N+1}(B, \omega) = 0 \tag{9}$$

is met. Note that (9) is an under-determined system $\mathbb{C}^{2K+1} \times \mathbb{R} \rightarrow \mathbb{C}^{2K+1}$, because the non-autonomous system (8) is assumed to be time-invariant. In a first step, (8) is solved keeping stimulus B and ω fixed until convergence is reached. In the next step, the approximate $(B^{(\nu)}, \omega^{(\nu)})$ of (9) will be updated. The algorithm proceeds with the first step until convergence of the outer loop is achieved. The under-determined system (9) can be solved by fixing the phase of the k-th harmonic of the stimulus, i.e. ${e_k^K}^T B - {e_{-k}^K}^T B = 0$. Using the affine invariance technique as damping strategy to Newtons method increases the convergence domain of the two stage algorithm drastically [8], but still the success of the two stage algorithm depends on the user estimated initial amplitude of the probe voltage source.

3 Initialization techniques

The problem in oscillator simulation consists of determining good initial values for the node voltages and the frequency of oscillation. Standard techniques for estimating the oscillation frequency are the Kurokawa condition method [7] and the generalized eigenvalue analysis.

3.1 The Kurokawa condition

The linearization of (7) at the operating point leads to

$$\begin{aligned} g_{DC}\, x(t) + c_{DC}\, \dot{x}(t) + e_n^m\, x_{N+1} = 0 \\ x_n - b(t) = 0, \end{aligned}$$

where $b(t)$ is a sinusoidal stimulus with unit amplitude. Kurokawa's method calculates the initial frequency in such a way that the impedance at the driving point has vanishing imaginary part. In other words, calculate the phasor X_{N+1} from

$$\begin{aligned} g_{DC}\, X + j\,\omega\, c_{DC}\, X + e_n^N\, X_{N+1} &= 0 \\ X_n &= 1 \end{aligned} \qquad (10)$$

such that $\mathrm{Im}\{X_{N+1}(\omega)\} = 0$. This can be done by a sweep of evaluations of (10) in a predefined frequency domain, or by Newton's method if the initial estimate of the frequency is close enough to the exact solution. In the latter case the following system of equations has to be solved

$$\begin{aligned} J_{X_{N+1}}\, \Delta\,\omega^{(\nu)} &= -\mathrm{Im}\{X_{N+1}(\omega)\} \\ \omega^{(\nu+1)} &= \omega^{(\nu)} + \Delta\,\omega^{(\nu)} \end{aligned}$$

where the Jacobian $J_{X_{N+1}}$ is obtained from calculating

$$g_{DC}\,\frac{\mathrm{d}X}{\mathrm{d}\omega} + j\,\omega\, c_{DC}\,\frac{\mathrm{d}X}{\mathrm{d}\omega} + e_n^N\,\frac{\mathrm{d}X_{N+1}}{\mathrm{d}\omega} = -j\, c_{DC}\, X.$$

After a few iterations a good estimate of the oscillation frequency is achieved, but there is no information gained about the initial node voltages of the circuit.

3.2 The generalized eigenvalue analysis

Because an autonomous circuit only can start up oscillating when the operating point is unstable (Andronov-Hopf bifurcation theorem, [3]), all generalized eigenvalues having $\mathrm{Re}\{\lambda\} > 0$ are interesting. The generalized eigenvalues can be calculated by

$$g_{DC}\, X_E + c_{DC}\, X_E\, \Lambda = 0, \qquad (11)$$

where X_E is the coefficient matrix of the N generalized eigenvectors. Under the assumption that there is only one dominating oscillation, there exist a complex pair of corresponding eigenvalues, $\lambda = \delta \pm j\,\omega$. From the above $\delta > 0$. For an index of the DAE larger than zero the matrix c_{DC} is singular, therefore generalized eigenvalues $\lambda \to \infty$ exist which lead to numerical problems using the QZ algorithm. For this reason, the inverse generalized eigenvalue problem is prefered

$$g_{DC}\, X_E\, \Lambda + c_{DC}\, X_E = 0 \qquad (12)$$

which is more stable because commercial circuit simulators reject circuits where g_{DC} is singular, i. e. there exists no DC path to ground from all circuit nodes. Therefore g_{DC} is a regular matrix and the eigenvalues λ are bounded. Also in this case, a good estimate of the oscillation frequency is achieved, but no information about the initial node voltages of the circuit.

4 Optimization based initialization

The initialization techniques mentioned in section 3 only deliver a crude estimate of the oscillation frequency of the autonomous circuit. The initial amplitude of the probe source that leads to a successful simulation has to be guessed. The one dimensional optimization based initialization procedure overcomes this problem. The solution of this fast procedure is a probe source amplitude value in the convergence domain of the two stage algorithm.

For some high Q oscillator circuits, the probe current will exhibit a minimum only if the initial frequency is extremely close to the true oscillation frequency [2]. In these cases, it is necessary to optimize the initial frequency in order to lead the two stage approach to a successful calculation of the limit cycle. This holds for low Q oscillators, too. The two dimensional optimization based initialization procedure determines a solution for the frequency and the amplitude of the probe source in the convergence domain of the two stage algorithm.

4.1 One dimension

For most high Q oscillators, the eigenvalue analysis and the Kurokawa condition techniques exhibit good oscillation frequency estimations, whereas the initial amplitude of the probe source has to be guessed. To become free of guessing the initial amplitude of the probe source, the following optimization based initialization procedure has been developed.

Imagine a two dimensional coordinate system with the axis representations

- x: amplitude $\|B\|$ of the additional probe source
- y: current $\|X_{N+1}\|$ through the probe source

where the amplitude axis is bounded by the lower and upper supply voltage of the oscillator circuit. The initial frequency is fixed. This leads to a curve with several local minima and one global minimum which is in the basin of attraction of the limit cycle of the oscillator circuit[1]. Newton methods are likely to get stuck in local minima or fail completely because of a singular Jacobian if the initial guess for the probe source amplitude is not in the basin of attraction of the global minimum. To overcome this problem, a derivative free global optimization technique can be used to deliver a crude initialization within the range of convergence of Newton methods. The crude solution of the optimization algorithm is the initialization for the fast two stage algorithm.

To detect the basin of attraction of the global minimum of a one dimensional function $f(x)$, defined on the interval $[l, u]$, the easy to implement Dividing Rectangles (DIRECT) algorithm [5, 1] - an enhancement of the standard

[1] one limit cycle presupposed

Lipschitzian approach - is of first choice. The basics of the DIRECT algorithm are as follows. Standard Lipschitzian algorithms assume that the rate of change of the function $f(x)$ is bounded. They assume that a positive constant K_L exists, called the Lipschitz constant, such that

$$|f(x) - f(x^{'})| \leq K_L|x - x^{'}|, \quad x, x^{'} \in [l, u] \qquad (13)$$

is fulfilled. The evaluation of $f(x)$ at the center point $c = x^{'} = (l + u)/2$ leads to the following inequalities

$$f(x) \geq f(c) + K_L(x - c), \quad \text{for } x \leq c, \qquad (14)$$

$$f(x) \geq f(c) - K_L(x - c), \quad \text{for } x \geq c. \qquad (15)$$

These inequalities can be regarded as two lines with slopes $+K_L$ and $-K_L$. The function has to be above the inverted V formed by the intersection of the two lines. At the endpoints of the interval, the lowest function values that can attain occur. The lower bound can be calculated to

$$\text{lower bound} = f(c) - K_L(b - a)/2. \qquad (16)$$

In the next step, the interval is divided into thirds, and the function $f(x)$ is evaluated at the center points of the left and right third. The original center point becomes the center of a smaller interval. The next step is to select one of these intervals for further sampling. For futher details concerning the interval selection procedure, see [5].

4.2 Two dimensions

The true frequency of oscillation can be different from the initial frequency obtained by using linear analysis techniques like the Kurokawa condition techniques or the generalized eigenvalue analysis. The difference can be explained by capacitances and conductances of semiconductor devices which are nonlinear functions of the terminal voltages. The difference between the estimated and the exact frequency of oscillation is not large for most high Q oscillator circuits, but they are large when compared to the range of convergence of the solution. It is possible, that the probe current $\|X_{N+1}\|$ will exhibit a minimum only if the frequency is extremely close to the exact oscillation frequency [2]. In these cases, the two stage algorithm will fail. To overcome this problem, the one dimensional optimization based initialization procedure explained in section 4.1 can be extended as follows. Imagine now a three dimensional coordinate system with the axis representations

- x: amplitude $\|B\|$ of the additional probe source
- y: frequency
- z: current $\|X_{N+1}\|$ through the probe source

where the amplitude axis is bounded by the lower and upper supply voltage and the frequency axis by an estimated frequency (Kurokawa, eigenvalue analysis) plus and minus a tolerance factor. This leads to a bounded plane with several local minima and one global minium, the limit cycle of the oscillator circuit. Figure 1 illustrates the current through

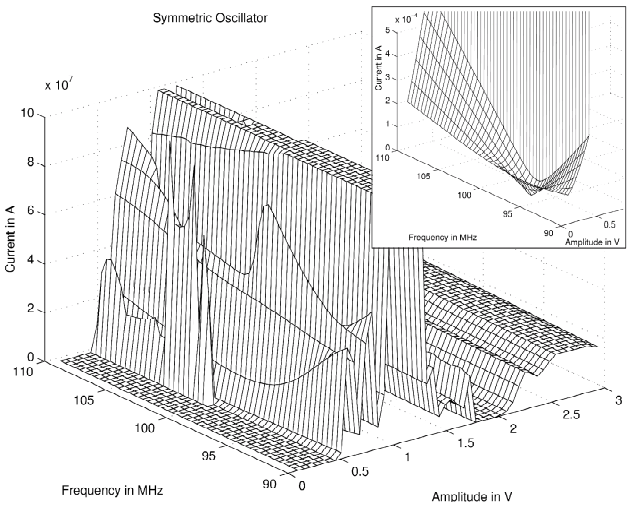

Figure 1. Current through additional probe source in bounded box. Top right: Figure zoomed in z-direction

the additional probe voltage source in dependence on the frequency and the amplitude of the probe source for a symmetric oscillator circuit. To detect the basin of attraction of the global minimum, the derivative free global optimization technique DIRECT can be used in this two dimensional case as well. For further details, see [5].

5 Results

The effectiveness of the one and two dimensional optimization based initialization procedures will be shown by means of simulation results for two typical oscillator circuits, a symmetric oscillator and a VHF-oscillator. For both oscillators, the obtained convergence range and the number of function and Jacobian evaluations will be compared for the two stage algorithm, the affine invariance damped two stage algorithm [8] and the optimization based initialized two stage algorithm. In all cases, the initial frequency is calculated by the Kurokawa condition. The optimization based initialization procedure switches to the two stage algorithm if $\|X_{N+1}\| < 10^{-3}$ is reached. The simulation results of the symmetric oscillator are listed in Table 1. The one (two) dimensional optimization based initialization procedure needs 5 (13) function and Jacobian evalutions and the following two stage algorithm 15 (15) evaluations, in

total 20 (28). That is as fast as the pure two stage algorithm but now without any convergence range restrictions. The

Table 1. Symmetric Oscillator

Initialisation	Damping	Convergence Range	Func.
Kurokawa	/	$0.1V \leq U \leq 0.4V$	21
Kurokawa	Affine Inv.	$0.1V \leq U \leq 0.7V$	65
One Dim.	/	$0.1V - 3V(vdd)$	20
Two Dim.	/	$0.1V - 3V(vdd)$	28

number of function evaluations for the Kurokawa initialized versions of the two stage algorithm are averaged values over the indicated convergence range. This holds for Table 2, too.

Table 2 presents the results of the VHF-oscillator. After 5 (13) function and Jacobian evaluations the one (two) dimensional initialization procedure switches to the two stage algorithm and after another 19 (19) evaluations, the limit cycle is calculated. That is in total 24 (32) Newton iterations, only slightly more overhead compared to the two stage algorithm initialized by Kurokawa, but faster than the damped version and with a convergence range over the whole supply voltage.

Table 2. VHF-Oscillator

Initialisation	Damping	Convergence Range	Func.
Kurokawa	/	$0.3V \leq U \leq 0.9V$	18
Kurokawa	Affine Inv.	$0.1V \leq U \leq 1.2V$	37
One Dim.	/	$0.1V - 5V(vdd)$	24
Two Dim.	/	$0.1V - 5V(vdd)$	32

The one dimensional initialization procedure fails in case of an eigenvalue analysis initialized voltage controlled oscillator circuit. The initial frequency is about 2.072 GHz, the true frequency of oscillation is about 2.224 GHz. The two dimensional intitialization procedure overcomes this problem by optimizing the initial frequency and the probe source amplitude. After 29 function and Jacobian evaluations the two dimensional optimization based initialization procedure switches to the two stage algorithm and after another 13 evaluations, the limit cycle is calculated. That is in total 42 Newton iterations.

6 Conclusions

A one and two dimensional optimization based initialization procedures were presented which overcome the problem of determining good initial values for the two stage algorithm. These are fast and reliable procedures as the examples have shown. No multiple runs of simulations are needed because of the presented deterministic optimization technique.

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