Generation of Optimum Test Stimuli for Nonlinear Analog Circuits Using Nonlinear Programming and Time-Domain Sensitivities

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Abstract

In this paper a novel approach for the generation of an optimum transient test stimulus for general analog circuits is proposed. The test stimulus is optimal with respect to the detection of a given fault set by means of a predefined fault detection criterion. The problem of finding an optimum test stimulus detecting all faults from the fault set is formulated as a nonlinear programming problem. A functional describing the differences between the good and all faulty test responses of the circuit serves as a merit functional for the programming problem. A parameter vector completely describing the test stimulus is used as the optimization vector. The gradient of the merit functional required for the optimization is computed using time-domain sensitivities. Since in this approach the evaluation of the fault detection criterion represented by the merit functional flows directly into the computation of the test stimulus, optimal test stimuli for hard to detect faults can be generated. If more than one input terminal is used for testing, several test stimuli can be generated simultaneously.

1. Introduction

Due to the rapid evolution of integrated circuits there is a great demand for tools and efficient techniques to test and diagnose these circuits. Although the area of the analog part of a mixed-signal IC is much smaller than the digital one, the test costs are dominated by the analog part because of its more complex specifications. Only the calculation of specific tests, which are generated for each circuit individually, allows short test times and therefore a low-cost test. Whereas in the digital domain many efficient techniques for test generation exist, test generation in the analog domain is still subject of intensive research.

Early work in the area of test generation for analog circuits concentrated on efficient application for specification tests. Approaches determining the optimal sequence of tests, which minimizes average test time, and methods dropping redundant test by exploiting high correlations between tests, are proposed in [1] and [2]. Cost saving DC based tests proposed in [3-4] are used at wafer-probe stage to quickly identify defective devices. DC tests are limited by their inability to test the dynamic behavior of a circuit and thus do not have the capability to find all faults in a circuit. Tests based on an AC stimulus, are proposed in [5-9]. These AC tests excite the circuit in the vicinity of the operating point and therefore are suitable for circuits, where the specifications are strongly related to the behavior of the circuit in the frequency domain. Transient tests [10-14] can be subdivided into tests, that suppose linear properties of the circuit, and tests, which are only applicable to circuits with non-linear behavior. For linear circuits a quadratic programming approach in [10] is used to maximize the difference between good and faulty test responses, while in [12] hyperplanes are calculated to distinguish between good and faulty circuits. In [13] an alternate test stimulus is calculated from the eigenvalues of the sampled device. The methods in [10], [12] and [13] require some features of the state-space representation of the circuit to be known, like the impulse response or the system matrix. For nonlinear circuits a divide and conquer strategy is used in [14] to sequentially synthesize the test stimulus for the entire duration of the test. This approach is very fast and yields a test stimulus of minimum length. A minimax formulation is used in [11] to maximize the difference between a good and a faulty circuit. Transient tests for analog circuits are supersets of DC and AC test. Since transient waveforms are able to test the complete dynamic behavior of a circuit, they have the capability to detect a larger number of faults than DC and AC tests do.

In this paper a new test generation method based on an optimization procedure is proposed to generate optimum transient test stimuli for a general analog circuit. A merit functional required for the optimization problem is used as the fault detection criterion. The merit functional, which depends on the parameters of the test stimulus, values the difference of the good test response to all faulty test responses of the circuit. The goal of the optimization process is to maximize the merit functional with respect to parameters of the test stimulus and thus to increase the fault detection capability of the stimulus. The computation of the gradients of the circuits, which are represented by time-domain sensitivities and required for optimization, is performed using the generalized adjoint network approach [15]. The rest of the paper is organized as follows: In the next section, the test generation method is presented. Implementation and experimental results are described in section 3. A conclusion of the paper is given in section 4.

2. Test generation method

The proposed test generation method presented in this paper is described in the next three subsections. In the first subsection the test generation problem is formulated as a nonlinear programming problem (NLP). The second subsection describes the computation of the gradients for the optimization problem, which constitutes the essential part of the test generation method. The presentation of the optimization procedure is given in the last subsection.

2.1. Problem statement

Given the netlist of the circuit under test (CUT), a fault list containing catastrophic and parametric faults, the duration of the test designated by the simulation time T_{sim} and the allowed range of values for the test stimulus. For simpler presentation we introduce some notations and assume without loss of generality, that test stimulus and test response are voltages and only one stimulus shall be calculated. Throughout the paper the good device is denoted with the index g and the faulty devices are denoted with the index f.

$F=\{f_1,...,f_k\}$	fault set with k faults
$\mathbf{w},\ \mathbf{w}\in\Re^k$	weighting vector for faults
$v_{inp}(t),\ 0\le t\le T_{sim}$	test stimulus
$v_{out}(t),\ 0\le t\le T_{sim}$	test response
$\mathbf{x},\ \mathbf{x}\in\Re^n$	parameter vector

The test stimulus is described entirely by the parameter vector **x**, as shown in Figure 1. A parametric representation of the input voltage source describing the test stimulus is given by (1). If more than one stimulus is used for testing, e.g. when testing circuits with multiple inputs, the parameter vector **x** contains the parameters of all test stimuli. The step waveform is chosen as building block for the test stimulus because of its high spectral content and hence its ability to excite a large range of faulty conditions. The parameter T_{step} is a measurement for the resolution of the test stimulus and should be chosen properly (n > 10).

In (2) and (3) the test generation problem is formulated as a NLP-problem. The merit functional $\psi(\mathbf{x})$ values the difference between the good test response and all faulty test responses of the CUT, whereas $\Phi^{f_j}(\mathbf{x})$ represents the difference arising from the fault f_j. For the estimation function $f(\cdot)$ the absolute value and the square function are used. The weighting factors w_j are used to distinguish hard to detect faults in the merit functional $\psi(\mathbf{x})$. The inequality constraints of the NLP-problem, used for restriction of the test stimulus, are described by the vector $\mathbf{g}(\mathbf{x})$. The solution of the NLP-problem is given by the parameter vector **x**, which minimizes the functional $-\psi(\mathbf{x})$ in consideration of the inequality constraints $\mathbf{g}(\mathbf{x})$ and hence maximizes $\psi(\mathbf{x})$ to enhance the fault detection capability. The computation of $\psi(\mathbf{x})$ is performed by a fault simulation, simulating each fault of the fault set F sequentially.

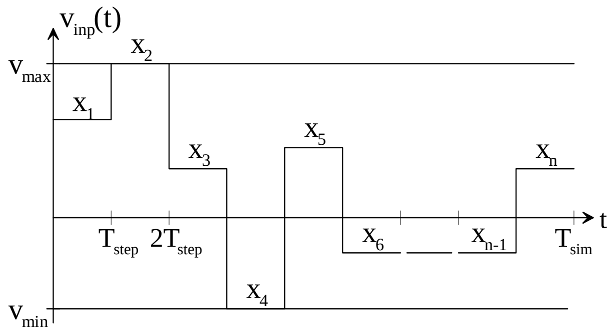

Figure 1. Parametric representation of the test stimuli

$$v_{inp}(t)=\sum_{i=1}^{n}x_i[1(t-(i-1)T_{step})-1(t-iT_{step})]\ ,\ n=\frac{T_{sim}}{T_{step}}$$
$$=v_{inp}(\mathbf{x},t)\ ,\ v_{min}\le v_{inp}\le v_{max} \quad (1)$$

$$\text{NLP:}\quad \min_{\mathbf{x}\in\mathbb{R}^n} -\psi(\mathbf{x}) \quad (2)$$
$$\mathbf{g}(\mathbf{x})\ge 0,\ \mathbf{g}\in\mathbb{R}^m$$
$$g_1=v_{max}-x_1$$
$$g_2=-v_{min}+x_1$$
$$\vdots$$
$$g_m=-v_{min}+x_n$$

$$\Phi^{f_j}(\mathbf{x})=\int_0^{T_{sim}} f(v_{out}^g(t)-v_{out}^{f_j}(t))\,dt$$
$$=\int_0^{T_{sim}} f(v_{out}^g(\mathbf{x},t)-v_{out}^{f_j}(\mathbf{x},t))\,dt \quad (3)$$
$$\psi(\mathbf{x})=\sum_{j=1}^{k}w_j\Phi^{f_j}(\mathbf{x})$$

Through parameterization of the control variables, which are the voltages and currents of the test stimuli sources, a constrained optimal control problem is transformed into a finite dimensional NLP-problem. A SQP-method [16] is used to solve the NLP-problem. Sequential quadratic programming (SQP) is known to be the most efficient computational method to solve the general NLP-problem.

2.2. Sensitivity Calculation

In order to solve the NLP-problem using the SPQ-method the gradient of the merit functional $\psi(\mathbf{x})$ is required. For computation of the gradient the generalized adjoint network approach [15] is applied to the good and all faulty circuits. To calculate parameter sensitivities with the aid of the adjoint network, a parametric representation of the network elements is needed. The parametric representation of the voltage source describing the test stimulus is given in (1). The following part of the paper explains, how to calculate $\nabla\psi(\mathbf{x})$.

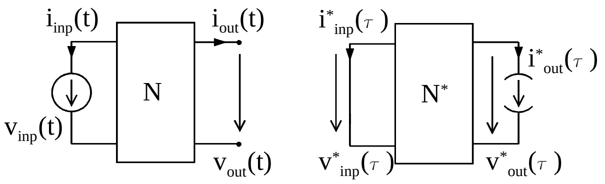

Figure 2. The network N and the adjoint network N* of the CUT

Given the network N and the adjoint network N^* of the CUT, shown in Figure 2. The variables in the adjoint network N^* are denoted with an asterisk. Since the requirement is made, that both N and N^* have the same topology but not necessarily the same element types in the corresponding branches, Tellegen's theorem can be applied, consider (4a) and (4b).

$$\sum_B v_b(t)\, i_b^*(\tau) \equiv 0 \quad (4a) \qquad \sum_B i_b(t)\, v_b^*(\tau) \equiv 0 \quad (4b)$$

The summation is taken over all branches of the networks N and N^*. Since Tellegen's theorem is independent of element values of N, it can be written in the way of (5). Variations in the branch voltages and currents caused by pertubation in the element values of N are denoted by $\Delta v_b(t)$ and $\Delta i_b(t)$.

$$\int_0^{T_{sim}} \sum_B \Delta v_b(t)\, i_b^*(\tau) - \Delta i_b(t)\, v_b^*(\tau)\, dt \equiv 0 \qquad (5)$$

Since the choice of the element types in the adjoint network is arbitrary, they are chosen in such a way as to render (5) independent of all $\Delta v(t)$ and $\Delta i(t)$ terms except for the output variable $\Delta v_{out}(t)$ and $\Delta i_{out}(t)$. The reason for this choice is that we are interested in the variation of the output variable with respect to element variations. The time variable in the adjoint network denoted by τ is chosen as $\tau = T_{sim} - t$, what is a arbitrary choice and will become clear later. By suitable choice of the elements in the adjoint network, e.g. by setting an independent voltage source to zero in N^*, and the fact that we are only interested in the variation of the output variable with respect to variations of the test stimulus $v_{inp}(\mathbf{x},t)$, (5) reduces to the following term.

$$\int_0^{T_{sim}} \Delta v_{out}(t)\, i^*_{out}(\tau) + \Delta v_{inp}(t)\, i^*_{inp}(\tau)\, dt + \Delta Q \equiv 0 \quad (6)$$

The transversal conditions, which result from the initial and final conditions of the networks N and N^*, are denoted by ΔQ. In many situations they do not have to be calculated. Using the parametric representation of $v_{inp}(\mathbf{x},t)$ we obtain (7) form Equation (6).

$$\int_0^{T_{sim}} -\Delta v_{out}(t)\, i^*_{out}(\tau)\, dt = \int_0^{T_{sim}} i^*_{inp}(\tau) \frac{d v_{inp}(t)}{d x_i} \Delta x_i\, dt + \Delta Q \quad (7)$$

The variation of the left side of (7) with respect to the i-th component of the parameter vector $\mathbf{x}$ is given by (8).

$$\frac{\Delta \int_0^{T_{sim}} -i^*_{out}(\tau)\, v_{out}(t)\, dt}{\Delta x_i} = \frac{\int_0^{T_{sim}} -i^*_{out}(\tau)\, \Delta v_{out}(t)\, dt}{\Delta x_i} = \int_0^{T_{sim}} i^*_{inp}(\tau) \frac{d v_{inp}(t)}{d x_i}\, dt + \frac{\Delta Q}{\Delta x_i}, \quad 1 \le i \le n \qquad (8)$$

Since the waveform of $i^*_{out}(\tau)$ can be chosen arbitrarily, we set $i^*_{out}(\tau)$ in such a manner, that the left side of (8) represents the desired sensitivity term. For example, if we want to obtain the sensitivity of the output voltage v_{out} with respect to the parameter x_i at time $t = T_{sim}$, we choose i^*_{out} as $i^*_{out}(\tau) = -\delta(\tau) = -\delta(T_{sim}-t)$.

For the computation of the gradient of $\psi(\mathbf{x})$ first only the gradient of $\Phi^{fj}(\mathbf{x})$ as shown in (9) is considered, representing the part of $\nabla\psi(\mathbf{x})$ introduced by the fault f_j.

$$\frac{\Delta\Phi^{f_j}(\mathbf{x})}{\Delta x_i} = \frac{\Delta \int_0^{T_{sim}} f(v^g_{out}(t) - v^{f_j}_{out}(t))\, dt}{\Delta x_i} = \frac{\Delta \int_0^{T_{sim}} f(e^{f_j}(t))\, dt}{\Delta x_i} = \frac{\int_0^{T_{sim}} \frac{df}{de}(t)\, \Delta v^g_{out}(t)\, dt}{\Delta x_i} - \frac{\int_0^{T_{sim}} \frac{df}{de}(t)\, \Delta v^{f_j}_{out}(t)\, dt}{\Delta x_i}, \quad 1 \le i \le n \qquad (9)$$

With the aid of Equation (8) both partial gradients of the right side of (9) are calculated and consequently the calculation basis for $\nabla\Phi^{fj}(\mathbf{x})$ is given by (10).

$$\frac{\Delta\Phi^{f_j}(\mathbf{x})}{\Delta x_i} = \left[\int_0^{T_{sim}} i^*_{inp}(\tau) \frac{d v_{inp}(t)}{d x_i}\, dt + \frac{\Delta Q}{\Delta x_i}\right]^g - \left[\int_0^{T_{sim}} i^*_{inp}(\tau) \frac{d v_{inp}(t)}{d x_i}\, dt + \frac{\Delta Q}{\Delta x_i}\right]^{f_j}, \quad 1 \le i \le n \qquad (10)$$

The first part of the gradient of $\Phi^{fj}(\mathbf{x})$ is calculated with the current $i^*_{inp}(\tau)$ from the adjoint network N^{g*} of the fault free device. The second term is calculated in a similar manner with the faulty device. Both adjoint networks N^{g*} and N^{fj*}

are excited with the same current source $i^*_{out}(\tau)$, which is given by (11).

$$i^*_{out}(\tau) = -\frac{df}{de}(T_{sim} - \tau)\ ,\ e^{f_j}(t) = v^g_{out}(t) - v^{f_j}_{out}(t) \qquad (11)$$

In the first step of the computation of $\nabla\Phi^{fj}(\mathbf{x})$ the transient analysis of the fault free network N^g and the faulty network N^{fj} is carried out for the time interval $t=[0,T_{sim}]$. The node voltages of both networks are stored at each timepoint of the forward simulations, because they are needed for calculation of the elements in the adjoint networks, see Equation (11). This step is already achieved by evaluating the term of $\Phi^{fj}(\mathbf{x})$. In the second step the transient analyses of the adjoint networks N^{g*} and N^{fj*} are performed for the time interval $\tau=[0,T_{sim}]$, although the evaluation of the sensitivity components obtained from (10) is carried out during analysis. Since the adjoint networks are linear and time-variant, their analysis can be performed very fast compared to those of the original networks. The main advantage of the adjoint method is given by the fact, that all parameter sensitivities can be obtained from only two circuit analyses. Consequently the gradient of $\psi(\mathbf{x})$ can be written as the weighted sum of the gradients of $\Phi^{fj}(\mathbf{x})$.

$$\nabla_{\mathbf{x}}\psi(\mathbf{x}) = \sum_{\mathbf{j=1}}^{\mathbf{k}} w_j \nabla_{\mathbf{x}} \Phi^{f_j}(\mathbf{x}) \qquad (12)$$

2.3. Optimization process

In the two previous subsections it has been described how to calculate the merit functional $\psi(\mathbf{x})$ and its gradient $\nabla\psi(\mathbf{x})$, which are both required for the optimization process. In this subsection the essential steps of the optimization process are explained more closely. Figure 3 shows the essential steps of the SOP-algorithm, which is used to solve the NLP-problem formulated in (2).

The NLP-problem is solved iteratively. Starting with a given parameter vector $\mathbf{x_0}$, which represents the test stimulus to be optimized, the $(l+1)^{st}$ iterate $\mathbf{x_{l+1}}$ is obtained from $\mathbf{x_l}$ by evaluating block 7, where $\mathbf{d_l}$ is the search direction within the l^{th} step and α_l is the step length. The search direction is determined by a quadratic programming subproblem (QP), which is formulated by a quadratic approximation of the Lagrange function $L(\mathbf{x},\boldsymbol{\lambda})$ of the NLP-problem and a linear approximation of the constraints $\mathbf{g}$, see block 2. The search direction $\mathbf{d}$ has a strong analogy to the search direction in Newton's method for solving systems of nonlinear equations and hence is optimal with stepsize $\alpha = 1$. For calculation of $\mathbf{d_l}$ the gradients of ψ and $\mathbf{g}$ are needed. The calculation of the stepsize α_l takes place in block 4 using line search. Starting with $\alpha = 1$ block 4 is processed as long as the merit functional $\varphi(\mathbf{x})$ has a value greater than $\varphi(\mathbf{x_l})$ along the search direction. The line search is used to modify the stepsize α for parameter vectors $\mathbf{x}$ far from the optimum and thus to guarantee global convergence of the method.

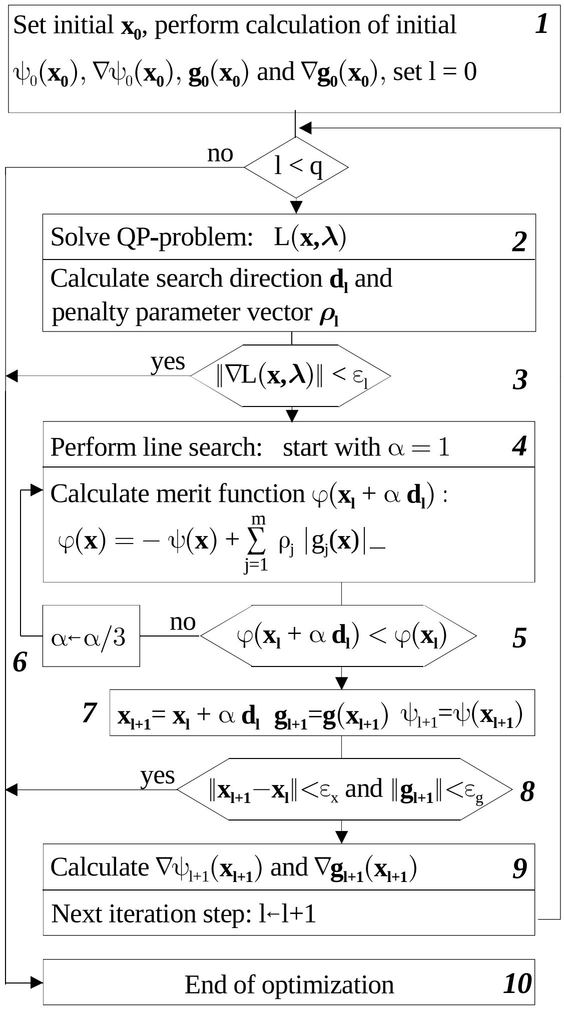

Figure 3. Optimization procedure

The penalty parameters denoted by the vector $\boldsymbol{\rho}_\mathbf{l}$ are required to consider violations of the inequality constraints $\mathbf{g}$. In every iteration through block 4 a complete fault simulation takes place to calculate the merit functional $\psi(\mathbf{x})$ and the merit functional $\varphi(\mathbf{x})$ from $\psi(\mathbf{x})$. Given that in presence of a fault a circuit cannot be simulated using the current test stimulus $v_{inp}(\mathbf{x_l},t)$ due to convergence problems, the fault will be removed from the fault list. Since removing a fault from the fault list changes the structure of the merit function $\psi(\mathbf{x})$ as defined in (3), a new optimization has to be performed, starting with the initial guess $\mathbf{x_0} = \mathbf{x_l}$ for the parameter vector. The gradients of ψ and $\mathbf{g}$, which are needed to determine the search direction $\mathbf{d_{l+1}}$ in the next iteration step, are computed in block 8. The optimization process terminates, either when a local minimum of $-\psi$ is found, or when a minimum of $-\psi$ is found at $\mathbf{g}$ near zero, or when the maximum iteration limit denoted by q is reached. In all three cases the fault detection capability of the test stimulus is improved.

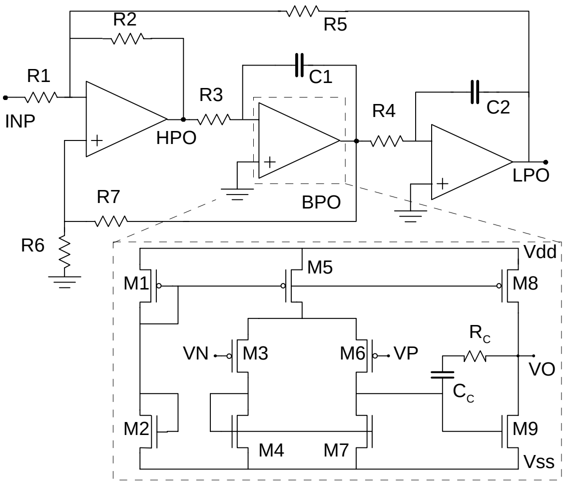

Figure 4. State-variable active filter (LPO: Low-pass output)

3. Implementation and results

The approach proposed in this paper has been implemented in a program named TORAD (**T**est Generat**or** for **A**nalog **D**evices). Several circuit analyses, like transient analysis and transient sensitivity analysis, which were required for the test generation process, have been implemented in the program.

Fault	$\Phi^{fj}(\mathbf{x_0})$	$\Phi^{fj}(\mathbf{x_{opt}})$	w_j
f1 : F(p,r7:resistance,5%)	8.374e-06	2.644e-05	1
f2 : F(p,r6:resistance,5%)	8.347e-06	3.317e-05	1
f3: F(p,c2:capacitance,5%)	1.551e-05	1.034e-04	1
f4 : F(p,c1:capacitance,5%)	9.818e-06	9.602e-05	1
f5 : F(p,m2_op1:l:,5%)	5.887e-08	7.431e-04	1
f6 : F(p,m2_op2:l,5%)	1.623e-08	6.528e-04	1
f7 : F(p,m5_op1:l,5%)	4.747e-09	3.998e-06	1
f8 : F(p,m5_op2:l,5%)	1.506e-08	9.289e-07	1
f9 : F(p,m5_op3:l,5%)	1.597e-08	6.967e-07	1
$\psi(\mathbf{x_0})$ = 4.21625288e-05 $\psi(\mathbf{x_{opt}})$ = 1.66071701e-03	**Number of iterations l :** 25		
$\mathbf{x_0}$ = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] $\mathbf{x_{opt}}$ = [-5, -5, -5, 1.1, 5, 5, 5, 5, -4.8, -4.2, -5, -1, 0.7]			

Table 1. Results of the optimization process

In this section the test generation method is applied to a time continuous state-variable active filter shown in Figure 4. The filter belongs to one of the ITC mixed-signal test benchmark circuits presented in [17]. The low-pass output of the filter denoted by LPO is selected as the output node

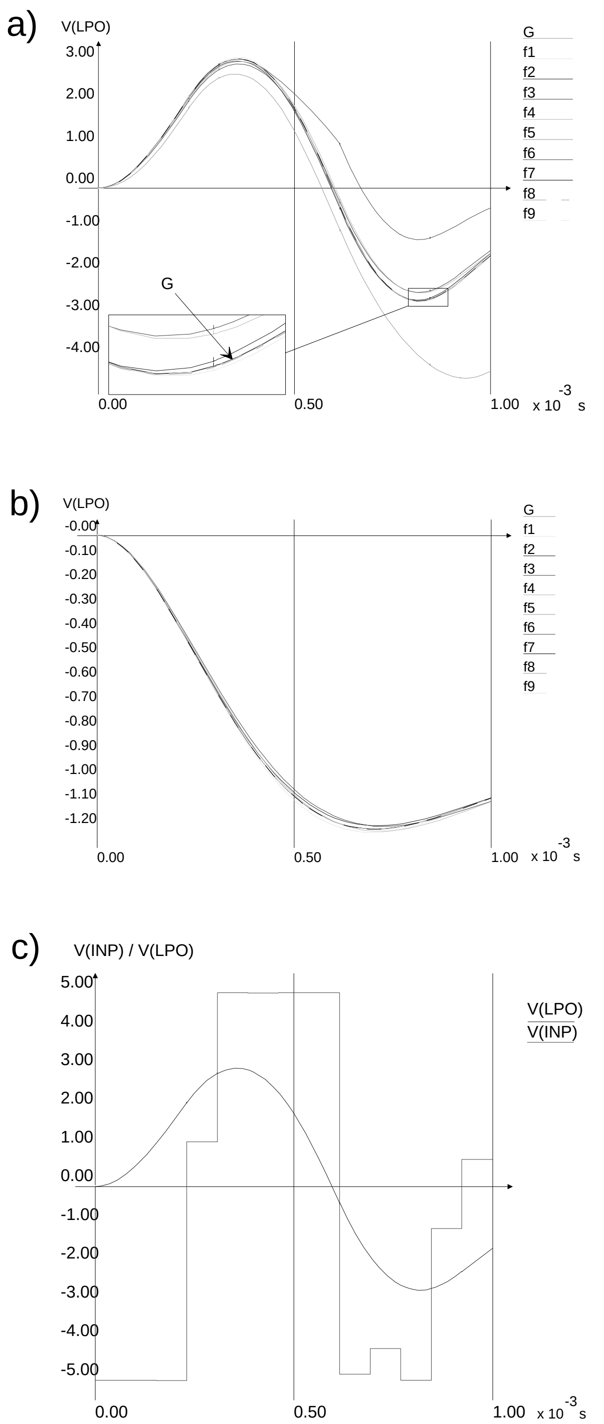

Figure 5. a.) Test responses after optimization b.) Test responses before optimization c.) Optimal test stimulus and test response of the good device

for test response measurements. To reduce the computational effort for the test generation, a small fault set F, which contains nine hard to detect parametric faults shown in Table 1, is used. The weighting factors for the faults are set manually or they are calculated from the results of the $\Phi^{fj}(\mathbf{x})$, which are determined by a previous fault simulation. For the state-variable filter the weighting factors are set to 1, because no improvement for the optimization process was established by changing these factors. The simulation results of the test generation are listed in Table 1. The test responses of the CUT before and after optimization using $\mathbf{x_0}$ and $\mathbf{x_{opt}}$ as test stimuli are shown in Figure 5. Figure 5b. shows the test responses of the good and all faulty circuits excited with the test stimulus $\mathbf{x_0}$ and Figure 5a. shows the test responses of the good and all faulty circuits excited with the test stimulus $\mathbf{x_{opt}}$. The unit step function was chosen as the initial test stimulus represented by $\mathbf{x_0}$ to demonstrate the global convergence of the test generation method. One optimum test stimulus for the circuit generated within 25 optimization iterations is presented by $\mathbf{x_{opt}}$, see Figure 5c. As one can see from the values of the merit functionals $\psi(\mathbf{x_0})$ and $\psi(\mathbf{x_{opt}})$ the generated test stimulus $\mathbf{x_{opt}}$ significantly enhances the fault detection capability. Table 2 lists the parameters used by the test generator.

Parameter	Value
Simulation time Tsim	1 ms
Vmax	5V
Vmin	-5V
Optimization variables n	13

Table 2. Parameters of the test generator used for the filter

After the test generation a fault simulation of 120 faults was performed to determine the effectiveness of the generated test stimulus, which resulted in a fault coverage of 99 percent.

4. Conclusion

In this paper a new systematic procedure to generate optimum test stimuli for general analog circuits was presented. Reasonable constraints on the input stimuli and a properly chosen fault detection criterion described by a merit functional have allowed to formulate the test generation procedure as a nonlinear programming problem. Compared to other test generation methods, in this approach time-domain sensitivities were used for the computation of the test stimuli. Only this fact has enabled the generation of optimum test stimuli. Since in this approach optimum test stimuli were generated, the method is best suited for the generation of test stimuli for hard to detect faults. The application of the proposed method was demonstrated at a realistic circuit.

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