Explicit Formulas and Efficient Algorithm for Moment Computation of Coupled RC Trees with Lumped and Distributed Elements

Qingjian Yu and Ernest S.Kuh

Electronics Research Lab. Univ. of California at Berkeley, Berkeley CA94720, USA
Email: {qjyu,kuh}@eecs.berkeley.edu

Abstract

In today's deep submicron technology, the coupling capacitances among individual on-chip RC trees have essential effect on the signal delay and crosstalk, and the interconnects should be modeled as coupled RC trees. We provide simple explicit formulas for the Elmore delay and higher order voltage moments, and a linear order recursive algorithm for the voltage moment computation for lumped and distributed coupled RC trees. By using the formulas and algorithms, the moment matching method can be efficiently implemented to deal with delay and crosstalk estimation, model order reduction and optimal design of interconnects.

1 Introduction

The RC tree is a typical model for on-chip interconnects. The Elmore delay and the moment matching method have been successfully used for the delay estimation, model order reduction and interconnect optimization [1]. These tasks can be very efficiently implemented as there are explicit formulas and a linear order algorithm for the moment computation of such RC trees [2]. In today's deep submicron technology, due to the dense placement of the interconnect wires and the large aspect ratio of wire height over wire width, the coupling capacitance between two wires may be even larger than the ground capacitance of each wire and can never be neglected. In this case, the interconnects should be modeled by several RC trees with floating capacitors connected among the tree nodes. We call such kind of interconnect model coupled RC trees.

It is well known that the capacitive coupling has significant effects on the signal delay and crosstalks. Unfortunately, no simple formulas and linear order algorithms have been published for the computation of the Elmore delay and higher order voltage moments of such coupled RC trees so that these effects cannot be as efficiently dealt with as in the single RC tree case. In the literature, three ways are often used. The first one relys on the decoupling technique, which is typically restricted to some very simple cases [6, 7]. The second one relys on the general methods for RC interconnect networks [8, 9], where the advantage of the tree structure is not taken so is not efficient enough. The third one uses very rough model, e.g., using the length of overlap between two wires as the metric of the crosstalk[10], which is far from accurate.

In this paper, we provide explicit formulas and a linear order recursive algorithm for the computation of the Elmore delay and higher order voltage moments of coupled RC trees consisting of lumped and/or distributed elements. They are based on the moment model of a capacitor, which is a current source with a known value. We use the technique of source splitting to split each floating current source into two ground sources so that the coupled RC trees are decoupled during moment computation and the algorithm for a single RC tree can be applied. For distributed RC lines, by using the same model and technique for distributed capacitances, we show that the moments of voltage and capacitive current along a line are polynomials of the coordinate of the line. We derive recursive formulas for the coefficient of these polynomials and set up an exact moment model for RC lines. With these models, we extend the linear moment computation algorithm for a single RC tree with lumped elements to coupled RC trees with both lumped and distributed elements without the need of discretization of the distributed lines. This algorithm is exact and very efficient. It is useful in the delay and crosstalk estimation, model order reduction and optimal design of such type of interconnects.

2 Coupled RC trees with lumped elements

2.1 Definitions

A set of coupled RC trees is an RC network, which consists of a number of RC trees with floating capacitors connected among the non-ground nodes of individual trees. Let $v_j^i(t)$ be the node voltage at the j-th node of the i-th tree, and $V_j^i(s)$ its Laplace transform. Let $V_j^i(s) = \sum_{k=0}^{\infty} V_{jk}^i s^k$, then V_{jk}^i is called the k-th order moment of $V_j^i(s)$. The moments of currents are similarly defined.

2.2 The Elmore delay

2.2.1 Formula

It is well known that when a unit impulse voltage is applied to the root of the i-th tree and all other roots are grounded, the negative of the first order moment $-V_{j1}^i$ is called the Elmore delay of node voltage $v_j^i(t)$. From the i-v characteristic of a capacitance C: $I(s) = sCV(s)$, and let I_k and V_k be the k-th order moment of $I(s)$ and $V(s)$, respectively, we have $I_0 = 0$ and $I_k = CV_{k-1}$; i.e., the capacitor behaves as an open circuit when the 0-th order moment is concerned, and a current source CV_{k-1} when the k-th ($k \geq 1$) order moment is concerned. Therefore, when we are concerned about the Elmore delay of $v_j^i(t)$, all the 0-th order voltage moments on the trees are equal to their root voltages. Let N^i be the tree node set of the i-th tree, and n_p^i be the p-th node in N^i; then, $V_{p0}^i = 1 \; \forall n_p^i \in N^i$ and $V_{p0}^h = 0 \; \forall n_p^h \in N^h, h \neq i$. For the first order moment, each capacitor C connected to n_p^i, either grounded or floating, behaves as a current source valued C. Let C_{Tp}^i be the total capacitance connected to the node n_p^i, and $I_{Cp}^i(s)$ be the total capacitive current flowing out of n_p^i, then its first order moment $I_{Cp,1}^i = C_{Tp}^i$. Let P_p^i be the path from the root of the i-th tree to node n_p^i, $P_{jp}^i = P_j^i \bigcap P_p^i$ and R_{jp}^i the total resistance on the path P_{jp}^i. Then, the current $I_{Cp,1}^i$ flows along path P_p^i and causes a component $-R_{jp} C_{Tp}^i$ for V_{j1}^i. Also, note that $V_{s1}^i = 0$. Therefore, we have

$$V_{j1}^i = -\sum_p R_{jp}^i C_{Tp}^i \tag{1}$$

Denote the Elmore delay of $v_j^i(t)$ as T_{Dj}^i, then

$$T_{Dj}^i = \sum_p R_{jp}^i C_{Tp}^i \tag{2}$$

It can be seen that the coupling capacitances in the active tree have the same contributions to the Elmore delay as the ground capacitances do, and the formulas of the Elmore delay are the same for both the single tree and coupled RC trees.

2.2.2 Effect of non-zero initial states on signal delay

It is well known that the signal delay in a net is affected by the signals at its neighbour[3], especially when the two nets are supplied by complement digital signals at the same time, i.e., one net's source voltage goes up from 0 to 1, and another one's from 1 to zero. In the extreme case that the source voltage of the second net is a step down function, it is equivalent to the case that the voltage supply for the second net is zero but its initial states are 1. Now we give a formula to show the effect of the non-zero initial states on the Elmore delay of the signal propagation in the first net.

Suppose that the i-th tree is in the zero initial state and supplied by a unit impulse voltage, and let S^i be its capacitively coupled trees with unit valued initial states and zero valued voltage supplies. Recall that from the i-v characteristic of a capacitor C: $i = C\frac{dv}{dt}$, when $v(0) = 1$, $I(s) = sCV(s) - Cv(0) = sCV(s) - C$, where a current source valued C and in the opposite direction of the capacitor voltage represents the effect of the initial state. Let C_{cp}^i be the total coupling capacitance connected among n_p^i and the nodes in the trees in S^i, i.e., $C_{cp}^i = \sum_{i_1 \in S^i} C_{pp_1}^{ii_1}$, where $C_{pp_1}^{ii_1}$ is the coupling capacitance connected between nodes n_p^i and $n_{p_1}^{i_1}$; then there are total capacitive current C_{cp}^i going along the path P_p^i from its source to node n_p^i, whose contribution to the zero-th order moment of V_{j0}^i is $-R_{jp}^i C_{cp}^i$, and the component of the zero-th order moment of V_{j0}^i caused by the effect of the nonzero initial states of the coupled neighbouring trees is

$$V_{j0}^{i,init} = -\sum_p R_{jp}^i C_{cp}^i \tag{3}$$

and the component of V_j^i caused by the source is

$$V_j^{i,s} = 1 - \sum_p R_{jp}^i C_{Tp}^i s + \ldots \tag{4}$$

where the superscripts "init" and "s" refer to the effect of the initial states and the source voltage, respectively.

Now suppose that a unit step voltage is applied to the i-th tree, then by superposition,

$$\begin{aligned} V_j^i &= \frac{1}{s} V_j^{i,s} + V_j^{i,init} = \\ &= \frac{1}{s}(1 - \sum_p R_{jp}^i C_{Tp}^i s + \ldots) - \sum_p R_{jp}^i C_{cp}^i + \ldots \\ &= \frac{1}{s}(1 - s \sum_p R_{jp}^i (C_{Tp}^i + C_{cp}^i) + \ldots) \end{aligned} \tag{5}$$

Now the negative coefficient of s in the parentheses $\sum_p R_{jp}^i (C_{Tp}^i + C_{cp}^i)$ is an "equivalent Elmore delay" of $v_j^i(t)$. Note that for each $C_{pp_1}^{ii_1} \in C_{cp}^i$, it is in the set of C_{Tp}, too, and the effect of the coupling capacitance to the Elmore delay due to the nonzero initial states in the worst case is equivalent to doubling the capacitance value.

Example 1.

For the coupled trees show in Fig.1, tree 1 is supplied by $V_s^1 = 1$ and tree 2 is supplied by $V_s^2 = 0$. When tree 2 is in zero initial states, the Elmore delays of v_1, v_2 and v_3 are as follows:

$$T_{D1} = R_1(C_1 + C_7 + C_2 + C_8 + C_3 + C_9)$$

$$T_{D2} = R_1(C_1 + C_7) + (R_1 + R_2)(C_2 + C_8 + C_3 + C_9)$$

$$T_{D3} = R_1(C_1 + C_7) + (R_1 + R_2)(C_2 + C_8) + (R_1 + R_2 + R_3)(C_3 + C_9)$$

When the initial states of tree 2 are 1, the "equivalent Elmore delays" (T'_D) of v_1, v_2 and v_3 are as follows:

$$T'_{D1} = R_1(C_1 + 2C_7 + C_2 + 2C_8 + C_3 + 2C_9)$$

$$T'_{D2} = R_1(C_1 + 2C_7) + (R_1 + R_2)(C_2 + 2C_8 + C_3 + 2C_9)$$

$$T'_{D3} = R_1(C_1 + 2C_7) + (R_1 + R_2)(C_2 + 2C_8) + (R_1 + R_2 + R_3)(C_3 + 2C_9)$$

Compared with the first case, it can be seen that the coupling capacitances C_7, C_8 and C_9 are doubled in the second case.

2.3 Higher order moments

Consider the k-th order voltage moments with $k \geq 1$. As mentioned in Sec.2.2, each ground capacitance C^i_j connected to node n^i_j is equivalent to a current source $C^i_j V^i_{j,k-1}$, and each coupling capacitance $C^{i_1 i_2}_{j_1 j_2}$ connected between nodes $n^{i_1}_{j_1}$ and $n^{i_2}_{j_2}$ is equivalent to a current source $C^{i_1 i_2}_{j_1 j_2}(V^{i_1}_{j_1,k-1} - V^{i_2}_{j_2,k-1})$ with direction from node $n^{i_1}_{j_1}$ to node $n^{i_2}_{j_2}$. Using the source splitting technique in circuit theory, this floating current source can be split into two ground sources, with one from node $n^{i_1}_{j_1}$ to ground valued $I_k = C^{i_1 i_2}_{j_1 j_2}(V^{i_1}_{j_1,k-1} - V^{i_2}_{j_2,k-1})$, and the other from node $n^{i_2}_{j_2}$ to ground valued $-I_k = C^{i_1 i_2}_{j_1 j_2}(V^{i_2}_{j_2,k-1} - V^{i_1}_{j_1,k-1})$, as shown in Fig.2. It is obvious that the two sources contribute to the k-th order voltage moments in the i_1-th and i_2-th tree respectively in the way similar to that the ground sources $C^{i_1}_{j_1} V^{i_1}_{j_1,k-1}$ and $C^{i_2}_{j_2} V^{i_2}_{j_2,k-1}$ do. The splitting of the floating current source into two ground sources is equivalent to decoupling the coupled RC trees during moment computation, and for each decoupled RC tree, the moment formulas and moment computation algorithm for a single RC tree can be applied. Based on this reasoning, we have the formulas for the moments of a set of coupled trees as follows, where CC^i_p is the set of coupling capacitances connected to node n^i_p:

$$V^i_{j0} = V^i_s, \qquad 1 \leq i \leq m \tag{6}$$

$$V^i_{jk} = \sum_p -R^i_{pj}\{C^i_p V^i_{p,k-1} + \sum_{C^{ii_1}_{pp_1} \in CC^i_p} C^{ii_1}_{pp_1}(V^i_{p,k-1} - V^{i_1}_{p_1,k-1})\}$$

$$= \sum_p -R^i_{pj}(C^i_{Tp} V^i_{p,k-1} - \sum_{C^{ii_1}_{pp_1} \in CC^i_p} C^{ii_1}_{pp_1} V^{i_1}_{p_1,k-1}), \qquad k \geq 1 \tag{7}$$

In the last expression of the above equation, the first term in the summation $-R^i_{pj} C^i_{Tp} V^i_{p,k-1}$ is the same as in the single RC tree case, and, in addition, each coupling capacitance has a contribution $R^i_{pj} C^{ii_1}_{pp_1} V^{i_1}_{p_1,k-1}$ to the moment $V^i_{j,k}$, which is particular for the coupled RC trees.

Example 2.
For the coupled RC trees shown in Fig.1, for $k \geq 1$, we have

$$V_{1,k} = -R_1[(C_1+C_7)V_{1,k-1} - C_7 V_{4,k-1} + (C_2+C_8)V_{2,k-1} - C_8 V_{5,k-1} + (C_3 + C_9)V_{3,k-1} - C_9 V_{6,k-1}]$$

$$V_{2,k} = -R_1[(C_1 + C_7)V_{1,k-1} - C_7 V_{4,k-1}] - (R_1 + R_2) [(C_2+C_8)V_{2,k-1} - C_8 V_{5,k-1} + (C_3+C_9)V_{3,k-1} - C_9 V_{6,k-1}]$$

$$V_{3,k} = -R_1[(C_1 + C_7)V_{1,k-1} - C_7 V_{4,k-1}] - (R_1 + R_2)[(C_2 + C_8)V_{2,k-1} - C_8 V_{5,k-1}] - (R_1 + R_2 + R_3)[(C_3 + C_9)V_{3,k-1} - C_9 V_{6,k-1}]$$

3 Coupled RC trees with distributed lines

3.1 Moment model of distributed RC lines

An RC line located in Tree i and connected between nodes n^i_j and its father node $n^i_{F(j)}$ is denoted by $Line^i_j$. For simplicity, we consider two coupled lines $Line^{i_1}_{j_1}$ and $Line^{i_2}_{j_2}$, and the result is easily extended to multiple coupled lines. Assuming that the length of the two lines is normalized to 1, and $z = 0$ and $z = 1$ correspond to the near and far end of each line. For $Line^i_j$, let C^i_j and R^i_j be its total ground capacitance and resistance, respectively, $V^i_{jk}(z)$ the k-th order voltage moment at coordinate z, $I^i_{C,jk}(z)dz$ the total k-th order moment of capacitive current at coordinate z with an infinitesimal interval dz, $I^i_{jk}(0)$ and $I^i_{jk}(1)$ the k-th order current entering and leaving the line. Let $C^{i_1 i_2}_{j_1 j_2}$ be the total coupling capacitance between $Line^{i_1}_{j_1}$ and $Line^{i_2}_{j_2}$.

We first show by induction that $V^i_{jk}(z)$ and $I^i_{C,jk}(z)$ are polynomials of z, and give recursive formulas to compute the coefficients of the polynomials.

Starting from order 0, it is known that $I^i_{C,j0}(z) = 0$ which is denoted by $\alpha^i_{j,00} = 0$. Similarly, $V^i_{j0}(z) = V^i_{j0}(0) = V^i_s$ is a constant, and is denoted by $\beta^i_{j,00}$.
For the $k-th$ order moment with $k > 0$, for $Line^i_j$, we have

$$I^i_{C,jk}(z) = C^i_j V^i_{j,k-1}(z) + C^{ii'}_{jj'}(V^i_{j,k-1}(z) - V^{i'}_{j',k-1}(z)) \tag{8}$$

where when $i = i_1$ and $j = j_1$, $i' = i_2$ and $j' = j_2$, and vice versa, and

$$V^i_{jk}(z) = V^i_{jk}(0) - R^i_j(I^i_{jk}(1)z + \int_0^z xI^i_{C,jk}(x)dx$$

$$+ z\int_z^1 I^i_{C,jk}(x)dx) \tag{9}$$

It can be seen from the formulas that when $V^i_{jk}(z)$ is a polynomial of z of order m, $I^i_{C,j,k+1}(z)$ is a polynomial of z of the same order, and $V^i_{j,k+1}(z)$ is of order $m+2$. It can be derived that $m = 2(k-1)$. Let $I^i_{C,jk}(z) = \sum_n^{2(k-1)} \alpha^i_{j,kn} z^j$ and $V^i_{jk}(z) = \sum_{j=0}^{2k} \beta^i_{j,kn} z^j$. From Eq(8), we have

$$\alpha^i_{j,kn} = C_i\beta^i_{j,(k-1)n} + C^{ii'}_{jj'}(\beta^i_{j,(k-1)n} - \beta^{i'}_{j',(k-1)n}) \tag{10}$$

and from Eq(9), we have we have

$$\beta^i_{j,k0} = V^i_{jk}(0) \tag{11}$$

$$\beta^i_{j,k1} = -R^i_j(I^i_{jk}(1) + \sum_{n=0}^{2(k-1)} \frac{\alpha^i_{j,kn}}{n+1}) \tag{12}$$

and

$$\beta^i_{j,k(n+2)} = \frac{R^i_j\alpha^i_{j,kn}}{(n+1)(n+2)}, \quad 0 \le n \le 2k-2 \tag{13}$$

Eqs(10)-(13) form a set of recursive formulas to compute the parameters α's and β's. When the k-th order 's α's are known,

$$I^i_{jk}(0) = I^i_{jk}(1) + J^i_{jk} \tag{14}$$

where

$$J^i_{jk} = \int_0^1 I^i_{C,jk}(z)dz = \sum_{n=0}^{2(k-1)} \frac{\alpha^i_{j,kn}}{n+1} \tag{15}$$

is the total k-th order capacitive current moment contributed by $Line^i_j$, and

$$V^i_{jk} = V^i_{F(j),k} - R^i_jI^i_{jk}(1) - E^i_{jk} \tag{16}$$

where

$$E^i_{jk} = \int_0^1 zI^i_{C,jk}(z)dz = \sum_{n=0}^{2k} \frac{\alpha^i_{j,kn}}{n+2} \tag{17}$$

is the total k-th order moment of the voltage drop on the line contributed by its k-th order capacitive current moment. From the above two equations, the moment model of an RC line can be presented by Fig.3 .

3.2 Equation of voltage moments

The voltage moment V^i_{jk} is contributed by both lumped and distributed elements. The component corresponding to the contribution of lumped elements is given by Eq(7). From Fig.3, it can be seen that the current source J^i_{pk} is connected to node $n^i_{F(p)}$ so that the contribution of the current source J^i_{pk} is $-R^i_{F(p),j}J^i_{pk}$, and for each $Line^i_p$ on the path P^i_j, the voltage source E^i_{pk} contributes an amount of $-E^i_{pk}$. We introduce a function $line(i,p)$ such that $line(i,p) = 1$ iff the branch connected between n^i_p and $n^i_{F(p)}$ is a line; and $line(i,p) = 0$, otherwise. We define a function $path(i,j,p)$ such that $path(i,j,p) = 1$ if $P^i_p \in P^i_j$; and $path(i,j,p) = 0$, otherwise. Then, we have the formula for the voltage moments with order $k \ge 1$ as follows, where C^i_{p0} is the lumped ground capacitance connected to n^i_p.

$$V^i_{j,k} = \sum_p \{-R^i_{pj}[C^i_{p0}V^i_{p,k-1} + \sum_{C^{ii_1}_{pp_1} \in CC^i_p} C^{ii_1}_{pp_1}(V^i_{p,k-1} - V^{i_1}_{p_1,k-1})]$$

$$+ line(i,p)[-R^i_{F(p),j}J^i_{pk} - path(i,p,j)E^i_{pk}]\} \tag{18}$$

4 Algorithm for moment computation

From the principles described in the previous sections, we provide an efficient recursive algorithm for the computation of moments in coupled RC trees. The computation is done from order 0 up to some specified maximum order K with the use of KCL and KVL of the moment model of the circuit. With order k, the computation is carried out by computing current moments from the leaves of each tree upstream to its root, then computing the voltage moments from the root of each tree downstream to its leaves. The algorithm is described as follows.

$Moment\_Computation(MAXORDER)$
$\{Zero\_Order\_Moment();$
  $for(k = 1; k \le MAXORDER; k++)$
    {if(RC lines exist) $\alpha\beta(k)$;
     for each tree T^i do
       $\{Current(i, Root(i), k);$
        $Voltage(i, Root(i),, k);$ } }
}
$Zero\_Order\_Moment()$
{for each tree T^i with source voltage V^i_s do
  for each node n^i_j in T^i do
    $V^i_{j0} = V^i_s;$
}
$\alpha\beta(k)$
{for each $Line^i_j$ do
   Compute $\beta^i_{j,(k-1)n}$, $n = 0, \ldots, 2(k-1)$;
  for each $Line^i_j$ do

{Compte $\alpha^i_{j,kn}$, $n = 0, \ldots, 2(k-1)$;
Compute J^i_{jk} and E^i_{jk}; }
}
$Current(tree\ i, node\ j, order\ k)$
{$current$=0;
if($n^i_j \neq Root(i)$)
{ $current = C^i_{j0} * V^i_{j,k-1}$;
for each coupling capacitance $C^{ii_1}_{jj_1} \in CC^i_j$ do
$curent+ = C^{ii_1}_{jj_1} * (V^i_{j,k-1} - V^{i_1}_{j_1,k-1})$; }
for each n^i_{jj} of the son node of n^i_j do
$current+ = Current(i, jj, k)$;
$I^i_{jk}(1) = current$;
if($line(i,j) == 1$)
$current+ = J^i_{jk}$;
return($current$); }
$Voltage(tree\ i, node\ j, node\ F(j), order\ k)$
{if($n^i_j == Root(i)$)
$V^i_{jk} = 0$;
else
{$V^i_{jk} = V^i_{F(j),k} - R^i_j * I^i_{jk}(1)$;
if($line(i,j) == 1$) $V^i_{jk} - = E^i_{jk}$; }
for each n^i_{jj} of the son node of n^i_j do
$Voltage(i, jj, j, k)$; }

When applying the algorithm to lumped RC trees, for each order, each grounded capacitor is visited once and each floating capacitor visisted twice in the call of function $Current$, and each floating node is visited once in the call of function $Voltage$. Therefore, the computation cost is proportional to the total number of capacitors times the maximum order of interest. In practical cases, the number of capacitors connected to each node is limited by a constant, and the computation complexity of the algorithm can be expressed as $O(nK)$, where n is the number of the nodes in the network, and K is the maximum order required. This is a linear order algorithm and is very efficient.

When applying the algorithm to distributed RC trees, suppose that there is an RC line connected between each floating node and its father node, then the computation complexity is $O(nK^2)$, as the number of α's and β's grows linearly with the order. However, if discrete model is used to represent each RC line as used in RICE [11], if for each line there are m sections in the model, then the computation cost will be $O(mnK)$. It has been shown [2] that in order to get exact moment matching by a nonuniform discrete model, $m > K$, and it is often seen in the literature, e.g. in [12], that a large number of uniform RC sections are used to model a line and the number of sections is proportional to the length of the line, but in our algorithm the computation cost is independent of the line length. Therefore, this algorithm runs both more accurately and much faster than the using of discrete model for RC lines.

error	1	2	A1	S2P	NEW
max(%)	25.9	18.4 (∞)	67.2	65.3	25.1
average(%)	17.5	8.93 (∞)	12.4	17.7	5.63
large items	29	0(5)	9	17	2

Table 1. Test data for coupled RC lines

error	1	2	A1	S2P	NEW
max(%)	66.4	41.7 (∞)	93.2	154.3	39.9
average(%)	23.5	6.57 (∞)	15.0	37.3	7.41
large items	45	2(10)	14	56	4

Table 2. Test data for coupled RC trees

5 Examples

Based on the efficient algorithm for moment computation, we developed a new crosstalk model for coupled RC trees. The model is based on the moment matching model with slight modifications so that the model is more accurate and always stable. Because of the limitation of the paper, there is no space for its detailed description, which can be found in [5]. We have tested 140 examples, and the results are summerized in Table 1 and 2 for coupled RC lines and trees, respectively. In the first line of each table, "1", and "2" refer to the one pole and two pole model generated by $Padé$ approximation with moment matching up to the order of 2 and 3. "$A1$" refers to the model by [13], which is an approximated second order $Padé$ model. "$S2P$" refers to the model of [12], and "NEW" refers to our new model. The item "large items" refers to the number of tests that the absolute error exceeding 20%. It can be seen that our new model works better than the models marked "1", "A1" and "S2P". Compared with the model marked "2" (the 3rd order $Padé$ model), note that there are 5 cases in the RC line tests and 8 cases in the RC tree tests that the 2-pole model is unstable, the data listed outside the parentheses only refer to the stable tests and those inside the parentheses refer to all the tests. It can be seen that the new model performs better than existing models thus far.

6 Conclusions

We have provided simple explicit formulas for Elmore delay and higher order voltage moments and a linear order recursive algorithm for the moment computation for coupled RC trees with lumped and/or distributed elements. As there is no discretization for distributed EC lines, it is exact and

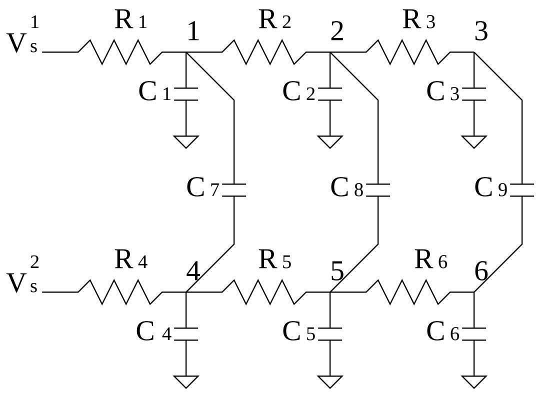

Figure 1. Coupled RC Trees

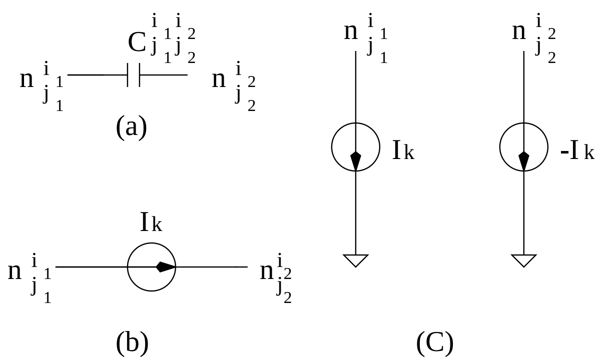

Figure 2. Moment Model of Floating Capacitor

very efficient. With the explicit formulas, the formulas for the sensitivity of moments w.r.t. the circuit parameters can be easily derived, which will be very useful when interconnect design and optimization is concerned. These formulas and algorithms provide an efficient way to deal with the delay and crosstalk estimation and model order reduction, and will be beneficial for the interconnect design, optimization and simulation. These formulas and algorithms can also be easily extended to RLC trees with both capacitive and inductive coupling.

References

[1] J.Cong, et al, "Interconnect design for deep submicron IC's", Proc. ICCAD'97, Nov. 1997, pp.478-485.

[2] Q.Yu & E.S.Kuh, "Exact moment matching model of transmission lines and application to interconnect delay estimation," IEEE Trans. on VLSI Sys., vol.3, pp.311-322, June 1995.

[3] J.S.Yim & C.M.Kyung, "Reducing cross-coupling among interconnect wires in deep-submicron datapath design," Proc. of 36-th DAC, June, 1999, pp.485-490.

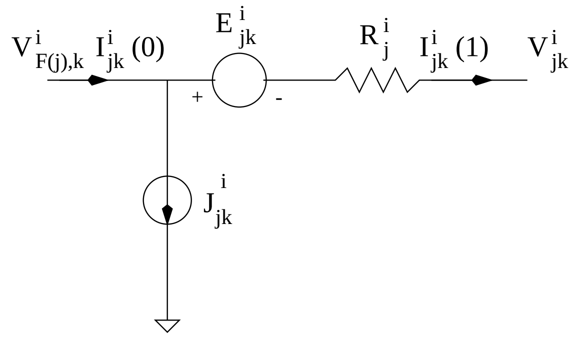

Figure 3. Moment Model of RC Line

[4] K.J.Kerns, et al, "Stable and efficient reduction of substrate model networks using congruence transforms," Proc. ICCAD'95, Nov. 1995, pp.207-214.

[5] Q.Yu and E.S.Kuh, "New efficient and accurate moment matching based model for crosstalk estimation in coupled RC trees," Proc. ISQED, March 2001.

[6] T.Sakurai,"Closed form expressions for interconnection delay, coupling and crosstalk in VLSIs," IEEE Trans. on Electron Devices, vol.40, No.1, pp.118-24, 1993.

[7] H.Kawaguchi and T.Sakurai, "Delay and noise formulas for capacitively coupled distributed RC lines," Proc. ASP-DAC'98, pp.35-43, Feb. 1998.

[8] P.Feldmann & R.W.Fruend, "Circuit noise evaluation by *Padé* approximation based model order-reduction techniques," Proc. ICCAD'97, pp.132-38, Nov. 1997.

[9] K.L.Shepard et al, "Global Harmony: Coupled noise analysis for full-chip RC interconnect networks," Proc. ICCAD'97, pp.139-46, Nov.1997.

[10] H.Zhou and D.F.Wang, "An optimal algorithm for river routing with crosstalk constraints," Proc. ICCAD'96, pp.310-15, Nov. 1996.

[11] C.L.Ratzlaff and L.T.Pillage, "RICE: rapid interconnect evaluation using AWE," IEEE TCAD, vol.13, pp.763-76, June 1994.

[12] E.Acar et al, "S2P: a stable 2-pole RC delay and coupling noise metric," Proc. 9-th Great Lakes Symp. on VLSI, pp.60-3, March 1999.

[13] A.Vittal et al, "Crosstalk in VLSI interconnects," IEEE Trans. on CAD, vol.18, No.12, pp.1817-24, Dec.1999.