

Efficient and Passive Modeling of Transmission Lines by Using Differential Quadrature Method *

Qinwei Xu and Pinaki Mazumder

EECS Dept., University of Michigan, Ann Arbor, MI 48109

Phone: (734) 763-2107; Fax: (734) 763-1503; email: {qwxu,mazum}@eecs.umich.edu

Abstract

This paper introduces a new transmission line modeling approach that employs an efficient numerical approximation technique called the Differential Quadrature Method (DQM). The transmission line has been discretized and the approximation framework is constructed by using the 5th order differential quadrature method, consequently an improved discrete equivalent-circuit model is developed in the paper. The DQM-based modeling requires far fewer intervening grid points for building an accurate discrete model of the transmission line than numerical methods like FD requires. It introduces far less state variables than FD-based models; therefore, it has higher efficiency. The DQM technique can be integrated in a circuit simulator since it preserves the passivity.

1 Introduction

In the past decades, the integrated circuits and systems have become both larger in silicon area and faster in operation. With VLSI feature sizes continuously shrinking to the ultimate nanometric dimensions, extremely large scale integrated circuits, fabricated in the forms of multi-chip modules (MCMs) and system-on-chips (SOCs), now contain more longer interconnects (measured in terms of signal wavelengths) running all over chips in the form of parallel busses. Signals carrying digital data at several GHz speed have time of flight through long interconnects comparable to the rise and fall times of the signals. Therefore, in modern high-speed integrated systems like MCMs and SOCs, the metal interconnects should be treated as lossy transmission lines with distributed RLCG parameters. It is imperative now to account for such transmission line effects in circuit simulation so that chips can be accurately

simulated in terms of signal speed as well as signal integrity due to crosstalk between interconnects, distortions due to impedance mismatching, and electromagnetic scattering and dispersions causing signal attenuations. Hence, precise modeling of interconnects is an important field of research in VLSI design automation.

As interconnections usually have large size, heavy density, and high order of RLCG circuit elements, reduced order models are needed for efficient circuit simulation. Asymptotic Waveform Evaluation (AWE) and its extensions are the most well-known methods for the approximation of general linear networks [1, 2]. The Krylov subspace techniques have been afterwards developed addressing the issue of passivity giving rise to efficient passive reduced order algorithms [3]. More recently, the congruence transformations have been applied to reduced-order modeling [4]. An extended technique based on Arnoldi's method with congruence transformations is presented in the literature [5], in which the PRIMA algorithm was demonstrated as an effective approach for developing passive reduced-order models.

Although the algorithms of model reductions are well developed and are being continuously improved, they can only handle the finite order systems in the forms of state equations. Transmission lines, however, are governed by nonlinear partial differential equations, which are actually infinite order systems. Therefore, it is inevitable to represent the transmission lines with approximate models involving finite state variables [6]. In order to efficiently perform the reduction algorithms on the finite system, the transmission line modeling needs to involve as few state variables as possible, while keeping required accuracy. Most effort to develop finite order models of distributed transmission line is focused on direct discretization approaches, which generally select grid points along the lines. As partial differential equations have been long approximately solved by finite difference (FD) methods [7], the discretization of transmission lines is far from a

*This work was partially supported by ONR grant under the Dual-Use Program.

novel topic. Despite its simplicity, it has, however, the disadvantage that the number of grid points, depending on the minimum wavelength, is generally 12 or more. Consequently, such an approach results in very large numbers of lumped elements for accurate modeling and thus sharply increase the number of state variables of the whole circuit. A compact difference method is employed in the literature [8], which has fourth order accuracy. In this discretization approach, the number of unknowns per wavelength required for highly accurate modeling is smaller and its dependence on the electrical length of the line is weaker.

The drawback of low order finite methods can be removed by using the high order finite methods or pseudospectral methods [9]. The mathematical fundamental of finite difference schemes is the Taylor series expansion. The scheme of low order finite method is determined by low order Taylor series, while the scheme of high order finite method is determined by high order Taylor series. The advantage of high order schemes is twofold: they allow one either to increase accuracy while keeping the number of mesh points fixed or to reduce the computational cost by decreasing the grid dimension while preserving accuracy. In general, the high order schemes have a high order truncation error. Thus, to achieve required accuracy, the mesh size used by the high order schemes can be much less than that used by low order schemes. As a result, the high order schemes can obtain accurate numerical solutions using very few mesh points. Chebyshev polynomial representation, a kind of pseudospectral methods, has been used to model transmission lines and has shown high efficiency [10]. However, it cannot guarantee passivity.

In this paper, the Differential Quadrature Method (DQM) is employed for passive modeling of transmission lines. A numerical technique similar to the spectral method, DQM was originally developed by mathematicians to approximately solve nonlinear partial differential equations (PDE) [11]. The idea of the differential quadrature method is to quickly compute the derivative of a function at any grid point within its bounded domain by estimating a weighted linear sum of values of the *function* at a small set of points belonging to the domain. This paper applies the differential quadrature method in the following steps. At first, the fifth order DQM is investigated and the specific approximation frames are derived for the modeling of transmission lines. Then the discrete modeling are derived by using the DQM-based approximation frames, and the equivalent circuit models are obtained to represent transmission lines. Like FD-based models, the DQM-based discrete models can be incorporated into popular all-purpose simulators. Due to the

super efficiency of DQM, DQM-based discrete modeling gives high approximation accuracy by using moderately few grid points, which improves the computational efficiency. The generated discrete models are theoretically proved to be passive, and have the linear form, which is compatible to the reduced order algorithm for linear circuit reduction. However, this presentation focuses on the DQM-based discrete modeling, and will not cover the issue of model order-reduction techniques.

2 Approximation framework

The following approximation framework is based on the Differential Quadrature Method (DQM), which is a mathematical interpolation technique proposed originally by Bellman in 1972 with a view to solving differential equations numerically [11]. We employ the differential quadrature method in order to approximate the first order derivatives of the distributed voltages and currents $u = u(x, s) = \{V(x, s), I(x, s)\}$ along the transmission line as shown below:

$$\frac{\partial}{\partial x} u(x_i, s) = \sum_{j=1}^N a_{ij} u_j(s), \quad (1)$$

where $x \in [0, 1]$, $0 = x_1 < x_2 < \dots < x_N = 1$, and $i, j = 1, 2, \dots, N$. Eqn.1 is called the N th order differential quadrature approximation.

The key procedure to this technique is to accurately determine the DQ coefficients a_{ij} by using the minimal value of the order (N) of DQM, since N , the number of grid points where the values of the function are sampled, decide the efficiency of the method in terms of computation time required by the DQM. Therefore, the technique applies a tradeoff between accuracy and computation time by selecting an appropriate test function to yield the values of DQ coefficients. Bellman applied the concept of Weighting Residual Method and showed that the following power series works as a good test function:

$$g(x) = \{1, x, x^2, \dots, x^{N-1}\}. \quad (2)$$

in order to determine the DQ coefficients in Eqn.1 [11]. By substituting each item in the function set into Eqn. 1, a set of equations having Vandermonde matrix is obtained, and the coefficients a_{ij} can be calculated by solving the equations. The DQ coefficients are given by the following closed-form formulas:

$$a_{ij} = \frac{1}{(x_i - x_j)} \frac{\prod_{k \neq j} (x_j - x_k)}{\prod_{k \neq i} (x_i - x_k)},$$

$$a_{ii} = \sum_{i \neq j} \frac{1}{x_i - x_j} \quad i \neq j \quad (3)$$

Such an approach is called the polynomial-based differential quadrature (PDQ) method. The above process shows that the PDQ method is closely related to the collocation or pseudo-spectral method [12].

Another way to determine the DQ coefficients is to employ harmonic (triangular) functions, called the harmonic differential quadrature (HDQ) method. An N th order Fourier expansion is a linear combination in an N -dimensional linear subspace which is spanned by the following orthogonal base,

$$g(x) = \{1, \dots, \sin \frac{N-1}{2} \pi x, \cos \frac{N-1}{2} \pi x\}, \quad (4)$$

where N is the number of grid points that is normally an odd number. The explicit formulas for estimating the values of DQ coefficients in this case are given by [13]:

$$\begin{aligned} a_{ij} &= \frac{\pi/2}{\sin((x_i - x_j)\pi/2)} \frac{\prod_{k \neq j} \sin((x_j - x_k)\pi/2)}{\prod_{k \neq i} \sin((x_i - x_k)\pi/2)}, \\ a_{ii} &= -\sum_{k \neq i} a_{ik} \quad i \neq j \end{aligned} \quad (5)$$

All the DQM coefficients in Eqn.3 (or Eqn.5) form an $N \times N$ matrix, called the DQM operator. For the transmission line problem, the DQM operator reveals the property of inverse symmetry as stated in Theorem 1.

Theorem 1: If the grid points are equally spaced, then the DQM operator is inverse symmetric with respect to the central point of the matrix, i.e.

$$a_{ij} = -a_{(N-i)(N-j)} \quad (6)$$

Eqn.6 can be proven by using either Eqn.3 or Eqn.5.

For simplicity and without any loss of generality, a single transmission line is modeled by DQM at first. Let AB be a section of the single transmission line. If it is uniformly segmented into 4 subsections, then 5 grid points are obtained as shown by Fig.1.

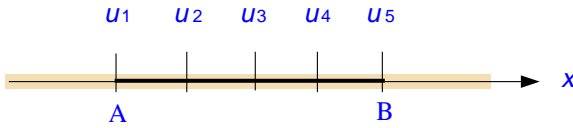


Figure 1: Approximation framework.

After normalizing the section AB by unity in length, the DQM approximation for the transmission line problem is represented by

$$\begin{bmatrix} u'_1 \\ u'_2 \\ u'_3 \\ u'_4 \\ u'_5 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} \quad (7)$$

By using Theorem 1 and after tedious mathematical manipulation, the DQM-based approximation framework for obtaining voltages (or currents) distributed along the transmission lines can be derived and is given by:

$$u_4 - u_2 = au'_1 + cu'_3 + au'_5, \quad (8)$$

where

$$\begin{aligned} a &= \frac{a_{31}}{a_{11}a_{32} + a_{32}a_{51} - a_{12}a_{31} - a_{31}a_{52}} \\ c &= \frac{a_{11} + a_{51}}{a_{12}a_{31} + a_{31}a_{52} - a_{11}a_{32} - a_{32}a_{51}} \end{aligned} \quad (9)$$

At the end points, the above approximation formulas Eqn.8 cannot be used. In this case, let the left-end approximation frames have the following form:

$$\begin{aligned} u_2 - u_1 &= b_1 u'_1 + au'_3 \\ u_3 - u_1 &= b_2 u'_2 + au'_4 \end{aligned} \quad (10)$$

We use the test functions in Eqns.2 and 4 to determine the coefficients for b_1 and b_2 , corresponding to PDQ and HDQ, respectively. Let Eqns.10 be exact to as many functions in Eqns.2 or 4 as possible. Specifically, in this case, enforce Eqns.10 being exact when the first two test functions in Eqns.2 or 4 are taken. Then the coefficients can be determined as:

$$b_1 = \frac{1}{4} - a \quad b_2 = \frac{1}{2} - a \quad (11)$$

for PDQ, and

$$b_1 = \frac{\sqrt{2}}{2\pi} \quad b_2 = a + \frac{\sqrt{2}}{\pi} \quad (12)$$

for HDQ.

Similarly, the right-end approximation formulas can be obtained as:

$$\begin{aligned} u_5 - u_4 &= au'_3 + b_1 u'_5 \\ u_5 - u_3 &= au'_2 + b_2 u'_4 \end{aligned} \quad (13)$$

Eqns.8, 10, and 13 constitute the complete approximation framework.

3 Discrete Modeling

Assume that a single transmission line stretches from 0 to d along the x axis, where d is the length of the line. Let the distributed per-unit-length (PUL) parameters of the line be denoted by R, L, G, C representing resistance, inductance, conductance and capacitance, respectively. The Telegrapher's equations can be written as:

$$\begin{aligned}\frac{d}{dx}V(x, s) &= -(sL + R)I(x, s) \\ \frac{d}{dx}I(x, s) &= -(sC + G)V(x, s).\end{aligned}\quad (14)$$

If the transmission line is uniformly segmented into $2N$ sections, then the discrete circuit has $2N + 1$ nodes. At each node, the Telegrapher's equation is discretized as

$$\begin{aligned}V'_i &= Z_i I_i \\ I'_i &= Y_i V_i \quad i = 1, \dots, 2N + 1,\end{aligned}\quad (15)$$

In order to apply the above DQM approximation framework to the nodal voltages and nodal currents, the length of transmission line should be normalized to match with the condition of Eqn. 7. Specifically, the length of each small section is normalized to be $1/4$ unit so that 5 consecutive grid points constitute one unit length over which the 5th order DQM can be performed. Therefore, it gives the following discretized equations:

$$\begin{aligned}Z_i &= \frac{4d}{2N}(R + sL) \\ Y_i &= \frac{4d}{2N}(G + sC) \quad i = 1, \dots, 2N + 1,\end{aligned}\quad (16)$$

Applying Eqns. 8, 10 and 13 to the nodal voltages and nodal currents, we obtain the DQM-based discrete modeling framework:

$$\begin{aligned}V_i - V_{i-1} &= aZ_{i-1}I_{i-1} + cZ_i I_i + aZ_{i+1}I_{i+1} \\ &\quad i = 2, \dots, N \\ I_{i+1} - I_i &= aY_{i-1}V_{i-1} + cY_i V_i + aY_{i+1}V_{i+1} \\ &\quad i = 2, \dots, N - 1\end{aligned}\quad (17)$$

for center grid points, and

$$\begin{aligned}V_1 - V_0 &= b_1 Z_1 I_1 + aZ_2 I_2 \\ V_{N+1} - V_N &= aZ_N I_N + b_1 Z_{N+1} I_{N+1} \\ I_2 - I_1 &= b_2 Y_1 V_1 + aY_2 V_2 \\ I_{N+1} - I_N &= aY_{N-1} V_{N-1} + b_2 Y_N V_N\end{aligned}\quad (18)$$

for boundary points.

Defining the current controlled voltage sources (CCVS) by

$$\begin{aligned}V_i^e &= aZ_{i-1}I_{i-1} + cZ_i I_i + aZ_{i+1}I_{i+1} \\ &\quad i = 2, \dots, N \\ V_1^e &= b_1 Z_1 I_1 + aZ_2 I_2 \\ V_{N+1}^e &= aZ_N I_N + b_1 Z_{N+1} I_{N+1}\end{aligned}\quad (19)$$

and the voltage controlled current sources (VCCS) by

$$\begin{aligned}I_i^e &= aY_{i-1}V_{i-1} + cY_i V_i + aY_{i+1}V_{i+1} \\ &\quad i = 2, \dots, N - 1 \\ I_1^e &= b_2 Y_1 V_1 + aY_2 V_2 \\ I_N^e &= aY_{N-1}V_{N-1} + b_2 Y_N V_N\end{aligned}\quad (20)$$

the DQM-based discrete model can be schematically represented by Fig.2.

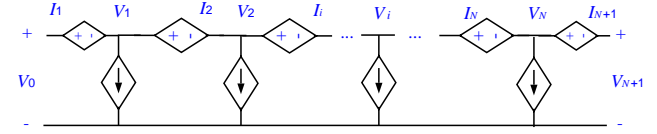


Figure 2: DQM-based discrete model.

Fig. 2 shows that DQM-based modeling has explicit physical meaning. The DQM-based discrete model consists of a chain of current controlled voltage sources (CCVS) and voltage controlled current sources (VCCS), compared to the FD-based discrete model consisting of a cascade of RLC elements.

It is well known in classical circuit theory that interconnections of individually stable systems may not necessarily constitute a collectively stable system. However, interconnections of passive circuit elements guarantee passivity and stability of the overall system. When multiport models are connected together, the resulting overall circuit can guarantee to be stable only if each of the multiport models is passive [5]. In this view, it is extremely important to investigate the passivity of the discrete model that results from the discretization of Telegrapher's equations. In order to do this, the following definitions and results due to [14] are used.

Lemma 1: Necessary and sufficient conditions for a transfer function $n \times n$ matrix $\mathbf{Y}(s)$ to be passive (i.e., $\mathbf{Y}(s)$ is positive-real) are given by:

- (1) each element of $\mathbf{Y}(s)$ is analytic in $\Re(s) > 0$,
- (2) $\mathbf{Y}(s^*) = \mathbf{Y}^*(s)$ and
- (3) $(\mathbf{Y}^*)^T(s) + \mathbf{Y}(s)$ is non-negative definite for all $\Re(s) \geq 0$.

Lemma 2: An n -port network is passive if and only if its admittance matrix $\mathbf{Y}(s)$ is positive-real.

Lemma 3: If $\mathbf{A}(s)$ is positive-real, then $\mathbf{A}^{-1}(s)$ is positive-real, if it exists.

Lemma 4: If $\mathbf{A}(s)$ is positive-real and \mathbf{B} is real, then $\mathbf{B}^T \mathbf{A}(s) \mathbf{B}$ is positive-real.

With two independent voltage excitations at the two ends, the state equation of the discrete model shown in Fig. 2 can be formulated by using the modified nodal analysis (MNA):

$$\left(\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_3 \\ -\mathbf{P}_3^T & \mathbf{P}_2 \end{bmatrix} + s \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2 \end{bmatrix} \right) \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix} = \mathbf{b}_i \quad (21)$$

where

$$\begin{aligned} \mathbf{P}_1 + s\mathbf{Q}_1 &= \begin{bmatrix} b_1 Y_i & a Y_i & & & \\ a Y_i & c Y_i & a Y_i & & \\ & \ddots & \ddots & \ddots & \\ & & a Y_i & c Y_i & a Y_i \\ & & & a Y_i & b_1 Y_i \end{bmatrix} \\ \mathbf{P}_2 + s\mathbf{Q}_2 &= \begin{bmatrix} b_2 Z_i & a Z_i & & & \\ a Z_i & c Z_i & a Z_i & & \\ & \ddots & \ddots & \ddots & \\ & & a Z_i & c Z_i & a Z_i \\ & & & a Z_i & b_2 Z_i \end{bmatrix} \\ \mathbf{P}_3 &= \begin{bmatrix} -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \end{bmatrix} \end{aligned}$$

and where

$\mathbf{V} = [V_1 \ V_2 \ \dots \ V_N]^T \in R^N$ is the vector of nodal voltages;

$\mathbf{I} = [I_1 \ I_2 \ \dots \ I_{N+1}]^T \in R^{N+1}$ is the vector of branch currents;

and the matrices $\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{Q}_1$ and \mathbf{Q}_2 are derived from Eqn.21:

$\mathbf{P}_1 \in R^{N \times N}$ and $\mathbf{P}_2 \in R^{(N+1) \times (N+1)}$ are symmetric and nonnegative definite, having units of conductance and resistance, respectively;

$\mathbf{Q}_1 \in R^{N \times N}$ and $\mathbf{Q}_2 \in R^{(N+1) \times (N+1)}$ are symmetric and nonnegative definite, having units of capacitance and inductance, respectively;

$\mathbf{P}_3 \in R^{N \times (N+1)}$, connecting matrix, is comprised of 1 or 0.

The matrix $\mathbf{b}_i \in R^{2N+1}$ contains two independent voltage sources connected to the two ends of transmission

line, which can be represented as

$$\mathbf{b}_i = \begin{bmatrix} 0 & \dots & 0 & 1 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & -1 \end{bmatrix}^T \begin{bmatrix} V_0^s \\ V_{N+1}^s \end{bmatrix} = \mathbf{B} \mathbf{V}^s \quad (22)$$

Noting that the original port variables are V_0, I_1, V_{N+1} and $-I_{N+1}$, the admittance matrix is obtained as:

$$\mathbf{Y}(s) = \mathbf{B}^T \left(\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_3 \\ -\mathbf{P}_3^T & \mathbf{P}_2 \end{bmatrix} + s \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2 \end{bmatrix} \right)^{-1} \mathbf{B} \quad (23)$$

Theorem 2: The matrix $\mathbf{Y}(s)$ in Eqn. 23 is positive-real.

Proof: Using Lemmas 1-4, the matrix $\mathbf{Y}(s)$ being positive-real ascribes to that the following matrix is positive-real:

$$\mathbf{W} = \begin{bmatrix} \mathbf{P}_1 & -\mathbf{P}_3^T \\ \mathbf{P}_3 & \mathbf{P}_2 \end{bmatrix} + s \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2 \end{bmatrix}. \quad (24)$$

Referring back to Lemma 1, the first two conditions are automatically satisfied for matrix \mathbf{W} . In proving that matrix \mathbf{W} satisfies condition (3), noting that matrices $\mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}_1$ and \mathbf{Q}_2 are all symmetric, it follows:

$$\mathbf{W} + (\mathbf{W}^*)^T = 2 \begin{bmatrix} \mathbf{P}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_2 \end{bmatrix} + 2\Re(s) \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2 \end{bmatrix}. \quad (25)$$

Since matrices $\mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}_1$ and \mathbf{Q}_2 are all non-negative, $\mathbf{W} + (\mathbf{W}^*)^T$ is therefore non-negative. Thus, the matrix $\mathbf{Y}(s)$ being positive-real, the DQM-based modeling for single transmission line preserves passivity.

The discrete model of multi-conductor transmission lines (MTL) can be straightforwardly obtained by extending the above formalization of the single transmission line (STL) and also the passivity of the MTL can be guaranteed.

4 Numerical results

Similar to FD-based discrete modeling, DQM-based modeling leads to filter-like multiport devices. These kind of models have the approximate transfer functions which exactly match in the low frequency band, and match poorly at higher frequencies with errors increasing as the frequency increases.

A heuristic rule for the resolution of the 5th order DQ methods is shown to segment the transmission line into 8 equal sections, where the sizes of the segment depend on the minimum wavelength in the spectrum. Therefore, for

a transmission line with length d , the number of sample points for the 5th order DQ approximation is

$$N_p = 8 \frac{d}{\lambda_{min}} + 1, \quad (26)$$

where λ_{min} is the minimum wavelength. Accordingly, the number of state variables of discrete modeling of a line with length d is $N_p + 3$ for DQ modeling.

Referring to a formula adopted by HSPICE [15], the maximum frequency of interest can be evaluated by

$$f_{max} = \frac{0.35}{t_r}, \quad (27)$$

where t_r is the rise time of the input waveform. The maximum frequency determines the minimum wavelength within the spectral range of interest.

The *first* example is a single transmission line having the following PUL parameters: $l = 360 \text{ nH/m}$, $c = 100 \text{ pF/m}$, $r = 36 \text{ } \Omega/\text{m}$, and $g = 0.01 \text{ S/m}$. Assuming that the digital signal has rise time 50 ps , the maximum frequency of interest is calculated by Eqn.27 to be 7 GHz . As the phase velocity is determined by $v_p = 1/\sqrt{lc} = 5/3 \times 10^8 \text{ m/s}$, the minimum wavelength is approximately 2.4 cm . The applied input is a step voltage whose rise time is 50 ps . By Eqn. 26, the number of sections using the 5th order DQ methods is approximately calculated as 8. The analysis shows that the frequency response of this modeling gives accurate results within the band from 0 to as high as 8 GHz . The transient results are shown in Fig.3, compared to the results of the Method of Characteristics (MMC). Noting that the transmission line in this example is an undistorted line, the transient simulation response calculated by MMC is the exact value if the round errors are ignored [16].

The *second* example consists of three coupled transmission lines as shown in Fig. 4. The length of the transmission line is 4 cm , and its RLCG parameters are represented in the following: $R_{11} = R_{22} = R_{33} = 3.448 \text{ } \Omega/\text{cm}$, $L_{11} = L_{22} = L_{33} = 4.976 \text{ nH/cm}$, $L_{12} = L_{23} = 0.765 \text{ nH/cm}$, $C_{11} = C_{33} = 1.082 \text{ pF/cm}$, $C_{22} = 1.124 \text{ pF/cm}$, $C_{12} = C_{23} = -0.197 \text{ pF/cm}$, $G_{11} = G_{22} = G_{33} = 1 \text{ mS/cm}$, $G_{12} = G_{23} = -0.01 \text{ mS/cm}$. The input excitation is a trapezoidal pulse with 100 ps rise/fall time whose magnitude is 1 V . The transient waveforms are shown in Figs. 5 and 6, which demonstrates that the results of both PDQ and HDQ match well with the result of HSPICE.

In this example, HSPICE uses the FD method and segments the transmission line into 20 sections, while the DQ methods need only 8 sections. Furthermore, the 20 sections of the FD method lead to $42 \times 3 = 126$ state

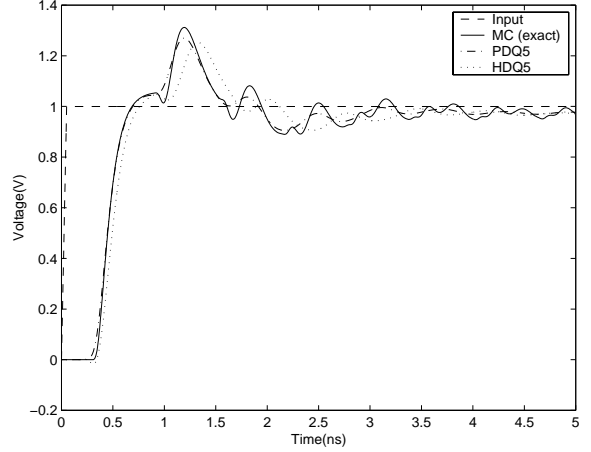


Figure 3: Transient responses of single transmission line.

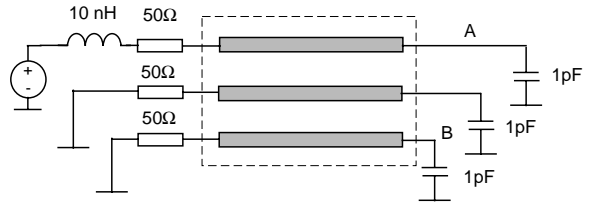


Figure 4: Circuit of coupled transmission line.

variables, while the 8 sections of DQ modelings lead to only $11 \times 3 = 33$ state variables. The whole circuit has 29 nodes by using the DQM modeling, in contrast to 89 nodes using HSPICE. Cast in MNA matrices, the state equation of DQ modelings has the size of 33×33 , while that of FD method has the size of 126×126 . As the run time of solving $n \times n$ linear equations is generally proportional to n^3 , the 5th order DQ methods have much higher efficiency than the FD method. Taking the same time step, the run time on an Ultra-1 SUN workstation by the DQM modeling is 0.108 s , while that of HSPICE is 0.7 s .

5 Conclusions

The passive discrete interconnect modeling is performed by employing a numerical technique called the differential quadrature method (DQM). The resulting equivalent circuit based on the DQM modeling is derived by using the 5th order DQM, and can be theoretically shown to preserve the passivity. The DQM discretizes the transmission line into few grid points across the entire length of the

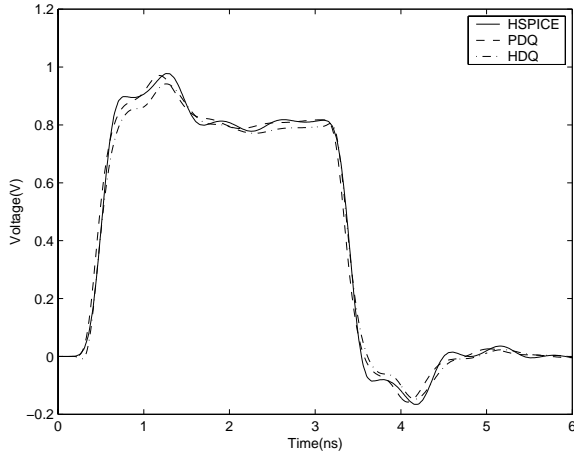


Figure 5: Transient responses at A.

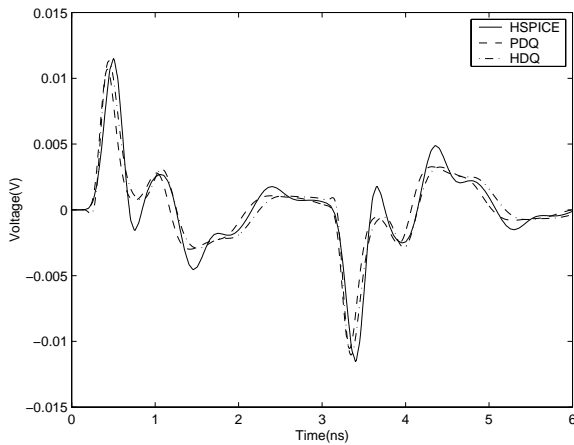


Figure 6: Transient responses at B.

line and computes the electrical parameters at those points in order to derive accurate and efficient discrete approximation. The discrete approximation framework is determined by two different mechanisms, which adopt polynomials and harmonic functions. These are called the polynomial differential quadrature (PDQ) and the harmonic differential quadrature (HDQ), respectively. An s -domain DQM-based discrete modeling has matrix representation whose elements are first degree with respect to s , which is compatible with Krylov subspace techniques for circuit reduction. Like the FD modeling, the DQM-based discrete modeling has explicit physical meaning, and results in equivalent circuits which can be directly incorporated into circuit simulators such as SPICE. The DQ modeling

has been shown to produce highly accurate delay models, and can handle both single and multi-conductor transmission lines. For the minimum wavelength, the DQM-based modeling generates as 9 internal voltage nodes, and 11 state variables as opposed to 21 nodes and 42 states generated by HSPICE. The DQM-based modeling preserves the passivity and generates fast solutions.

References

- [1] L. T. Pillage and R. A. Rohrer, "Asymptotic waveform evaluation for timing analysis," *IEEE Trans. Computer-Aided Design*, vol. 9, no. 4, pp. 352–377, 1990.
- [2] E. Chiprout and M. S. Nakhla, "Analysis of interconnect networks using complex frequency hopping," *IEEE Trans. Computer-Aided Design*, vol. 14, no. 2, pp. 186–200, 1995.
- [3] P. Feldman and R. W. Freund, "Efficient linear circuit analysis by Pade approximation via the Lanczos process," *IEEE Trans. Computer-Aided Design*, vol. 14, no. 5, pp. 639–649, 1995.
- [4] K. J. Kerns and A. T. Yang, "Stable and efficient reduction of large, multiport RC networks by pole analysis via congruence transformations," *IEEE Trans. Computer-Aided Design*, vol. 16, no. 7, pp. 734–744, 1997.
- [5] A. Odabasioglu, M. Celik, and L. T. Pileggi, "PRIMA: Passive reduced-order interconnect macromodeling algorithm," *IEEE Trans. Computer-Aided Design*, vol. 17, no. 8, pp. 645–653, 1998.
- [6] Q. Yu, J. M. L. Wang, and E. S. Kuh, "Passive multipoint moment matching model order reduction algorithm on multiport distributed interconnect networks," *IEEE Trans. Circuits and Systems-I*, vol. 46, no. 1, pp. 140–160, 1999.
- [7] A. R. Djordjevic, T. K. Sarkar, and R. F. Harrington, "Time-domain response of multiconductor transmission lines," *Proc. IEEE*, vol. 75, no. 6, pp. 743–764, 1987.
- [8] A. C. Cangellaris, M. Celik, S. Pasha, and L. Zhao, "Electromagnetic model order reduction for system-level modeling," *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 6, pp. 840–849, 1999.
- [9] C. Canuto, *Spectral methods in fluid dynamics*. New York: Springer-Verlag, 1988.

- [10] M. Celik and A. C. Cangellaris, "Simulation of multiconductor transmission lines using Krylov subspace order-reduction techniques," *IEEE Trans. Computer-Aided Design*, vol. 16, no. 5, pp. 485–496, 1997.
- [11] R. Bellman, B. G. Kashef, and J. Casti, "Differential quadrature: a technique for the rapid solution of nonlinear partial differential equations," *J. Comp. Phys.*, vol. 10, no. 1, pp. 40–52, 1972.
- [12] B. Fornberg, *A practical guide to pseudospectral methods*. Cambridge university press, 1996.
- [13] C. Shu, "Fourier expansion-based differential quadrature and its application to Helmholtz eigenvalue problems," *Commun. Numer. Meth. Engng.*, vol. 13, 1997.
- [14] R. W. Newcomb, *Linear Multiport Synthesis*. New York: McGraw Hill, 1966.
- [15] Meta-Software, *HSPICE User's Manual*. Meta-Software, Inc, 1996.
- [16] Q. Xu, Z. F. Li, P. Mazumder, and J. F. Mao, "Time-domain modeling of high-speed interconnects by modified method of characteristics," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 2, pp. 323–327, 2000.