Efficient Bit-Error-Rate Estimation of Multicarrier Transceivers

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Abstract

Multicarrier modulation schemes are widely used in several digital telecommunication systems, such as Asymmetric Digital Subscriber Lines (ADSL) and Wireless Local Area Network (WLAN) based on Orthogonal Frequency Domain Multiplexing (OFDM). An estimate of the Bit-Error-Rate (BER) degradation due to non-idealities in the transceiver (e.g. nonlinear distortions in the analog front-ends, digital clipping,...) is much more complicated in a multicarrier system than in a single-carrier system due to the large number of carriers and the huge number of possible transmitted symbols. This paper proposes a method for estimating the BER of such OFDM modulation schemes in a CPU time that is two orders of magnitude smaller than a Monte-Carlo method, as confirmed by simulations on a 5 GHz IEEE 802.11 WLAN receiver front-end.

1. Introduction

OFDM modulation schemes are widely used in telecommunication systems. Examples are ADSL modems, WLAN applications,...[1]. A figure of merit for the quality of a digital telecom transceiver is the bit-error-rate (BER). This is typically determined using an end-to-end Monte-Carlo simulation, that comprises the digital part of the transmitter, the analog front-end of the transmitter, the transmission channel, the receiver front-end and the digital part of the receiver. The input at the transmit side is a set of symbols. At the receiver side it is checked whether these symbols have been transmitted correctly. For low bit-error-rates (values of 10^{-5} or lower), a large amount of symbols needs to be transmitted, leading to lengthy simulations [2]. This paper presents a technique that significantly reduces the number of required symbols for an accurate BER simulation. It extends the results of [3] on analog ADSL measurements towards mixed analog/digital transceivers and gives more insight in the choice of the simulation parameters onto the accuracy of the BER estimate.

The efficiency of the method is illustrated using high-level simulation of a mixed signal receiver front-end of a 5GHz IEEE 802.11 WLAN transceiver. This simulation shows that the described method is

- 100 times faster than a pure Monte-Carlo approach
- applicable on mixed analog/digital transceiver were both the analog and the digital circuits suffer from nonlinear distortions and/or clipping.

The structure of the paper is as follows: after a description of an OFDM signal in Section 2, the global ideas of the method are explained in Section 3. The mathematical details of the method are presented in Section 4 and an example is given in Section 5.

2. Characteristics of an OFDM signal

An OFDM signal consists of a sum of N carriers

$$x(t) = \sum_{i=1}^{N} a_i(t)\cos(\omega_i t) + b_i(t)\sin(\omega_i t), \qquad (1)$$

where all carriers are modulated using a separate modulation scheme such as Phase Shift Keying (PSK), Quadrature Amplitude Modulation (QAM)... The modulations are represented by $a_i(t)$ and $b_i(t)$.

For some values of $a_i(t)$ and $b_i(t)$, large peaks can occur. The large signal peaks can lead to severe nonlinear distortion (e.g. clipping), which can increase the BER significantly. A practical measure to characterize an OFDM signal is the crest factor (CF). This is the ratio of the maximum amplitude of the transmitted symbol x over the root-mean-square value of all possible symbols:

$$CF = \frac{\max|x|}{\text{rms}} \qquad (2)$$

Clearly, OFDM symbols with a large crest factor give rise to a large nonlinear distortion.

3. Global idea of the method

An efficient way for accurate BER estimations in multicarrier systems requires a dedicated approach. Indeed, measurement and/or simulation of the nonlinear effects on all possible symbols is not feasible. For example, the IEEE standard 802.11 which uses 52 carriers and a 64-QAM modulation, has 64^{52} possible symbols to transmit. A Monte-Carlo approach would still require the generation of many symbols, since the ones that give rise to the high signal peaks, and hence to bit errors, have a fairly low probability of occurrence.

In this paper, we present a BER estimation technique for which the number of symbols that have to be generated (and simulated) for a given BER accuracy is much lower than for a Monte-Carlo approach. The idea of the method is as follows: prior to simulation a large number of OFDM symbols are generated. These are classified in sets according to their crest factor. For each set a BER estimation is performed. The resulting BER values together with the probability of occurrence of each crest factor value are combined to obtain the overall BER.

The BER for a set of symbols with the same crest factor is computed either with a Monte-Carlo method or with a quasi-analytical method [2]. The latter approach - which is used for low crest factors - considers the in-band distortion as Gaussian distributed noise. The power of this noise is computed from simulation results. This power estimate is then used in an analytical formula to compute the BER. For high crest factors, however, the assumption of having independent Gaussian distributed noise can be violated. Then a Monte-Carlo approach can be used with a higher accuracy but with a comparable efficiency. Indeed, the number of required experiments is low, since the probability of bit errors is high for large crest factors.

4. Mathematical Framework

4.1. Basic principle

The OFDM signal described by (1) can be seen as a multisine with a large number of components. The amplitudes and phases of all carriers are determined by the transmitted symbol. They can be considered as independent realizations of a stochastic process. From [4] it is known that the overall response to such multisines can be characterized using a best linear approximation of the transfer function together with a stochastic nonlinear distortion. This best linear approximation can be seen as a generalization of the AM/AM and AM/PM characteristics [2] to broadband signals. The stochastic nonlinear distortion can be approximated as additive noise due to the interaction of the large number of independent amplitudes and phases of the carriers $a_i(t)$ and $b_i(t)$. The level of this additive noise is a function of

- the nonlinearity of the complete transmission path,
- the modulation scheme used (QAM, PSK,...),
- the number of carriers in the OFDM signal,
- the signal level.

Starting from the probability density function (pdf) of the crest factor, the best linear approximation and the ‘noise’ level of the in-band nonlinear distortion, the BER can be estimated using

$$\mathrm{BER} = \int \mathrm{BER}(x) f_{CF}(x) dx \qquad (3)$$

where $\mathrm{BER}(x)$ denotes the BER for a crest factor x while $f_{CF}(x)$ denotes the pdf of the crest factor. The following estimation scheme is used:

1. Generate a large number of OFDM symbols. These are divided into sets, that each correspond to a different value of the crest factor. For the definition of the sets, we consider a grid of M crest factors $x_1,...x_M$;
2. Determine the probability of occurrence for each crest factor using the pdf: $f_{CF}(x)$;
3. For each set corresponding to $x=x_1,...x_M$, store a number of symbols, typically about 100;
4. Determine the in-band distortion levels. These are obtained with an end-to-end simulation using the symbols stored in the previous step. The in-band distortion for low crest factors will be used later as the additive noise levels in the quasi-analytical method;
5. Compute the $\mathrm{BER}(x_i)$ as function of the crest factor either using either a standard quasi-analytical method or with a Monte-Carlo approach [2].
6. Approximate the total BER of equation (3) with a finite sum over the grid of crest factors:

$$\mathrm{BER} = \sum_{i=1}^{M} \mathrm{BER}(x_i) f_{CF}(x_i) . \qquad (4)$$

Steps 1 to 3 only depend on the number of carriers and their modulation. They do not require any simulation of the telecom system. As a result, both the pdf and the excitation signals can be generated off-line and can be reused for BER estimations with different transceiver architectures.

4.2. Determination of the pdf of the crest factor

Several theoretical studies can be found in the literature which give an approximation of the crest factor distribution [5], [6]. However, the assumptions made in these studies about the constellations of the carriers are violated for practical OFDM modulation schemes. Therefore, Monte-Carlo simulations are used to estimate the pdf. Fig. 1 shows the pdf for the IEEE standard 802.11 using a 64-QAM modulation. This pdf has been extracted from 50 million OFDM symbols. A limited number of symbols is stored for every crest factor for later simulations. This extraction only needs to be done once for a given number of carriers and their modulation.

5. Example: a 5 GHz IEEE 802.11 WLAN receiver front-end

The proposed methodology is applied to a system-level simulation of a 5 GHz WLAN receiver front-end, shown in fig. 2.

The OFDM modulated input signal uses a 5.25 GHz carrier and has a bandwidth of 20 MHz. It consists of 52, 64-QAM modulated carriers with a 312.5 kHz spacing and four pilot carriers. The consecutive OFDM symbols are separated using a cyclic prefix. Furthermore, it is assumed that the signal level is such that the clipping levels of the ADC’s equal four times the standard deviation (calculated

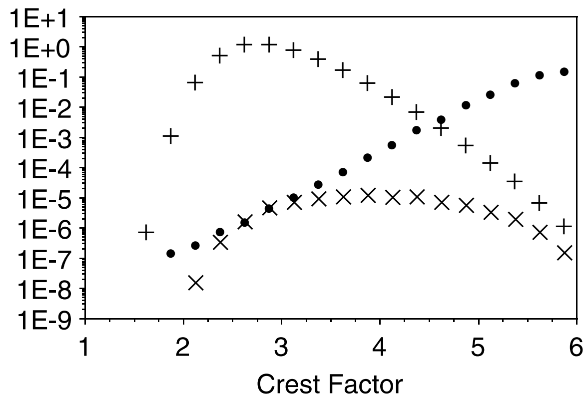

Figure 1: Analysis for an OFDM signal as function of the crest factor (IEEE 802.11 standard using 64-QAM modulation): '+': probability density function; '●': BER; '×': Product of the pdf and the BER.

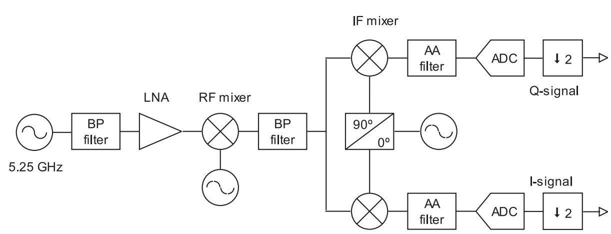

Figure 2: A 5 GHz WLAN receiver architecture.

over all possible symbols) of the output signals.

After passing through a third-order elliptic blocking filter, the signal is amplified using a low-noise amplifier which is characterized using its noise figure, gain, and second- and third-order intercept points IIP_2 and IIP_3. After downconversion with the RF mixer, the IF signal is filtered out using a fourth-order elliptic filter. A final downconversion is performed using two IF mixers which are also modelled using their conversion gain, IIP_2 and IIP_3. The IF signals are then filtered using a sixth-order elliptic anti-alias filter and digitized using a 8 bit ADC running at 80 MHz. This ADC includes both a uniform quantization and clipping. The latter introduces strong nonlinearities for signals with a high crest factor. Finally, the output of the ADC is sent to a decimation filter in order to stress the ability of handling mixed analog/digital simulations.

A large number of OFDM symbols needs to be simulated to obtain a good estimate of the BER. Hence, it is very important to have an efficient system-level simulator. To this purpose, the simulator FAST [7] is used. This simulator uses a multirate, multicarrier signal (MRMC) representation. This representation extends the complex lowpass signal representation [2] such that the effect of out-of-band distortions onto in-band distortions can be taken into account. For example, the out-of-band distortion from the LNA folds back inside the band of interest by the nonlinear operation of the RF mixer.

Thanks to the MRMC signal representation and the efficient dynamic dataflow scheduler the computational speed of FAST reaches the limits of the processor of the computer. As a figure of merit, it has been demonstrated that identical simulations in MATLAB are about ten times slower than in FAST [7]. The time required to compute the response of the WLAN receiver for one single OFDM symbol (256 complex data samples coming from the 64 carrier which are four times oversampled) is less than 3 ms on a PIII 500MHz. Computing the response on 4000 OFDM symbols takes approximately 12 s on a PIII 500 MHz.

5.1. Determination of the pdf of the crest factor

As already mentioned above, the pdf of the crest factor is determined prior to the actual simulation by randomly generating fifty million OFDM symbols (see also fig. 1). During this process, up to 500 symbols have been stored for each set of crest factors $x_1,...,x_M$ between 1.5 and 6 with a spacing of 0.1. The generation of fifty million OFDM symbols took 13 hours on a PIII. This computation of the pdf and the generation of test symbols has to be performed only once and can be reused for all BER estimates with the same number of carriers and the same modulation.

5.2. The Best Linear Approximation

The signals at the output of the front-end must be reconstructed in order to compute the BER. This is performed by the equalizer which is always present in the digital part of the receiver. This equalizer, which is an adaptive filter, normally converges to the best linear approximation of the complete transmission chain. Since the impact of the equalizer onto the BER is not analyzed here, it is assumed that the best linear approximation - shown in fig. 3 - can be used to reconstruct the signals at the output of the front-end.

The best linear approximation of the receiver architecture is computed for all considered crest factors. The cyclic prefix makes it possible to compute the best linear approximation by averaging the ratio of the Fourier transforms of the input and the output signals. If the length of the cyclic prefix is sufficiently long, then the leakage introduced by the FFTs can be neglected. From the computations it is seen that the linear approximation changes only slightly as function of the crest factor. This implies that the mean value over the crest factors of the best linear approximation (fig. 3) can be used for all crest factors.

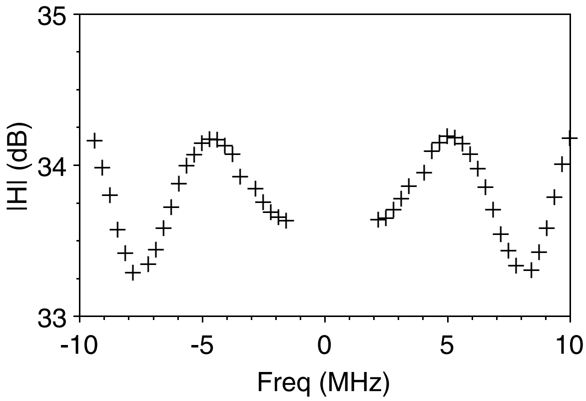

Figure 3: Mean value of the best linear approximation of the frequency response. The horizontal axis is the frequency offset of a carrier with respect to the 5.25 GHz center frequency. The gap around the carrier frequency comes from the zero carriers as defined by the IEEE standard 802.11.

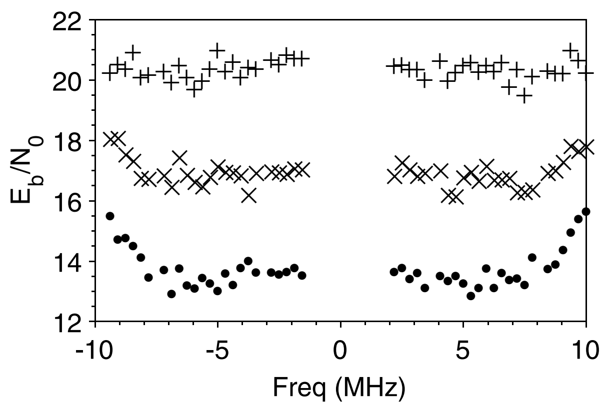

Figure 4: Energy per Bit to Noise Density Ratio (E_b/N_0) of the stochastic (nonlinear) distortion as function of the frequency offset for different levels of the crest factor. From upper to lower curves: crest factors 2, 4 and 5.

5.3. The Stochastic Nonlinear Distortion

The in-band nonlinear distortions of the large number of sinewaves behave as stochastic components. The RMS value of the stochastic (nonlinear) distortion is computed from the original input signal and the received signal (with the inclusion of the adaptive filter for compensating the linear dynamic system). Fig. 4 shows that the Energy per Bit to Noise Density Ratio (E_b/N_0) computed using this RMS value of the stochastic nonlinear distortion changes significantly as function of the crest factor. E_b/N_0 is minimal in the centre of the frequency band of interest. This is due to the contribution of a large number of frequency combinations to the nonlinear distortion near the center. This also influences the BER as function of frequency.

5.4. The Estimated BER

Having the stochastic nonlinear distortion in the form of additive noise, this noise, which is assumed to be independent complex normally distributed, is used for the computation of the BER for low crest factors with the quasi-analytical method. For high crest factors a short Monte-Carlo BER estimation is performed. Fig. 5 shows the estimated BER as function of the crest factor. The BER for

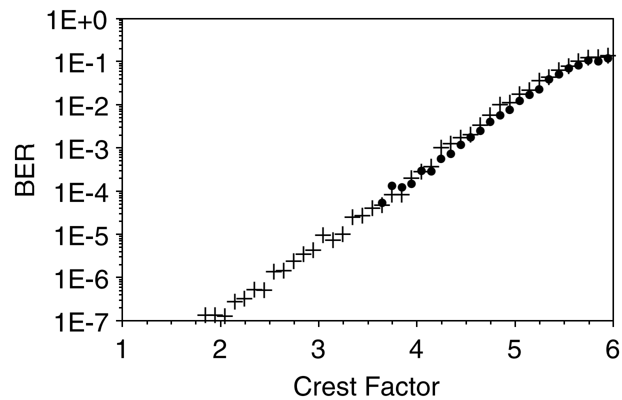

Figure 5: Verification of the estimated BER as function of the crest factor: '+': quasi-analytical method (100 experiments/point); '●': Monte-Carlo simulation (5000 experiments/point).

low crest factor symbols is dominated by the additive noise. The BER increases when the crest factor of the signal increases since in-band nonlinear distortion becomes more important than the influence of the additive noise. Fig. 5 illustrates that for the used signals the quasi-analytical method can be used for the complete range of considerd crest factors.

5.5. Computation of the total BER

For the total BER we combine the BER of each crest factor and the pdf according to (4), as visualized in fig. 1. Clearly, the product

$$BER(x_i)f_{CF}(x_i) \quad (5)$$

has a smooth behavior as function of the crest factor: the BER increases exponentially, while the pdf decreases exponentially. The smooth and fairly constant behavior of the product (5) makes it possible to use a coarse grid for the crest factors.

An accurate result for the total BER (i.e. with a low variance on the estimated BER) is obtained when the individual variances of the components of (5) are constant. This implies that the relative accuracy of the individual BER estimates $BER(x_i)$ needs to be constant as well. Therefore, we perform an identical number of simulations for each crest factor.

The accuracy of the BER estimate changes for varying grid sizes and number of symbols. From table 1 we see that

	25	50	100	250	500
0.1	3.44e-5	1.85e-5	1.66e-5	1.44e-5	
0.25	3.74e-5	2.03e-5	1.64e-5	1.46e-5	1.37e-5
0.50	3.78e-5	2.15e-5	1.76e-5	1.52e-5	1.41e-5

Table 1: Estimated BER for varying grid sizes of the crest factor (0.1, 0.25, 0.5) and varying number of symbols for each crest factor (from 25 up to 500). A Monte-Carlo analysis using 1 million OFDM symbols resulted in a BER of 1.49e-5.

the accuracy is more sensitive to the number of symbols for each crest factor than to the size of the crest factor grid.

The quality of a telecom link is often expressed in terms of the BER as function of additive noise (E_b/N_0). Typically, this is a waterfall-shaped curve [2]. For the WLAN system under consideration, this curve, shown in Fig. 6, reveals that for a small additive noise, the BER is determined by the nonlinearities in the system. This result

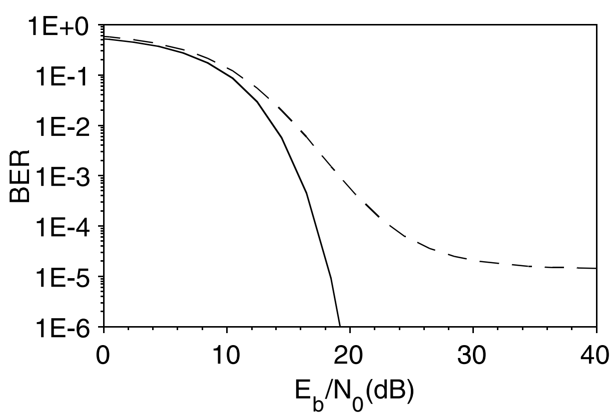

Figure 6: BER of the IEEE WLAN system as function of the additive noise (expressed in E_b/N_0). Solid line: no nonlinear distortion; dashed line: nonlinear distortion included.

has been computed off-line in a post-processing stage using the quasi-analytical method, requiring only a few minutes of CPU time.

6. Conclusions

A method has been presented for the efficient estimation of the BER in multicarrier systems such as OFDM-based WLAN transceivers. Thanks to a careful selection of the input symbols prior to simulation, it is possible to obtain a good estimate of the BER with a much smaller number of symbols than would be required with a Monte-Carlo analysis method. The selection of the excitation is performed using the crest factor of the signal. Experimental results, obtained from a high-level simulation of a 5 GHz IEEE 802.11 WLAN receiver front-end, have shown that with the new approach the BER in multicarrier systems can be estimated with the same accuracy as a Monte-Carlo method in a computation time that is two orders of magnitude lower.

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