Predicting Coupled Noise in RC Circuits

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Abstract

A novel method which can be regarded as the noise-counterpart of the celebrated Elmore's delay formula—both being based on the first two moments of the network's transfer function—efficiently and accurately predicts maximum noise between two capacitively coupled RC networks, without simulation. The method applies to general topologies (with significant simplification for coupled trees), accurately models how coupling varies with driver transition time, and quantifies the uncertainty in the calculated noise values. Efficient enough for large circuits, the new method can serve as a key ingredient in CAD methodologies to ensure that a layout is noise-problem free.

1. Introduction

Capacitively coupled noise can sabotage a deep sub-micron design, if not properly managed [1]. There is a clear need for efficient, accurate analysis of crosstalk, including its impact on timing [2]. A number of papers--[3][4][5][6]--propose formulas that predict or bound noise, but these papers usually postulate a simple topology, often a coupled T network (Figure 2). At the other extreme are papers that invoke the machinery of circuit simulation or general N-port reduction [7][8][9]. We seek a middle ground, a simple theory of crosstalk based on only the first two non-zero moments that can be regarded as the noise counterpart to the celebrated Elmore's delay formula.

Considering the coupled RC networks in Figure 1, our goal is to estimate *peak* noise—in closed form, without simulation—at each victim receiver, like R, due to a transition of the aggressor's driver d. We want to estimate how the noise varies from receiver to receiver depending on actual layout. Moreover, we want to predict how the magnitude of crosstalk varies with the rise or fall time Δ of the aggressor's source. Finally, we want to gauge how much approximation is involved in our estimate. This states our problem.

We attempt to solve the noise problem by analogy with the familiar Elmore delay formula used to predict interconnect delay [11]. The Elmore formula is based on the first two moments of the transfer function from the driver to a receiver node on the *same* net; we develop analogous noise equations based on the first two moments of the transfer function from the *aggressor* driver to the *victim* receiver. The resulting 'Elmore crosstalk' formulas give us greater generality than the coupled T formulas in the literature and greater efficiency than the general N-port reduction/simulation methods. The closest counterpart to our method is [10], which—as we shall see—is a *one* moment method.

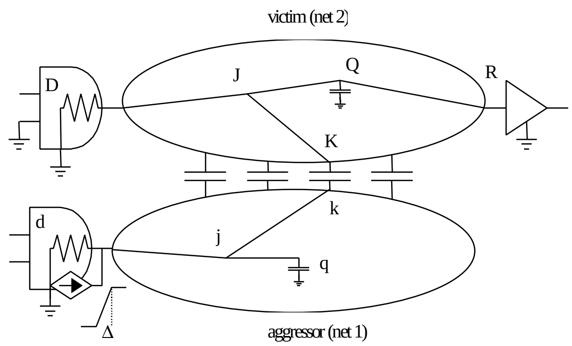

Figure 1 Two coupled RC Networks

Coupled Circuit Equations

The nodal equations for a pair of coupled RC networks like those in Figure 1 can be written in block form as

$$\left\{\begin{bmatrix} C_{11} & C_{21}^T \\ C_{21} & C_{22} \end{bmatrix} s + \begin{bmatrix} G_{11} & 0 \\ 0 & G_{22} \end{bmatrix}\right\} \begin{bmatrix} V^{(1)} \\ V^{(2)} \end{bmatrix} = \begin{bmatrix} e_d \\ 0 \end{bmatrix} J_d(s)$$

$$V_R(s) = \begin{bmatrix} 0 & e_R^T \end{bmatrix} \begin{bmatrix} V^{(1)} \\ V^{(2)} \end{bmatrix} \qquad (2.1)$$

The matrix partitions correspond to the aggressor and victim nets. Blocks C_{11} and G_{11} are the capacitance and conductance matrices of the aggressor and $V^{(1)}(s)$ is the Laplace transform of net 1's nodal voltages. C_{22} G_{22} and $V^{(2)}(s)$ are the corresponding quantities for net 2 (the victim net). Normally, C_{11} and C_{22} are diagonal. Block C_{21} and C^T_{21} constitute the coupling between the nets. $J_d(s)$ is the Laplace transform of the current source at d, $V_R(s)$ the transform of the noise voltage at R. Unit vector e_d has 1 in the row corresponding to the nodal equations for node d where the driver is attached; unit vector e_R has a 1 in the row corresponding to the receiver node. The driver conductance g_d is included in G_{11}; the conductance of the quiescent victim driver, g_D, is included in G_{22}.

3. Moment Calculation

We are interested in calculating the initial two coefficients in the expansion of the transfer impedance

$$Z_{dR}(s) \equiv \frac{V_R(s)}{J_d(s)} = z_1 s + z_2 s^2 + \ldots \qquad (3.1)$$

where z_0 is absent because there is no dc connection between the two circuits. We will first show how to calculate z_1 and z_2 and then show how to estimate coupling noise using these first two moments.

Noise feedback from the victim to the aggressor will only influence coefficients z_3 and higher; accordingly, since we are only interested in z_1 and z_2, we can ignore block C_{21}^T in (2.1). With this simplification, (2.1) becomes block lower triangular and is readily solved:

$$Z_{dR}(s) \approx \\ -e_R^T (C_{22}s + G_{22})^{-1} C_{21} s (C_{11}s + G_{11})^{-1} e_d \qquad (3.2)$$

this expression being exact up to the second moment z_2. From (3.1),

$$z_1 = \lim_{s\to 0}\{Z_{dR}(s)/s\}, z_2 = \lim_{s\to 0}\frac{d}{ds}\{Z_{dR}(s)/s\} \qquad (3.3)$$

Applying these formulas to (3.2), we get[1]

$$z_1 = -e_R^T G_{22}^{-1} C_{21} G_{11}^{-1} e_d \qquad (3.4)$$

$$z_2 = e_R^T G_{22}^{-1}(C_{22}G_{22}^{-1}C_{21} + C_{21}G_{11}^{-1}C_{11})G_{11}^{-1}e_d$$

4. Moments for Coupled Trees

We will now show that when the two coupled nets are trees, the calculation of z_1 and z_2 reduces to calculating several generalized Elmore delays.

To this end we first decompose (3.4) into several sub-calculations:

$$\begin{aligned} a &= G_{11}^{-1}e_d \\ b &= G_{11}^{-1}C_{11}a \\ c &= -G_{22}^{-1}C_{21}a \\ d &= -G_{22}^{-1}C_{21}b \\ e &= G_{22}^{-1}C_{22}c \\ z_1 &= e_R^T c, \quad z_2 = -e_R^T(d+e) \end{aligned} \qquad (4.1)$$

Vectors a and b are associated with net 1, while c, d, and e are associated with net 2.

To further simplify, we restrict ourselves to the case of RC trees in which only one node per tree (namely, the active or quiescent driver) has a resistance to ground. All other nodes may have series resistances between nodes, and capacitances to ground (or to other nets) but no resistances to ground. Such trees we will call **uniquely grounded**. It is also useful to introduce the following notation.

Definition 4.1. For any two nodes q and k in an uniquely grounded RC tree, $\mathbf{R}_{(q,k)}$ denotes the total resistance (including the driver resistance) that is *common* to the unique resistive paths from q to ground and from k to ground.

In Figure 1, $R_{(q,k)}$ would be the path resistance from node j to d.

In the case of uniquely grounded RC trees, the elements of G_{11}^{-1} (and similarly G_{22}^{-1}) have a surprisingly simple interpretation. To see this, apply a dc current source I at node q, say, and calculate the resulting voltage at node k, say, of net 1 (see Figure 1). This current I will flow along the path q-j-d to ground. The voltage v_k produced at node k is equal to $R_{(q,k)}I$. Alternatively, we get this voltage by solving

$$G_{11}v = Ie_q \quad v_k = e_k^T v \qquad (4.2)$$

that is, $v_k = Ie_k^T G_{11}^{-1} e_q$. We have established

Theorem 4.1 The kq'th element of G_{11}^{-1} is the voltage produced at node k from a unit dc current source at node q. For a uniquely grounded RC tree, this voltage is equal to $R_{(q,k)}$, the resistance common to the paths from q to ground and k to ground.

If net 1 is a uniquely grounded tree, then, by the same sort of reasoning,

$$a = G_{11}^{-1}e_d = g_d^{-1}[1 \quad 1 \quad \Lambda \quad 1]^T = g_d^{-1}1_n \qquad (4.3)$$

where g_d is the internal conductance of the driver and $\mathbf{1}_n$ is a column vector of all ones.

From Theorem 4.1, it is easy to see that vector b in (4.1) is nothing other than the vector of Elmore delays (divided by g_d) for net 1[2]:

$$b_k = e_k^T G_{11}^{-1} C_{11} a = g_d^{-1} \sum_{q\in Nodes(1)} R_{(q,k)} C_q^{gnd} \qquad (4.4)$$

Vector c and d are what might be called 'generalized Elmore delays', in that the capacitors used are the coupling capacitors (weighted by b_k in the case of d) rather than capacitors to ground, in contrast to the standard Elmore calculation.

[1] In calculating z_2 we have used the identity

$$\frac{d}{ds}(As+B)^{-1} = -(As+B)^{-1}A(As+B)^{-1}.$$

[2] For purposes of calculation (4.4)—and similarly (4.6)—coupling capacitors are treated as grounded. This is because these capacitances contribute to the diagonal terms of C_{11} and C_{22} in (2.1).

$$c_Q = g_d^{-1} \sum_{K,k \in Coupled} R_{(Q,K)} C_{Kk}^{cpl}$$
$$d_Q = g_d^{-1} \sum_{K,k \in Coupled} R_{(Q,K)} C_{Kk}^{cpl} b_k \tag{4.5}$$

The sums are over all coupling capacitors. Finally, e is the vector of Elmore delays for net 2, capacitors to ground being weighted by the elements of c:

$$e_R = g_d^{-1} \sum_{Q \in Nodes(2)} R_{(R,Q)} C_Q^{gnd} c_Q \tag{4.6}$$

It is clear from these equations that all components of a,b,c,d, and e are positive; hence, from (4.1), z_1 is positive and z_2 negative.

The quantities in (4.4)-(4.6) can be computed efficiently by doing post and pre-order traversals of the trees, the procedure being similar to the standard one used for Elmore delays. For non-tree circuits, the moments can be calculated from (4.1) by LU factoring G_{11} and G_{22}.

5. Noise From Moments

Having explained in detail how the first two moments of the transfer impedance can be calculated, we return to our primary task of considering how these moments can be used to predict coupling noise.

The general procedure for predicting noise or delay from a set of moments is this. First, judiciously select a family of functions $F = F^{(p_1,\ldots,p_m)}(s)$ with parameters $p_1,\ldots,p_m$. The form of F is chosen so that the inverse transform $f(t) \equiv L^{-1}[F]$ has a shape similar to the expected impulse responses of actual circuits. Next, calculate values for the parameters so that the Maclaurin series of *F(s)* has the same initial coefficients as $Z_{Rd}(s)$; in other words, **match moments**. Finally, for a given source $J_d(s)$, take the noise or delay of the approximate waveform $\hat{v}_R(t) \equiv L^{-1}[F(s)J_d(s)]$ as an estimate for the noise or delay of the true waveform $v_R(t) \equiv L^{-1}[Z_{Rd}(s)J_d(s)]$. Limitations of this procedure are that the choice of $F^{(p_1,\ldots,p_m)}(s)$ is usually rather *ad hoc* and there is no way of estimating or bounding the error $|v_R(t) - \hat{v}_R(t)|$.

Instead of selecting a particular matching form $F^{(p_1,\ldots,p_m)}(s)$, we first argue abstractly by considering the class of *all* suitable matching functions and considering to what extent the response is determined by choosing *any* element from this class. Evidently, there is an infinitude of coupled RC circuits having the same moments z_1 and z_2. As the response of these circuits will not all be the same, clearly $v_R(t)$ is not uniquely determined from z_1 and z_2. But to what extent *is* the response determined from z_1 and z_2?

Definition. A trial function *f(t)* is **admissible** if it satisfies the following conditions:

$$\begin{aligned}
&\text{(a)} \int_0^\infty f(t)dt = 1 \\
&\text{(b)} \int_0^\infty tf(t)dt = 1 \\
&\text{(c)}\ f(t) = 0, \quad t < 0 \\
&\text{(d)}\ f(t) \ge 0 \quad t \ge 0
\end{aligned} \tag{5.1}$$

Conditions (a) and (b) are normalization conditions, (c) expresses causality, and (d) captures the feature of RC circuits that the impulse response is non-negative [11].

Let $f_i \in S^a$ be an element from the class S^a of all admissible functions. Then

$$Z_i(s) \equiv z_1 s F_i(-\frac{z_2 s}{z_1}) = z_1 s + z_2 s^2 + \ldots. \tag{5.2}$$

where $F_i(s)$ is the Laplace transform of f_i. In other words, from any admissible function we can form an expression that matches the first two moments of $Z_{dR}(s)$. The quantity $t_{12} \equiv -z_2/z_1$ in (5.2), being positive with dimensions of time, can be though of as a **coupling time constant** from d to R.

For simplicity, assume the driver is a Norton circuit with a saturating ramp current source:

$$J_R(s) = g_d V_{DD} \frac{1 - e^{-\Delta s}}{\Delta s^2} \tag{5.3}$$

Δ is the rise time of the ramp. The response of (5.2) to this source is, for $0 \le t \le \Delta$,

$$\frac{\hat{v}_R(t)}{g_d V_{DD}} = L^{-1}\left[\frac{Z_i(s)}{\Delta s^2}\right] = \frac{z_1}{\Delta} \int_0^{t/t_{12}} f_i(u)du \tag{5.4}$$

For $t>\Delta$, the same expression applies except the lower limit of the integral is $t/t_{12} - \Delta$. In either case, without knowing the specific form of f_i, we cannot, of course evaluate (5.4). However, because of admissibility properties (a) and (b), we *can* say that, for any admissible f_i,

$$\hat{v}_R^{\max} \approx \frac{g_d V_{DD} z_1}{\Delta}, \text{ if } \Delta >> t_{12} \tag{5.5}$$

since the maximum value of the integral in (5.4) is approximately (but always somewhat less than) 1 for such functions and rise times. How much greater must Δ exceed τ_{12} for (5.5) to be accurate? Appendix A gives one answer to this question. What is the peak noise for smaller input rise-times? Knowledge of only z_1 and z_2 doesn't tell us much here; according to (5.4), the max noise predicted from test function f_i is proportional to the max area of a section of base Δ under the graph of f_i—a quantity which could vary greatly from one admissible function to another.

6. Fast Rise-Time Noise

To make further progress, we chose a specific trial function whose form is motivated by physical reasoning. Transfer function (3.2) suggests the trial form

$$F^{(x)}(s) \equiv \frac{1}{(1+t_{+}s)(1+t_{-}s)} \qquad (6.1)$$

where, for $F^{(x)}(s)$ to be admissible,

$$t_{\pm} = \frac{1 \pm x}{2} \quad |x| \leq 1 \qquad (6.2)$$

The physical motivation behind (6.1) is that the factor with τ_+, say, captures in some way the average response from the driver to the coupling capacitors and τ_- captures the response from the coupling capacitors to the receiver. If both circuits are approximately the same size, then we expect τ_+ and τ_- to be approximately the same, whereas for strongly imbalanced circuits, the time constants will differ significantly.

For each value of $|x| \leq 1$ we get a different prediction for the maximum coupled noise. In the limiting case of very fast drivers (Δ=0), it can be shown that

$$\frac{\max v_R^{(x)}}{\max v_R^{(1)}} = \frac{1}{x}\left\{\left(\frac{1-x}{1+x}\right)^{\left(\frac{1-x}{2x}\right)} - \left(\frac{1-x}{1+x}\right)^{\left(\frac{1+x}{2x}\right)}\right\} \qquad (6.3)$$

a quantity which increases from 2/e=0.7358 for x=0 to 1 for |x|=1.

7.'Elmore Noise' and its Uncertainty

One possibility is to choose x in (6.1) so as to match the third moment z_3 in the expansion (3.1) of $Z_{dR}(s)$. Alternatively, we can take (6.3) for x=0 and x=1 as estimating the uncertainty in our predicted maximum noise; this is the approach we take here.

Theorem 7.1. The maximum crosstalk between two RC nets driven by a saturating ramp source, as predicted by form (6.1) and the first two moments z_1 and z_2, is

$$v_R^{\max} = \frac{g_d V_{DD} z_1}{\Delta}\left(1 - e^{-\frac{z_2 \Delta}{z_1}}\right) \qquad (7.1)$$

The actual noise can be up to 2/e≈75% less that (7.1).

Proof. We substitute (6.1) with x=1 into (5.4). This gives us (7.1). The uncertainty is then taken from (6.3).

As already noted, another result on error bounds for $v_R^{\max}$ appears in appendix A. This result is rigorous and does not depend on form (6.1)

8. Examples

To test our theory, we consider two limiting cases of (7.1) and show that in these special cases (7.1) reduces noise formulas already published in the literature. We then give a numerical example.

Example 1: Slow driver transition. If the driver's transition time is very large compared to the coupling time constant, the exponential term in (7.1) can be ignored and we get

$$v_R^{\max} = \frac{g_d V_{DD} z_1}{\Delta} = \frac{V_{DD}}{\Delta}\sum_{K,k} R_{(R,K)} C_{Kk}^{cpl} \qquad (8.1)$$

where the sum is over all coupling capacitors. This is the result of [10], a one moment estimation of noise. According to our theory, (8.1) is valid only when $\Delta >> -z_2/z_1$.

Equation (7.1) is a generalization of [10], and an important one, since for short transition times, (8.1) utterly breaks down. In the extreme case of a step input, (8.1) predicts an *infinite* noise pulse—a wild unreality—whereas (7.1) predicts the sober limit

$$v_R^{\max}(\Delta \to 0) = -\frac{g_d V_{DD} z_1^2}{z_2} \qquad (8.2)$$

Example 2. Coupled T networks. Many crosstalk papers start from a circuit like that in Figure 2. Let us apply our theory to this simple coupled T topology. The expressions for z_1 and z_2 for this circuit, which has only one coupling capacitor, are

$$\begin{aligned} z_1 &= r_d X R_2 \\ z_2 &= -r_d X R_2 \{R_1(C_1 + X) + R_2(C_2 + X)\} \end{aligned} \qquad (8.3)$$

where

$$R_1 = R_x + r_d \quad R_2 = R_y + r_D \qquad (8.4)$$

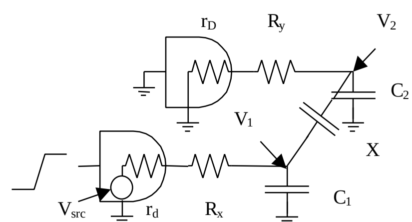

Figure 2 Coupled T Network

Substituting into (8.2),

$$v_2^{\max} = \frac{X R_2}{R_1(C_1 + X) + R_2(C_2 + X)} \qquad (8.5)$$

This agrees with the noise expression derived in [6]. Our theory, however, generalizes [6] in that (7.1) extends (8.5) to arbitrary coupled topologies and to finite ramp sources.

Incidentally, the coupled T circuit of figure 2 has a transfer function exactly of the form (6.1), an indication of the physical suitability of that parameterized form for predicting coupled noise.

Example 3. For a numerical example, consider the circuit of Figure 3. For simplicity, all resistors and capacitors have value 1. Using (7.1) to predict the peak noise coupled to nodes 1, 2, 3, and 4 for various driver transition times, we plot in Figure 4 actual (solid lines) and predicted noise (dashed lines) for driver ramp times (abscissa) from .1sec to 100 sec. The agreement is excellent for slow transistion times, and even for step inputs the uncertainty estimate of

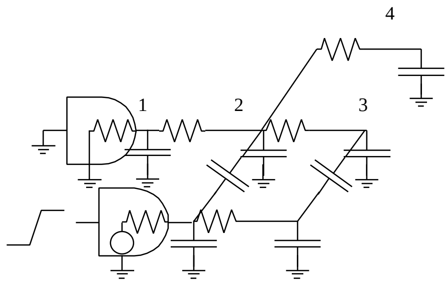

Figure 3 Test Circuit

Theorem 7.1 is adequate in this example, the actual values lying within the range of the theorem. Note that the formula of [6] do not apply to this more general topology, and [10] is applicable only for transition times that are large relative to the Elmore delays of the circuit.

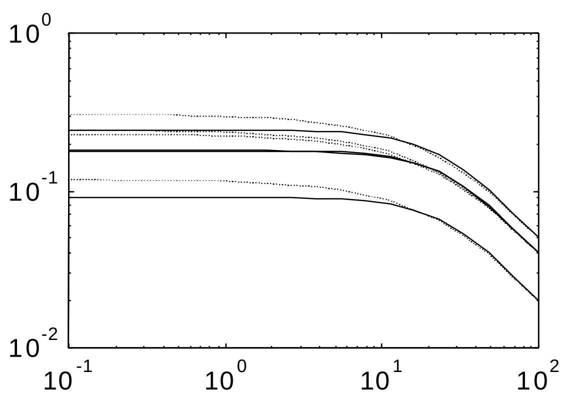

Figure 4. Peak Noise versus Driver Rise-Time

Conclusion

Calculating crosstalk based on two moments balances the engineering goals of efficiency, generality, and accuracy. The moment calculation is quite efficient, especially for trees, and no simulation is required. The method applies to *any* topologies and customizes noise for each receiver. By treating a coupled circuit as a high-pass filter (equation 7.1), it captures the rolloff of noise with driver rise time. Finally, the method is unusually careful in that it also estimates the uncertainty in the predicted values.

Appendix A

Theorem. The maximum crosstalk between two coupled RC circuits satisfies

$$\frac{g_d V_{DD} z_1}{\Delta} \ge v_R^{\max} \ge \frac{g_d V_{DD} z_1}{\Delta}\left(1-\frac{t_{12}}{\Delta}\right) \qquad \text{(a.1)}$$

where Δ is the driver rise/fall time , $t_{12} = -z_2/z_1$,and z_1, z_2 are the first two moments of the transfer impedance (3.1).

Proof. First, we establish some inequalities. For f an admissible function,

$$1 \ge \int_t^\infty u f(u)du \ge t\int_t^\infty f(u)du \qquad \text{(a.2)}$$

since $u \geq t$ in the range of integration. Hence

$$\frac{1}{t} \ge \int_t^\infty f(u)du \qquad \text{(a.3)}$$

and

$$1 \ge \int_0^t f(u)du = 1 - \int_t^\infty f(u)du \ge 1 - \frac{1}{t} \qquad \text{(a.4)}$$

The max noise over all positive time will be greater (or equal) to the max noise over the interval $0 \leq t \leq \Delta$. But in this later interval, max noise occurs at $t=\Delta$. Hence, combining (a.4) at $t=\Delta$ with (5.4), we establish our result.

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