Parametric Fault Diagnosis for Analog Systems Using Functional Mapping[1]

S. Cherubal and A. Chatterjee
Georgia Institute of Technology, Atlanta, GA-30332
e-mail: (sashi, chat)@ee.gatech.edu

Abstract

We propose a new Simulation-After-Test (SAT) methodology for accurate diagnosis of circuit parameters in large analog circuits. Our methodology is based on constructing a non-linear regression model using prior circuit simulation, which relates a set of measurements to the circuit's internal parameters. First, we give algorithms to select measurements that give all the diagnostic information about the Circuit-Under-Test (CUT). From these selected measurements, we solve for the internal parameters of the circuit using iterative numerical techniques. The methodology has been applied to several mixed-signal test benchmark circuits and has applications in process debugging for mixed-signal integrated circuits (ICs) as well troubleshooting and repair of board level systems.

1 Introduction and Previous Work

The test cost of mixed-signal systems is dominated by the test complexity of the analog part. Faults (violation of functional specifications) caused by variation of parameters in analog components are particularly difficult to test and diagnose. Currently, these devices are tested for their functional specifications. If the circuit fails any one of the functional specifications, it is necessary to determine what caused the failure for process debugging, repair and tuning the manufacturing process. In this paper, we present a comprehensive methodology for test selection and diagnosis of parametric faults in complex analog circuits. The proposed methodology (a) eliminates the need to perform circuit simulation for fault diagnosis from observed measurements, (b) is applicable to general non-linear circuits, (c) can diagnose single as well as multiple faults, (d) is able to diagnose large deviation of parameters where sensitivity based techniques are inaccurate (e) allows measurement selection that minimizes the number of nodes accessed during testing as well as test time.

Many approaches to fault diagnosis have been presented in literature. Bandler and Salama [1] have given a review of the early work on fault diagnosis. Existing approaches to fault diagnosis can be divided into two classes- (1) Simulation Before Test (SBT) methods and (2) Simulation After Test (SAT) methods.

SBT approaches are based on the principle of building a fault dictionary-i.e. a list of all possible behaviors of the circuit under fault. The behavior of the CUT is compared with the list of faulty behaviors stored in the fault dictionary for fault diagnosis [8-11]. SBT approaches have been mainly aimed at the diagnosis of digital systems and single catastrophic faults in analog systems.

SAT methodologies explicitly solve for the values of internal parameters of the CUT from a set of measurements on the CUT using on-line circuit simulations [3-7]. The application of these methods has been limited to linear systems and small non-linear circuits because of the need to perform costly circuit simulations in real time. Slamani and Kaminska [2] use the concept of incremental sensitivity for identification of circuit parameters. In this approach, the authors assume that the sensitivity matrix can be computed easily. In general, the computation of sensitivity will be as complex as a full-circuit simulation.

Stenbakken and Sounders [12] and Spaandonk and Kevenaar [13], have used matrix decomposition techniques on the sensitivity matrix, to select measurements for analog circuits. Here, the aim of measurement selection is computation of all the specifications of the CUT from a subset, and not diagnosis of parameters of the CUT.

2 Overview of the Methodology

The performance specifications of any analog CUT depends on a set of circuit parameters.(values of resistors, capacitors, gain of opamps etc.) Our goal is to compute (diagnose) the values of the circuit's parameters, given a set of measurements on the CUT. As noted in Section 1, the major obstacle in the extension of SAT methodologies to large analog circuits is the need to perform circuit simulations based on measurement data. We propose to build a non-linear regression model to approximate the functional relationship between parameters of the CUT and measure-

[1]This work was supported by DARPA, under contract number F33615-95-2-5562

ments made on it, which we denote by $\bar{f}^*_{pm}$.The regression model is given by

$$\bar{f}^*_{pm}(\bar{p}) = \bar{m} \qquad \bar{p} \in \Re^{n_p}, \bar{m} \in \Re^{n_m} \tag{1}$$

where n_p is the number of parameters of the CUT and n_m is the number of measurements. This regression model is constructed using prior circuit simulations. We use a non-linear regression tool -Multivariate Adaptive Regression Splines (MARS) [18] to generate the regression model. Given a set of measurements $\bar{m}$ from the CUT, we solve for a set of parameter values that satisfies Equation(1). This is a set of nonlinear equations in several variables and is solved using iterative techniques similar to Newton-Raphson's algorithm [16].

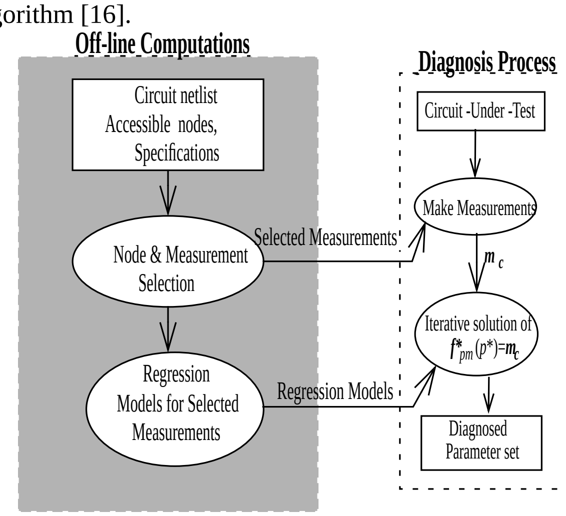

FIGURE 1. Overview of Diagnosis Methodology

Our technique can be seen as a 'SAT-like' technique in which time consuming circuit simulation has been replaced by a simple evaluation of the regression model. This means that our technique can be used to diagnose large analog circuits where simulation complexity makes conventional SAT methods difficult to apply. Since we explicitly solve for all the circuit parameters, we can diagnose single as well as multiple faults. An overview of the diagnosis technique is shown in Figure 1.

In many analog circuits, internal nodes will have to be probed to achieve the required diagnostic resolution. We propose algorithms to pick a minimum number of internal nodes to observe in order to achieve the required diagnostic resolution.

In Section 3, we discuss the algorithms used for choosing an minimum set of nodes, and measurements on the selected nodes, to obtain the required diagnostic resolution. In Section 4, we discuss the methodology for building $\bar{f}^*_{pm}$. In Section 5, we discuss the different diagnosis algorithms used to solve for the circuit parameters. In Section 6, we give diagnosis results for three of the mixed-signal test benchmark circuits [20]. Finally, conclusions are presented in Section 7.

3 Measurement Selection for Diagnosis

We assume that a list of all possible measurements that can be made on the CUT is given. The objectives of the measurement selection procedure are

1. Minimize the number of nodes accessed during testing and
2. Minimize the test time.

To achieve these goals, we examine the testing process. As an example, consider the frequency domain testing of a state variable filter shown in Figure 2. Sine-waves of differ-

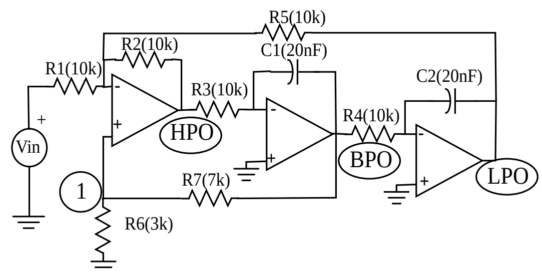

FIGURE 2. State-Variable Filter.

ent frequencies are applied to the input and the gains at the different nodes are measured. The set of gains at the different frequencies are the set of measurements on the CUT. Each measurement on the CUT has the following two attributes: (a) a *test signal* which consists of the state of all inputs to the CUT (in this case, frequency of the input signal) and (b) the node where the measurement is made. Since test signals are applied sequentially to the CUT, test time will be minimized by minimizing the number of test signals. An overview of the measurement selection algorithms is given in Figure 3.

3.1 Node and Signal Selection

Liu et. al. [15] have shown that the number of parameters that can be solved for from a given set of measurements is *rank(**S**)*, where ***S*** is the sensitivity matrix which is the *matrix derivative* of the measurements w.r.t the parameters. Therefore, we use the rank of the sensitivity matrix as a measure of the diagnostic information contained in a set of measurements. The rank of a matrix is defined as the number of its non-zero singular values [17]. In the case of approximate dependencies or low sensitivity values, instead of singular values that are zero, we will get singular values that are extremely small. In these cases, it is may not be possible to solve for as many parameters of the CUT as there are non-zero singular values, due to measurement noise, modeling errors etc. Therefore, while computing the rank of the sensitivity matrix, we consider all singular values which are less than a pre-specified fraction (for example 1%) of the first (largest) singular value to be zero. By doing this, we are considering nearly dependent columns of the sensitivity matrix to be dependent.

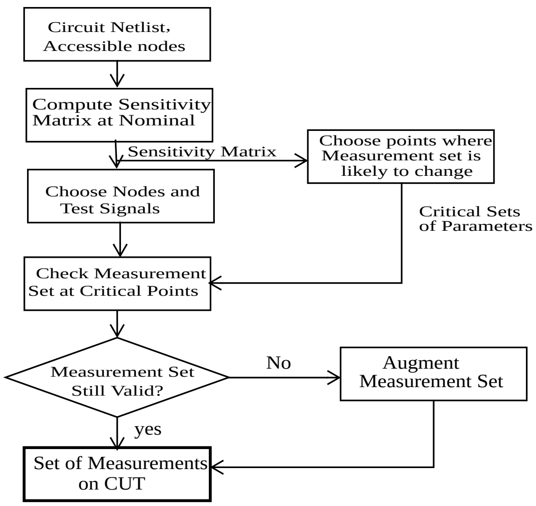

FIGURE 3. Overview of Measurement Selection Methodology

The aim of the measurement selection algorithm is to select a subset of measurements whose sensitivity matrix has the same rank as the whole set of measurements. Let $\overline{\boldsymbol{m}}$ be a subset of the set of all possible measurements on the CUT with sensitivity matrix= $\boldsymbol{S}_{\overline{\boldsymbol{m}}}$. If $rank(\boldsymbol{S}_{\overline{\boldsymbol{m}}}) = rank(\boldsymbol{S})$, where $\boldsymbol{S}$ is the sensitivity matrix of the whole set of measurements, we define the set $\overline{\boldsymbol{m}}$ to be a set having *complete diagnostic information*.

Our primary aim is to minimize the number of nodes that have to be accessed during testing. Therefore, we initially assume that *all* the test signals are applied on the CUT and try to find a minimum set of nodes that will give *complete diagnostic information*. We use the following greedy procedure to search for the set of nodes.

1. Selected Nodes = ϕ, Selected Measurements $\overline{\boldsymbol{m}}=\phi$.
2. Find node which, if added to the set of selected nodes, will cause a maximum increase in the $rank(\boldsymbol{S}_{\overline{\boldsymbol{m}}})$. Let this node be *k*.
3. Add node k to Selected Nodes.
4. Add all measurements made on node *k to* $\overline{\boldsymbol{m}}$.
5. If $rank(\boldsymbol{S}_{\overline{\boldsymbol{m}}}) < rank(\boldsymbol{S})$ go to step 2, else end.

Once a set of nodes has been selected, we try to minimize the number of test signals that have to be applied to minimize test time. In order to achieve this, we use a greedy search similar to the one used for node selection. For each test signal applied, the list of measurements consists of all the measurement(s) of the test signal which are on the selected nodes. Test signals are sequentially added to the set of selected test signals till $rank(\boldsymbol{S}_{\overline{\boldsymbol{m}}}) = rank(\boldsymbol{S})$. After signal selection we get a minimal set of nodes and test signals for the CUT. Since these nodes and test signals are selected using the sensitivity matrix computed with the nominal values of the CUTs parameters, these may not give *complete diagnostic information* with different values of parameters. Therefore, the set of measurements must be checked at points of the parameter space where the relationship between parameters and measurements is likely to have changed.

3.2 Augmenting Measurement Set

The relationship between parameters and measurements can change under fault. For example, an output of an opamp can saturate or a transistor can cut off due to parametric faults, making measurements made on the circuit insensitive to parameter variations in the circuit. Such change of behavior is most likely to occur when the measurements take extreme values. Therefore, we use the heuristic of identifying points in the parameter space where the selected measurements take extreme values. To find these points, for each selected measurement m_i, we compute two sets of parameters $\overline{U}_i=[u_1, u_2, .., u_j ,.., u_n]$ and $\overline{L}_i=[l_1, l_2, .., l_j, .., l_n]$. The parameters u_j and l_j are given by

$$u_j = p_j + sgn(S^{m_i}_{p_j}) \cdot \Delta p_j \tag{2}$$

$$l_j = p_j - sgn(S^{m_i}_{p_j}) \cdot \Delta p_j \tag{3}$$

respectively. p_j is the nominal value of the j^{th} parameter and Δp_j is the maximum deviation in the parameter p_j. $S^{m_i}_{p_j}$ is the sensitivity of m_i to p_j. The measurement m_i is taken to its upper extreme value by the parameter set by the parameter set $\overline{U}_i$ and it is taken to the lower extreme value by the parameter set $\overline{L}_i$. A set of points $\boldsymbol{P}=[\overline{U}_1, \overline{U}_2,.., \overline{U}_k, \overline{L}_1, \overline{L}_2, \overline{L}_k]$ is computed, where the selected measurements take extreme values. At each of the points in $\boldsymbol{P}$, the sensitivity of *all* the measurements is recomputed. Now, we check if the set of selected measurements is a set having *complete diagnostic information*. If not, additional nodes/signals are selected using the algorithms described in Section 3.1, till the set of selected measurements has *complete diagnostic information*.

4 Building the Regression Model.

Once the measurements to be made on the CUT are identified, a regression model is built relating the parameters of the CUT to the selected measurements. The regression model is built from a set of training data generated through simulation of the CUT. We generate instances of the CUT by varying the parameters of the CUT according to the fault statistics given. If no information is known about the specific kinds of faults that can occur, we assume uniform, independent distribution to the different parameters.

We have used Cadence circuit simulator *Spectre* for our circuit simulations. To extract measurements from the simulation data we used the Cadence waveform processing tool *Artil.*

We used a non-linear regression tool, MARS [18] to generate the regression model from the simulation data. The tool has the ability to accurately model highly non-linear functions with large dimensionality of input parameter space. Devarayanadurg et. al. [19] have used MARS to capture the output of mixed-signal modules as a function of different input signals for behavioral simulation. In our approach, we use MARS to approximate the variation of measurements made on a analog circuit with the variation of the circuit's parameters.

5 Diagnosis Procedure

Our methodology solves for the values of parameters of the circuit using a modified Newton-Raphson (N-R) method. For the system of non-linear equations $\bar{f}^*_{pm}=\bar{m}_c$, the iteration step is given by

$$\bar{p}_{k+1} - \bar{p}_k = J(\bar{p}_k)^{-1} \cdot (\bar{f}^*_{pm}(\bar{p}_k) - \bar{m}_c) \qquad (4)$$

where $\bar{m}_c$ are the measurements obtained from the CUT and $J(\bar{p}_k)$ is the Jacobian of $\bar{f}^*_{pm}(\bar{p})$ at p_k. Here the Jacobian is the sensitivity matrix described in Section 3.1. An overview of the diagnosis procedure is given in Figure 4. There are several issues involved in diagnosis using N-R iterations. They are

1. The set of measurements may not uniquely identify the parameters of the CUT
2. Convergence: N-R is only locally convergent- i.e. the iterations converge only if the starting point is close to the solution

We deal with these issues below

5.1 Dependent Input Variables

To solve for the parameters of the CUT, we need to invert the *sensitivity matrix* in each iteration. Dependent or approximately dependent variables can cause the sensitivity matrix to be ill-conditioned. Existence of dependent variables means that there exists infinitely many solutions to the diagnosis equations. In this case, we need to identify parameters that cannot be uniquely solved for, and find one possible solution. Groups of parameters that cannot be uniquely solved for from the given set of measurements are called *ambiguity groups* [14,15]. We use the approach given by Liu et. al. [15] to identify ambiguity groups. Once the ambiguity groups are identified, parameters in each ambiguity group are treated as reference parameters (i.e. they are not varied) so that the rest of the parameter set can be solved for.

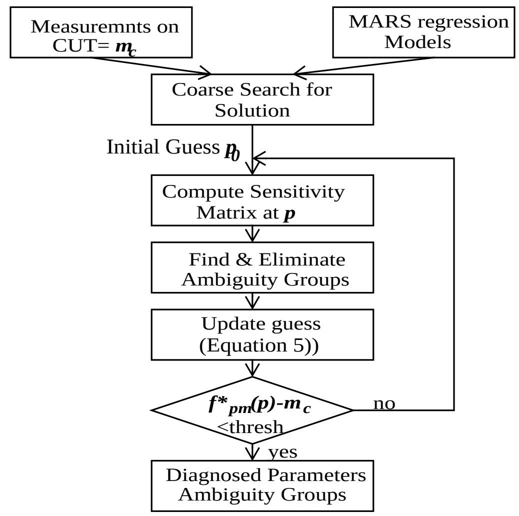

FIGURE 4. Overview of Diagnosis Procedure.

5.2 Convergence of N-R

We need a coarse search procedure to get fairly close to the actual solution to ensure convergence of N-R. We use a simple heuristic of varying one parameter at a time till the maximum value of the error $(\bar{f}^*_{pm}(\bar{p}) - \bar{m}_c)$ is minimized. We use these values of parameters as the initial value for N-R iterations. Also, damping of the iteration sequence, which is a very common way of improving convergence [16] has been implemented. In a damped N-R iteration, the iteration equation is given by

$$\bar{p}_{k+1} - \bar{p}_k = \lambda \cdot J(\bar{p}_k)^{-1} \cdot (\bar{f}^*_{pm}(\bar{p}_k) - \bar{m}_c) \qquad (5)$$

where $0 < \lambda \leq 1$. The starting value for λ is 1 and the value is geometrically reduced till the error in the present iteration becomes smaller than the error in the previous iteration.

6 Results

In this section, we apply our methodology to three ITC mixed-signal test benchmark analog circuits and show diagnosis results. The circuit schematics are shown in Figure 2, Figure 5 and Figure 6. The list of parameters for each circuit is given in Table 1. The values within the parentheses indicate the nominal values of the parameters. Since we are using a voltage input signal and measuring only voltages we cannot solve for the values of individual resistors or capacitors. Therefore, the parameters of the CUT are resistor ratios and R-C products. The list of all accessible node and test signals for the circuits is given in Table 2. The two linear filters are tested in the frequency domain using sine waves. For the DAC, the list of measurements consists of Integral Non-linearity (INL) for inputs words 0 to 255, volt-

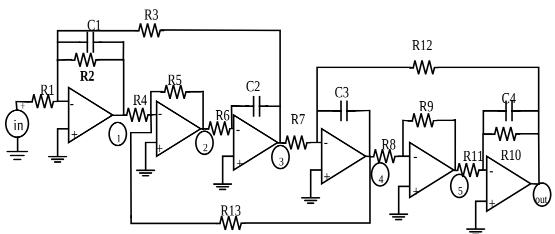

FIGURE 5. Leapfrog Filter

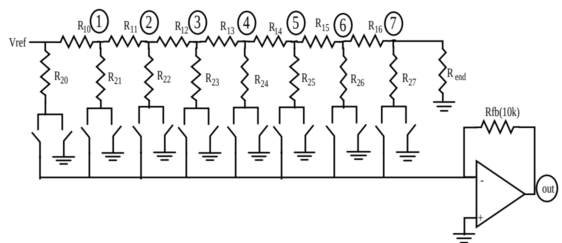

FIGURE 6. 8-bit Ladder D/A converter

ages at nodes 1 to 7 and the rise-time for a $0 \rightarrow 255$ transition. Nodes and test signals were chosen using the measurement selection algorithms. The selected nodes and test signals are shown in Table 3. Three test frequencies were chosen for the state variable filter and six test frequencies were chosen for the leapfrog filter. For the DAC, the measurements chosen were INL measurements for 9 input code words, the DC voltages at nodes 1, 2, 3, 4, 6 and 7 and the rise-time for a $0 \rightarrow 255$ transition. Regression models were built for the selected measurements. Prototypes of the circuits were built to test the diagnosis algorithms. Faults were injected into the circuit by varying the circuit parameters. In the DAC, an *lm346 programmable opamp* was used to vary the opamp's Gain-Bandwidth (GBW) and Slew Rate (SR). The GBW and SR of this opamp IC can be varied by changing an external bias resistor. The selected measurements were made on the CUTs. Test signals were generated using an HP33120A signal generator and measurements were made using an HP54645D digital storage oscilloscope. The 8-bit quantizer of the oscilloscope implies an error of about $\pm 0.5\,\%$ (1/256) in the measurements. The diagnosis algorithms were run on the measured data. The parameters of the circuit (values of resistors, capacitors, etc.) were measured using an HP974A 4.5-digit multimeter. The results of fault diagnosis are given in Table 4. The first column of the table shows the measured values of all the parameters of the CUT and the second column gives the values computed by the diagnosis algorithm. The average error in the diagnosed parameters for the state variable filter and leapfrog filter are 2.07% and 2.83%, respectively. For the ladder DAC, complete diagnosis was not possible from the given set of measurements. The parameters that cannot be solved for uniquely are marked by an asterisk (*). The gain-bandwidth and of the opamp does not affect any of the measurements, and hence it cannot be solved for. Also, the parameters which control the LSBs of the DAC cannot be identified uniquely because of

TABLE 1. Parameters of the three circuits

Circuit	Parameters (Nominal Values)
State Variable Filter	R2/R1(1), R5/R1(1), 1/R3C2(5000), 1/R4C2(5000), R6/R7(0.428)
Leapfrog filter	R2/R1(1), R3/R1(1), R5/R4(1), R13/R4(1), R9/R8(1), R12/R7(1), R10/R11(1), 1/R1C1(10000), 1/R6C2(5000), 1/R7C3(5000), 1/R11C4(10000)
8-bit ladder DAC	R10/Rfb(0.5), R20/Rfb(1), R11/Rfb(0.5), R21/Rfb(1), R12/Rfb(0.5), R22/Rfb(1), R13/Rfb(0.5), R23/Rfb(1), R14/Rfb(0.5), R24/Rfb(1), R15/Rfb(0.5), R25/Rfb(1), R16/Rfb(0.5), R26/Rfb(1), R27/Rfb(1),Rend/Rfb(1), (GBW(1.0e6), SR(0.4V/µs) and Vos(0)) of opamp

TABLE 2. Accessible nodes and tests for the circuits

Circuit	Test Signals	Accessible Nodes
State Variable Filter	400Hz, 800Hz, ...4kHz	HPO, LPO,BPO,1
Leapfrog filter	200Hz, 400Hz, 600Hz, ...4kHz	1, 2, 3, 4, 5, out
8-bit ladder DAC	INL(0 to 255), V(1), V(2), V(3), V(4), V(5), V(6), V(7), Risetime(0 ->255 transition)	out, 1,2, 3, 4, 5, 6, 7

TABLE 3. Nodes and Test signals chosen

Circuit	Nodes Chosen	Test signal chosen
State Variable Filter	HPO, BPO, LPO	400Hz, 800Hz, 3.6kHz
Leapfrog filter	1, 2, 3, 4, 5, out	200Hz, 600Hz, 1400Hz, 2000Hz, 2200Hs, 2400Hz
8-bit ladder DAC	out, 1, 2, 3, 4, 6, 7	INL for(128, 64, 32,31,16, 14, 8, 5, 3), V(1), V(2), V(3), V(4), V(5), V(7) Risetime(0 ->255)transition

low sensitivity values. It is seen that the methodology is able to accurately solve for parameters of the different circuits for single and multiple faults

6.1 Computational Requirements

To compute the average CPU time required for diagnosis, instances of faulty circuit parameters were generated and measurements for these instances were computed through simulation. The diagnosis algorithms were run on this simulation data to compute the average CPU time required for diagnosis. Experiments were run on a SUN Ultra-2 workstation. The average CPU time required for diagnosis of the three circuits is given in Table 5. The third column indicates the average number of NR iterations required by the diagnosis algorithm for each test circuit.

TABLE 4. Diagnosis results

Measured values of parameters	Diagnosed Values of parameters
State Variable Filter	
1.223, 1.226, 4280, 6079, 0.4049	1.232, 1.240, 4325., 6113., 0.4172
1.222, 1.226, 7545, 3724, 0.3326	1.241, 1.24, 7401., 3825, 0.3295
1.414, 1.232, 7350, 6063, 0.2703	1.393, 1.252, 7678, 6070, 0.2924
Leapfrog Filter	
1.230, 1.225, 1.001, 1.200, 0.8142, 1.225, 0.8119, 11970, 7403, 6157, 10220	1.226, 1.265, 0.977, 1.176,0.8193, 1.201, 0.7984, 12657, 7458, 6266, 10885
1.476, 1.225, 0.6807, 1.200, 0.8142, 1.225, 1.198, 11959, 7403, 6157, 15085	1.412, 1.331, 0.6818, 1.168, 0.8277, 1.180, 1.187,12728, 7301, 6050, 14536.
8-bit ladder DAC	
1.000, 0.450, 0.996 0.450, 0.997, 0.448, 1.000, 0.449, 1.005, 0.450, 0.997 0.449 0.996 0.449 0.998, 1.002, 1.10e6, 4.05e5, 3e-3	1.006, 0.468, 1.020, 0.4660, 1.039, 0.480, 1.037, ***0.490*, 0.984*, 0.410*, 0.931*, 0.515*, 0.894*, 0.581*, 1.25*, 0.886*, 2.18e6****, 4.745e5, 4.30e-3
1.131, 0.581, 0.996, 0.450, 1.129, 0.579, 1.000, 0.449, 1.005, 0.450 0.997, 0.449, 0.996, 0.449 0.998 1.002, 2.3e6, 0.937e6, 3e-3	1.141, 0.586, 1.029, 0.463, 1.197, 0.590, 1.155, ***0.486*, 1.251*, 0.562*, 1.091*, 0.520*, 1.11*, 0.639*, 1.249, 0.7009*, 1.87e6****, 1.079e6, 4.5e-3

TABLE 5. CPU time required for diagnosis

Circuit	Average CPU Time for Diagnosis (sec)	Average # of NR iterations
State Variable filter	0.0284	1.44
Leapfrog Filter	0.252	2.3
8-bit ladder DAC	0.2186	3.1

7 Conclusions and future work

In this paper, we have discussed a methodology for identification of a circuit's parameters from measurements made on it. The methodology has been shown to be applicable to a large class of analog circuits. It is seen that the extent to which diagnosis is possible depends on the set of measurements chosen. Our future research will concentrate on devising such efficient tests to aid diagnosis.

References

[1]J. W Bandler and A. E. Salama, "Fault Diagnosis of Analog Circuits," *Proceedings of IEEE*, Vol. 73, August 1985, pp 1279-1326.

[2]M. Slamani and B. Kaminska, "Analog Circuit Fault Diagnosis based on Sensitivity Computation and Functional Testing," *IEEE Design &Test of Computers*, Vol. 9, No. 1, March 1992, pp 30 -39

[3]A. Walker, W. E. Alexander and P. K. Lala, "Fault Diagnosis in Analog Circuits using Element Modulation," *IEEE design & test of computers*, Vol. 9, No. 1, March 1992, pp 19-29.

[4]H. Dai and M. Sounders, "Time Domain Testing Strategies and Fault Diagnosis for Analog Systems," *IEEE transactions on Instrumentation and Measurement*, Vol. 39, No. 1, February 1990, pp 157-162.

[5]N. Sen and R. Saeks, "Fault Diagnosis for Linear Systems via Multifrequency Measurements," *IEEE transactions on circuits and systems*, Vol. CAS-26, July 1979, pp 440-457.

[6]L. Rapisarda and R. A. Decarlo, "Analog Multifrequency Fault Diagnosis," *IEEE transactions on Circuits and Systems*, Vol. CAS-30, April 1983, pp 223-234.

[7]C. Wu, K. Nakajima, C. Wey and R. Saeks, "Analog Fault Diagnosis with Failure Bounds, "*IEEE transactions on Circuits and Systems*," Vol. CAS-29, No. 5, May 1982, p 277-284.

[8]R. Spina and S. Upadhyaya, "Linear Circuit Fault Diagnosis using Neuromorphic Analyzers," *IEEE transactions on Circuits and Systems-II: analog and digital signal processing*, Vol. 44, No. 3, March 1997, pp 190-196.

[9]S. S. Somayajulam, E. Sanchez-Sinencio and J. P. de Gyvez, "Power Supply ramping and Current Measurement based Technique for Analog Fault Diagnosis," *Proceedings of IEEE VLSI test symposium*, 1994, pp 234-239.

[10]Z. You, E. Sanchez-Sinencio and J. P. de Gyvez, "Analog System-level Fault Diagnosis based on a Symbolic Method in the Frequency Domain," *IEEE transactions on Instrumentation and Measurement*, Vol. 44, No. 1, February 1995, pp28-35.

[11]R.Voorakaranam, et. al., "Hierarchical Specification-Driven Analog Fault Modeling for efficient fault simulation and diagnosis" *Proceedings, International Test Conference*, 1997, pp 903-12.

[12] G. N. Stenbakken and T. M. Sounders, "Test point selection and testability measures via QR factorization of linear models," *IEEE transactions on Instrumentation and measurement*, June 1987, pp 813-817.

[13]J. van Spaandonk and T. A. M. Kevenaar, "Iterative test point selection for analog circuits," *Proceeding of the VLSI test symposium*, 1996, pp 66-71

[14]G. N. Stenbakken, M. T. Sounders and G. W. Stewart, "Ambiguity Groups and Testability," *IEEE transaction on Instrumentation and Measurement*, Vol. 38, No 5, October 1989, pp 941-945

[15]E. Liu, W. Kao, E. Felt and A. Sangiovanni-Vincentelli, "Analog testability analysis and Fault diagnosis using behavioral modeling," *Proceedings of IEEE Custom Integrated Circuits Conference*, 1994, pp 413-416.

[16]J. M. Ortega and W. C. Rheinbolldt, Iterative Solution of Nonlinear Equations in Several Variables, New York, Academic, 1970.

[17]D. S. Watkins, Fundamentals of Matrix Computations, New York: Wiley, 1991.

[18]J. H. Friedman, "Multivariate Adaptive Regression Splines," *The Annals of Statistics*, Vol. 19, No, 1, pp 1-141.

[19]G. Devarayanadurg, P. Goeteti and M. Soma, "Hierarchy Based Fault Simulation of Mixed-signal ICs," *Proceedings International Test Conference*, 1996, pp 521-527.

[20]B. Kaminska, K. Arabi, I. Bell, P. Goteti, J. L. Huertas, B. Kim, A. Rueda and M. Soma, "Analog and Mixed-Signal Benchmark Circuits- First Release," *Proceedings International Test Conference*, 1997, pp 183-190.