Probabilistic Standard Cell Modeling Considering Non-Gaussian Parameters and Correlations

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***Abstract*—Variability continues to pose challenges to integrated circuit design. With statistical static timing analysis and high-yield estimation methods, solutions to particular problems exist, but they do not allow a common view on performance variability including potentially correlated and non-Gaussian parameter distributions. In this paper, we present a probabilistic approach for variability modeling as an alternative: model parameters are treated as multi-dimensional random variables. Such a fully multivariate statistical description formally accounts for correlations and non-Gaussian random components. Statistical characterization and model application are introduced for standard cells and gate-level digital circuits. Example analyses of circuitry in a 28 nm industrial technology illustrate the capabilities of our modeling approach.**

I. Introduction

Corner-based design and analysis methods are perfectly justified when global variations dominate, but they lose accuracy when random local variations gain significance at deep submicron technology nodes. From this point of view, process variability is a known, yet not completely solved problem in integrated circuit design [1]. Other variation sources, such as layout-dependent proximity effects [2], may further worsen variability issues but are neglected in this paper.

For certain design styles and particular problems, methods to analyze variability have been developed. In digital design, statistical static timing analysis (SSTA) [3] is the most detailed method to consider timing fluctuations. It treats standard cell delays as analytical functions of varying process parameters. Although enhancements capture correlations or circuit hierarchy, consider non-Gaussian distributions, and apply higher-order polynomial models [4]–[7], SSTA is not widely utilized in industry. Furthermore, timing variations are merely combined with other performance characteristics, such as static and dynamic power. An exception is the variability-aware trade-off between timing and leakage in [8]. Nevertheless, the distributions of timing and other performance characteristics including their correlations are not addressed by existing approaches.

SRAMs are further circuits suffering from variability. They are especially critical due to the large number of bit cells. To analyze variability, high yields have to be estimated. Importance Sampling, most probable points of failure, and statistical blockade are algorithms to solve this task [9], [10]. However, these methods appear to be inappropriate for digital circuits since they focus on extreme and rare events.

We argue that a unified consideration of variability will be highly beneficial since it can be used on different levels of abstraction and in different design styles. To simultaneously handle multiple performance characteristics, such as timing and energy consumption, we propose a probabilistic modeling approach that treats model parameters as multi-dimensional random variables (RV). Combining rank correlation coefficients and generalized lambda distributions (GLD) [11], it allows non-Gaussian and correlated parameters.

After introducing the theoretical background, we will focus on statistical standard cell characterization and probabilistic modeling. Simulation examples of standard cells in a 28 nm industrial technology will illustrate the necessity of correlated non-Gaussian modeling and the benefits of our approach. Finally, the application of probabilistic models in gate-level circuit analyses will be briefly outlined.

II. Multivariate Statistics & Modeling

In [12], an algorithm to generate random samples from a k-dimensional RV $\mathbf{X}$ with its components $\mathrm{X_i}$ is described. It supports arbitrary correlations between the components and arbitrary distribution shapes while two prerequisites have to be fulfilled. (a) Quantile functions $\mathrm{Q_i(u)}$ with $0 \le \mathrm{u} \le 1$, inverses of cumulative distribution functions (CDF), describe the distributions of the random components $\mathrm{X_i}$ independently. (b) Rank correlation coefficients $\mathrm{r_{i,j}}$ capture statistical correlations between any two components $\mathrm{X_i}$ and $\mathrm{X_j}$. They are summarized in a $\mathrm{k} \times \mathrm{k}$ matrix $\mathbf{R}$ with $\mathrm{R_{ij}} = \mathrm{r_{i,j}}$.

To make use of this approach, probabilistic modeling has to address both prerequisites using sample data as inputs. We chose the generalized lambda distribution (GLD) to model the distributions of the random components $\mathrm{X_i}$. With $\mathrm{u} \in [0, 1]$, the GLD is defined by its quantile function [11],

$$x_i = Q_i(u) = \lambda_{i,1} + \frac{\frac{u^{\lambda_{i,3}}-1}{\lambda_{i,3}} - \frac{(1-u)^{\lambda_{i,4}}-1}{\lambda_{i,4}}}{\lambda_{i,2}}. \tag{1}$$

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The four distribution parameters for location ($\lambda_{i,1}$), scale ($\lambda_{i,2}$), and shape ($\lambda_{i,3}$, $\lambda_{i,4}$) can be adapted to approximate a variety of probability distributions, including uniform and Gaussian. In a different parametrization, the GLD was successfully applied to model inverter performance characteristics and local transistor variability already [13], [14].

Rank correlation coefficients $r_{i,j}$ between two random components X_i and X_j can be determined from sample data by

$$r_{i,j} = 1 - \frac{6 \cdot \sum_{k=1}^{N} d_k^2}{N \cdot (N^2 - 1)}, \tag{2}$$

with the sample size N and the rank differences d_k. The rank of a sample value x_i is its position in the ordered observations of the component X_i. Consequently, the rank difference d_k is the difference in ranks of the k-th observations of X_i and X_j.

In this paper, we determine rank correlation coefficients and GLD parameters from Monte Carlo sample data. More efficient approaches may be subject to future research. Data analysis and model application are performed in statistics software *R* [15] for which a method to map sample data to GLD parameters is available [16].

III. Setup for Statistical Cell Characterization

In statistical standard cell characterization, the data required to determine rank correlation coefficients and GLD parameters as described in Sec. II have to be generated. For our analyses, we assume that device compact models contain global and local process variability.

In the conventional approach, standard cell performance characteristics, such as delay and power, are determined for different input signals and output loads and potentially at different process corners. Clearly, to generate sample data, Monte Carlo SPICE simulations have to replace corner runs, as it is illustrated in Fig. 1.

Further adaptations have to be made due to the correlations for which three different types have to be considered:

(1) intra-cell correlations: between performance characteristics of a single cell – for instance, a fast standard cell usually has high static and dynamic power;
(2) inter-cell correlations: between performance characteristics of different cells – for instance, if a cell is fast, neighboring cells tend to be fast as well; and
(3) inter-instance correlations: between performance characteristics of different instances of a cell – for instance, if one inverter is fast, other inverters tend to be fast as well.

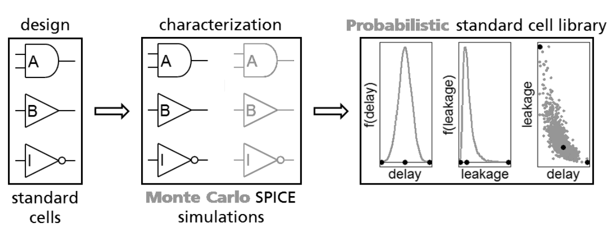

Fig. 1. Conventional cell characterization with corners (black) and extensions for statistical characterization (bold gray)

Monte Carlo SPICE simulations with identical global but cell-specific or instance-specific local variation parameters generate sample data to calculate the correlation types (1) and (2). To capture correlations of type (3), each cell has to be duplicated and simulated twice. The simulation overhead involved is required to obtain the fully multivariate description.

Based on the simulation results, GLD approximations can be found for the distributions of all performance characteristics. In the future, these parameters could replace raw numbers in table-based models, for instance non-linear delay models.

Correlation handling, however, is more complicated. We propose to keep the three types of correlations presented above and create several correlation sub-matrices:

(1) the matrix $\mathbf{R_X}$ stores intra-cell correlation coefficients of standard cell X;
(2) the matrix $\mathbf{R_{XY}}$ stores inter-cell correlation coefficients between performance characteristics of standard cells X and Y; and
(3) the matrix $\mathbf{R_{XX}}$ stores inter-instance correlation coefficients between performance characteristics of two instances of standard cell X.

With this approach, the correlation sub-matrices $\mathbf{R_A}$, $\mathbf{R_B}$, $\mathbf{R_I}$, $\mathbf{R_{AB}}$, $\mathbf{R_{AI}}$, $\mathbf{R_{BI}}$, $\mathbf{R_{AA}}$, $\mathbf{R_{BB}}$, and $\mathbf{R_{II}}$ have to be determined when we consider the three standard cells depicted in Fig. 1, an AND2 gate (A), a buffer (B), and an inverter (I).

IV. Probabilistic Modeling of a Set of Cells

As an illustrative example, probabilistic models are derived from 1000-sample Monte Carlo SPICE simulations for a set of standard cells in a 28 nm industrial technology: an AND2 gate, a buffer, an inverter, a NAND2 gate ($\overline{\text{A}}$), and a NOR2 gate ($\overline{\text{O}}$). State-dependent leakage power for all static input signal combinations as well as cell delay and dynamic energy for relevant input pin to output pin switching events are the performance characteristics of interest X_i. The dynamic performance characteristics are determined for exponential input voltage sources with time constants $\tau = 2.5$ ps and $\tau = 5$ ps as well as output loads $C_L = 5$ fF and $C_L = 10$ fF.

The Q-Q plots in Fig. 2 present the distributions of selected performance characteristics. Significant deviations from the diagonals indicate non-Gaussian behaviors in many cases; yet, empirical data and the GLD approximations are nearly identical. To further investigate this statement, with a significance level $\alpha = 0.05$, Shapiro-Wilk tests check raw data for Gaussianity, and Kolmogorov-Smirnov tests compare raw data with 10^5 samples from the approximated GLDs. The test results are summarized in the bar plots in Fig. 3 displaying the total number of performance distributions as well as the number of valid GLD and Gaussian approximations grouped by leakage power, cell delay, and dynamic energy. The distributions of leakage and cell delay are generally non-Gaussian. This is also true for approximately 56 % of the dynamic energy distributions. Most valid Gaussian approximations can be observed for fall energy distributions, for instance the NAND2 fall energy in the lower right corner of Fig. 2. On the contrary, nearly all GLD approximations represent raw data well, see

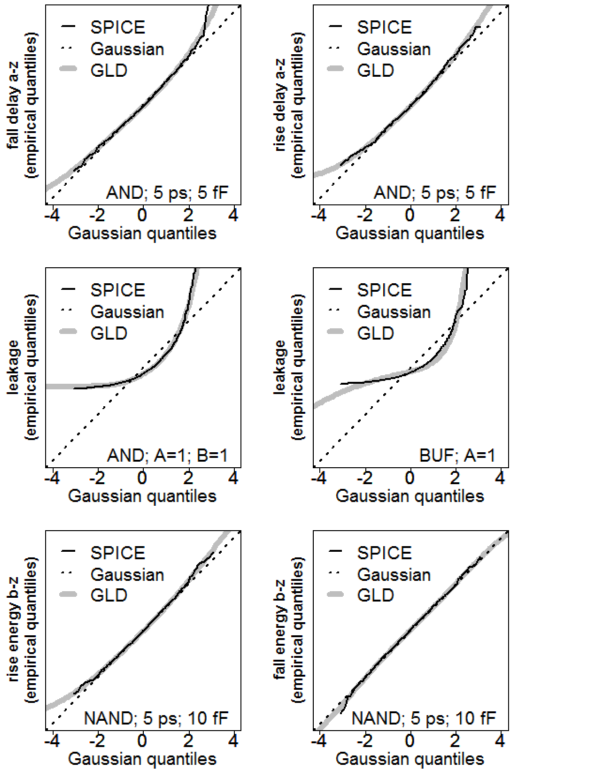

Fig. 2. Example Q-Q plots of performance distributions, sample size 1000

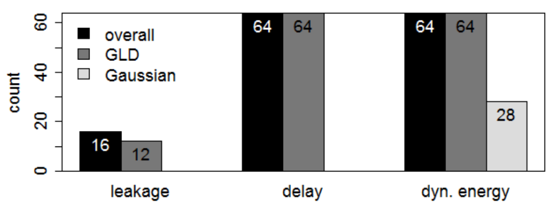

Fig. 3. Total number of performance distributions and numbers of valid GLD and Gaussian approximations

Fig. 2. There are four exceptions, exclusively leakage power distributions due to slight deviations in the tail regions: buffer leakage for input $\mathrm{A}=1$ (middle of right column in Fig. 2), NAND2 leakage for inputs $\mathrm{A}=0$ and $\mathrm{B}=1$ as well as $\mathrm{A}=1$ and $\mathrm{B}=0$, and NOR2 leakage for inputs $\mathrm{A}=\mathrm{B}=0$. In general, assuming Gaussian distributions introduces inaccuracies, but approximating raw data by GLDs performs well.

Preserving correlations is the second important task in probabilistic modeling. For a selected set of performance characteristics, we compare raw data and a sample of size 1000 from our probabilistic standard cell models. The scatter plots in Fig. 4 demonstrate the good agreement between the cluster of points. This holds for intra-cell correlations (inverter and buffer instances 1), inter-instance correlations (inverter instances 1 and 2), and inter-cell correlations (inverter and buffer). The maximum absolute difference between the rank correlation coefficients from raw data and from our test sample is in the range of 0.05 to 0.07 and hence very small. It can be reduced below 0.02 by increasing the sample size by an order of magnitude.

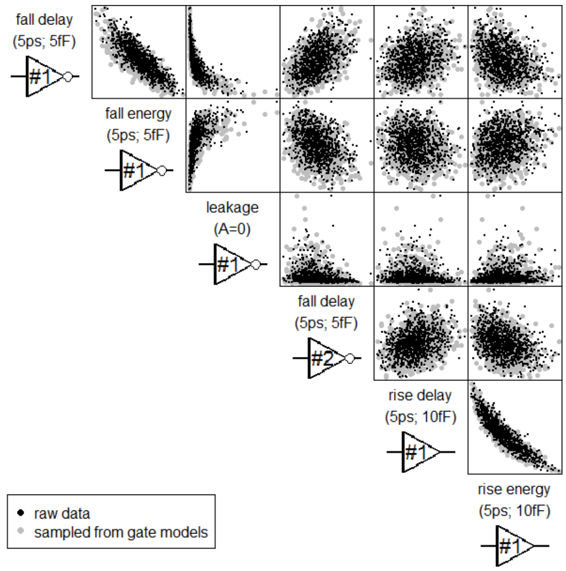

Fig. 4. Examples of preserved correlations; plots in a column share x-coordinates; plots in a row share y-coordinates; performance characteristics defined in corresponding rows and columns; #k defines the instance number of the shown logic gate; arbitrary units

Altogether, this example demonstrates that the GLD approach is very accurate in standard cell modeling. It captures well both correlations and non-Gaussian distributions.

V. Application Scenarios

The principal application of probabilistic standard cell models in gate-level analyses is schematically illustrated at the examples of an AND4 circuit and a NAND4 circuit composed by 2 NAND2 gates, a NOR2 gate, and an inverter, see Fig. 5.

To utilize the sampling approach in [12], all standard cells need to be considered when building the circuit correlation matrix $\mathbf{R}$. At this stage, we make use of our sub-divisions of correlations into intra-cell, inter-cell, and inter-instance components. For the AND4 circuit, NAND2 and NOR2 intra-cell correlations ($\mathbf{R}_{\overline{\mathrm{A}}}$ and $\mathbf{R}_{\overline{\mathrm{O}}}$), NAND2 inter-instance correlations ($\mathbf{R}_{\overline{\mathrm{A}}\,\overline{\mathrm{A}}}$), and inter-cell correlations between NAND2 and NOR2 ($\mathbf{R}_{\overline{\mathrm{A}}\,\overline{\mathrm{O}}}$) have to be taken into account to build the correlation matrix (black part only),

$$\mathbf{R}_{\mathrm{NAND4}} = \begin{pmatrix} \mathbf{R}_{\overline{\mathrm{A}}} & \mathbf{R}_{\overline{\mathrm{A}}\,\overline{\mathrm{A}}} & \mathbf{R}_{\overline{\mathrm{A}}\,\overline{\mathrm{O}}} & \mathbf{R}_{\overline{\mathrm{A}}\,\mathrm{I}} \\ \mathbf{R}^T_{\overline{\mathrm{A}}\,\overline{\mathrm{A}}} & \mathbf{R}_{\overline{\mathrm{A}}} & \mathbf{R}_{\overline{\mathrm{A}}\,\overline{\mathrm{O}}} & \mathbf{R}_{\overline{\mathrm{A}}\,\mathrm{I}} \\ \mathbf{R}^T_{\overline{\mathrm{O}}\,\overline{\mathrm{A}}} & \mathbf{R}^T_{\overline{\mathrm{O}}\,\overline{\mathrm{A}}} & \mathbf{R}_{\overline{\mathrm{O}}} & \mathbf{R}_{\overline{\mathrm{O}}\,\mathrm{I}} \\ \mathbf{R}^T_{\overline{\mathrm{A}}\,\mathrm{I}} & \mathbf{R}^T_{\overline{\mathrm{A}}\,\mathrm{I}} & \mathbf{R}^T_{\overline{\mathrm{O}}\,\mathrm{I}} & \mathbf{R}_{\mathrm{I}} \end{pmatrix}. \tag{3}$$

An inverter has to be appended to create a NAND4 circuit. Consequently, the corresponding sub-matrices, the gray sub-matrices in (3), have to be added to the AND4 correlation matrix to obtain the NAND4 correlation matrix $\mathbf{R}_{\mathbf{NAND4}}$.

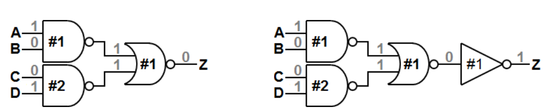

Fig. 5. Gate level schematics of an AND4 and a NAND4 circuit

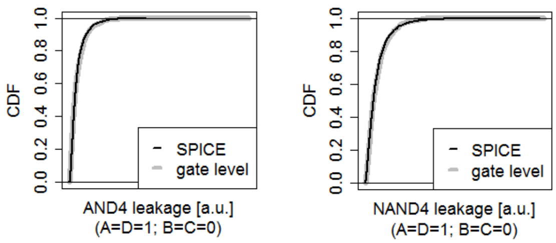

Fig. 6. Leakage power distributions of 4-input AND and NAND circuits for A=D=1 and B=C=0 determined by statistical gate level analyses and Monte Carlo SPICE simulations

In a statistical gate-level analysis, the circuit correlation matrix $\mathbf{R}$ and the GLD parameters are the inputs to the sampling algorithm in [12]. Subsequent parameter selections according to the current scenarios and suitable calculations can then be carried out utilizing the sample data.

If, for instance, the leakage power distributions of the AND4 and the NAND4 circuits in Fig. 5 are of interest for the input signals $\mathrm{A}=\mathrm{D}=1$ and $\mathrm{B}=\mathrm{C}=0$, the NAND2 leakage power for $\mathrm{A}=1$ and $\mathrm{B}=0$ (instance #1) as well as $\mathrm{A}=0$ and $\mathrm{B}=1$ (instance #2), the NOR2 leakage power for $\mathrm{A}=\mathrm{B}=1$, and the inverter leakage power for $\mathrm{A}=0$ (NAND4 circuit only) have to be summed. In Fig. 6, the good agreement between the results of the gate level calculations and Monte Carlo SPICE simulations demonstrates that, in principle, probabilistic standard cell models can be applied in statistical gate level circuit analyses. While the SPICE simulations took more than $30\,\mathrm{s}$ to complete, the gate level calculations finished in approximately $0.6\,\mathrm{s}$, which corresponds to a $50\,\mathrm{X}$ speed up.

Other circuit performance characteristics can be derived from the same set of sample data by selecting and summing the corresponding standard cell performance characteristics, for instance propagation delay as the sum of cell delays or energy consumption as the sum of dynamic energy consumptions of the cells. This allows a joint analysis of the variability in circuit performance and power including their correlations.

VI. Summary, Conclusions & Outlook

In this paper, we presented a probabilistic modeling approach that considers model parameters as multi-dimensional random variables. Combining rank correlation coefficients and generalized lambda distributions (GLD), the method

(1) simultaneously handles global and local process variations;
(2) efficiently approximates Gaussian and non-Gaussian distributions; and
(3) jointly considers multiple performance characteristics, such as delay and power, including their correlations.

Applied to an example set of standard cells in a 28 nm industrial technology, we could show that assumptions of Gaussian distributions of performance characteristics may be wrong while the GLD accurately models the data. Furthermore, the correlations in raw data could be captured. The proposed sub-division of correlations into intra-cell, inter-cell, and inter-instance correlations may be a step towards efficient correlation handling in digital circuit design and analysis.

The model application was demonstrated for simple gate-level digital circuits. Compared with SPICE reference simulations, we obtained a $50\,\mathrm{X}$ speed up and accurate results with the probabilistic gate-level analysis. How this sampling-based analysis approach can be applied to very complex circuits remains as future work, yet.

Although we applied our approach to an example set of standard cells in a 28 nm industrial technology, it can conveniently be adapted to other problems, such as probabilistic device compact modeling or SRAM performance modeling, which are mostly subject to current research. We work towards future publications demonstrating how this method can be successfully utilized in these areas as well.

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