Periodic Jitter and Bounded Uncorrelated Jitter Decomposition Using Incoherent Undersampling

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Abstract— Jitter measurement is an essential part for testing high speed digital I/O and clock distribution networks. Precise jitter characterization of signals at critical internal nodes provides valuable information for hardware fault diagnosis and next generation design. Recently, incoherent undersampling has been proposed as a low-cost solution for signal integrity characterization at high data rate. Incoherent undersampling drastically reduces the sampling rate compared to Nyquist rate sampling without relying on the availability of a data synchronous clock. In this paper, we propose a jitter decomposition and characterization method based on incoherent undersampling. Associated fundamental period estimation techniques along with properties of incoherent undersampling, are used to isolate the effects of periodic and periodic crosstalk jitter. Mathematical analysis and hardware experiments using commercial off-the-shelf components are performed to prove the viability of the proposed method.

Keywords - Incoherent Undersampling; Jitter Separation; Periodic Jitter; Bounded Uncorrelated Jitter; Crosstalk Jitter

I. INTRODUCTION

Reconstruction of periodic digital waveforms is utilized for testing high-speed systems. Even with multiple parallel ADCs, Nyquist rate sampling is very difficult to be implemented at high data rate, and is susceptible to distortion due to small amount of mismatch between the parallel paths. Various undersampling techniques are used to overcome the sampling bottleneck and reconstruct the signal waveforms. To increase effective sampling rate, equivalent time sampling (ETS) employs a fixed sampling clock and a swept delay increasing every trigger cycle. In source-synchronous design, to capture the noise information, the reference clock provided to the device under test (DUT) should be relatively clean. If the trigger signal provided by the DUT is already noisy, it is difficult to precisely characterize the signal integrity of the system. Incoherent undersampling (IUS) techniques utilize simple acquisition schemes using external uncorrelated clock and back-end DSP processing to reconstruct signal waveform [1-5]. One major benefit of IUS is that synchronization between the DUT clock and the sampling clock is not needed. The DUT circuit and the ADC used for waveform capture can be in two different clock domains. Clean measurement clocks can be provided to the ADC by precise clock generation equipment. Therefore, the DUT can be characterized with minimal bias. One of the key ideas of IUS is the precise estimation of the fundamental period. In this paper, we use the fundamental period estimation and correction technique proposed in [6]. Furthermore, we extend the fundamental

period estimation method to cover non-periodic signals with uncorrelated periodic components and separate the components.

Time domain waveform characterization is essential for jitter measurement and analysis. As the I/O interface data rate and clock frequency increases, precise jitter characterization can provide valuable feedback to designers of high-speed I/O, and clock generation and distribution circuit. In general, there are different types of jitter: random jitter, which is caused by device noises (e.g. thermal noise and flicker noise [7, 8]), is unpredictable electronic timing noise. It is a random process, which is divided into Gaussian and non-Gaussian distributions [9]. Usually, RJ is assumed to be a Gaussian distribution which is a good approximation for most practical cases. Data dependent jitter is caused by asymmetry in rise time and fall time, or bandwidth limitation of the channel. Crosstalk jitter, also called bounded uncorrelated jitter, is caused by signals from channels (aggressor) coupling into another channel (victim). It is bounded because of finite range coupling effect. It is uncorrelated because there is no correlation between the signal transitions of physically close links [10]. The exact model depends on the data pattern, signals at the aggressor nodes, and coupling mechanism. As the speed of data links increases, the signal, with shorter rise/fall time, contains more high frequency components which could easily leak into other links. This effect causes crosstalk jitter and reduces the timing margin in high speed link and I/O design. Periodic jitter can be caused by multiple mechanisms. For example, power supply switching noise could couple to data or clock signal lines and cause periodic jitter [11-14]. Especially in clock generation and distribution, periodic jitter and random jitter measurement is important. Low-frequency noise can easily couple into the clock through the power supply network. All the different types of jitter might be present in signals together, and therefore, there are different algorithms to separate and quantify them. For example, tail-fitting is a method to characterize the distribution of the random jitter when periodic jitter is present in the signal [15]. In this paper, we propose jitter separation methods based on incoherent undersampling and linear regression to separate periodic jitter, following a technique similar to that used in [16]. We exploit the fact that the aggressors and victim are uncorrelated to separate uncorrelated periodic crosstalk jitter. We provide detailed mathematical analysis and a proof of concept demonstration of the application of the proposed method using off-the-shelf circuit. The remainder of the paper is organized as follows: In section II, proposed jitter separation method is described. Hardware

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measurement for jitter separation method is provided in section III followed by conclusion in section IV. The contributions of this paper are listed as follows:

- Propose separation / reconstruction for non-periodic signals with uncorrelated periodic components
- Proposed method for the application of periodic jitter separation and periodic crosstalk jitter separation

II. PROPOSED METHOD

A. Periodic Signal Reconstruction and Fundamental Period Estimation

Incoherent undersampling is a signal waveform acquisition method used to reconstruct high speed periodic signals. The sampled waveform is digitized directly after track-and-hold circuit, which is used to increase the ADC bandwidth. The hardware acquisition does not require trigger signal or synchronization between the data clock and the sampling clock. The fundamental period T_0 is estimated by minimizing a metric calculated using the acquired samples. Let x(t) be a periodic signal with fundamental period $T_0 \cdot x(t)$ is sampled with sampling period T_s . Let the sample set $X = \{x[i] = x(iT_s)\}$. Fold all the samples to one estimated fundamental period \tilde{T}_0 .

$$t' = mod(t, T_0)$$

where t' is the new timing for the samples within the estimated fundamental period \tilde{T}_0 . Let Y be the sample set ordered with t'. When the estimation \tilde{T}_0 is incorrect, the sampling points would scattered across all the plots. The phenomenon is similar to incorrect triggering in a trigger-based oscilloscope. We use total variation as the metric to estimate the correctness of the estimation. The calculation of total variation given set Y is

$$TV(Y,T) = \sum_{i=2}^{N} |y[i] - y[i-1]|$$

where N is the number of samples. When estimated period \tilde{T}_0 is incorrect, the absolute value of the difference between adjacent sampling points would increase since the samples are scattered across the whole range in arbitrary order. Thus the estimated fundamental period \tilde{T}_0 can be obtained by minimizing total variation.

$$\widetilde{T}_0 = \arg\min_{m} TV(Y,T)$$

Given a roughly known fundamental period, we sweep around that value and obtain the estimated fundamental period. We refer to this estimate as coarse estimate since there is estimation error due to reduced local sensitivity of the total variation cost function to error in fundamental period estimation in presence of noise. A secondary step is required to obtain finer estimation of the fundamental period as discussed in the next paragraph.

Once the fundamental period is estimated, the waveform can be reconstructed by folding all the samples onto a time window of length equal to one fundamental period. If we do regression at the transition edges of the reconstructed signal and plot the deviation of the samples from the regression line along the sampling time before folding, the deviation will have a constant drift component because of the time period estimation error in the coarse estimation step. Therefore, after coarsely estimating the period, fine adjustment is performed using the following equation

$$\widetilde{T_0}_{fine} = \widetilde{T_0} + \varepsilon = \widetilde{T_0} + m\widetilde{T_0}$$

Where $\tilde{T}_{0_{fine}}$ is the fine estimation of the fundamental period, ε is the estimation error, and *m* is the slope of the regression line. The waveform can be reconstructed using this finer estimate of the fundamental period. The jitter separation technique is described in the following sub section.

B. Periodic Jitter Separation

Referring to the fine adjustment step in the last section, the timing deviation of the transition edge contains the information about the jitter. Therefore, after reconstructing the periodic waveform, we do regression at the transition edge and plot the deviation of the samples from the regression along the sampling time. In reality, the periodic jitter usually consists of lower frequency components compared to the signal itself. By analyzing the waveform using curve fitting or irregular FFT, deterministic components can be identified and accurately characterized. We can further compensate for these deterministic components to better understand the impact of the different jitter components on the device. The hardware measurement is shown in section III.

C. Separation of Signals with Periodic Uncorrelated Components

Bounded uncorrelated jitter, also called crosstalk jitter, is caused by signals coupling between different channels. The signal and the coupled noise are usually uncorrelated. It can be modeled as uncorrelated signals superimposed on the data. If both the crosstalk noise and the signal are periodic, then it is possible to separate out the signal and the noise components. Two signals are uncorrelated if their covariance is zero. Let us consider a very simple case. Let

$$f(t) = \sin t + \sin 2t$$

$$g(t) = \sin \sqrt{2}t + \sin 5\sqrt{2}t$$

$$h(t) = f(t) + g(t)$$

f(t) and g(t) are two zero-mean periodic signals and let the period be T_f and T_g respectively. On the other hand, h(t) is not a periodic signal, since the two fundamental periods are irrational relative to each other. Therefore, it is not possible to visualize the signal h(t) using oscilloscopes, since there is no correct trigger timing. If we sample h(t) at time t_i and use IUS to reconstruct the signal, it will not work because h(t) is not periodic. However, if we still fold the all the samples to the period T_f , by ordering with respect to

$$t' = mod(t, T_f)$$

Then the two components can be separated as follows. Let t_0 be such that $0 \le t_0 \le T_f$. If we select the samples within a small interval Δt , $\Delta t \ll T_f$ centered at t_0 , then the samples H_{t_0} $H_{t_0} = \left\{ h(t_h) \text{ where } t_0 - \frac{1}{2} \Delta t \le mod(t_h, T_f) \le t_0 + \frac{1}{2} \Delta t \right\}$

Where t_h is the sampling time of samples in *H* represent all samples falling within the window. Since Δt is small, we know the fact that

 $t_h \approx kT_f + t_0,$ $k \in integer$ Therefore for the samples in H

$$h(t_h) = h(kT_f + t_0) = f(kT_f + t_0) + g(t_h)$$

= f(t_0) + g(t_h)

If we average N number of samples in H_{t_0} , we have

$$\frac{1}{N}\sum_{h}h(t_{h}) = \frac{1}{N}\left(Nf(t_{0}) + \sum_{h}g(t_{h})\right)$$

Because f(t) and g(t) are uncorrelated, sample g(t) at time $kT_f + t_0$ is very likely to sample g(t) at random points. We have made assumption that the mean of q(t) and f(t) are zero. Therefore the second term of the equation above is close to zero when N is big. That is the average of samples set H_{t_0}

$$\frac{1}{N}\sum_{h}h(t_{h})\approx f(t_{0})$$

This tells us that when we estimate the fundamental period of one of the periodic component of a non-periodic signal correctly, the average of the samples within a small time period is the value of the periodic component at that small time period. Other uncorrelated components are averaged to zero. If the estimation is neither T_f nor T_g , the average would be close to zero since f(t) and g(t) are zero-mean signals. This indicates that when we estimate the fundamental period, we should modify the original metric (TV). Fold all the samples into one period T, and partition T into small equalsized time period. Let sample set Z be the average of samples in these different time periods in T

 $\widetilde{T}_0 = \arg \max_T Variance(Z)$ The variance of the averaged signal is used to measure the power of Z to distinguish if there are periodic components left after averaging, since signals with different periods are close to zero after averaging. If we collect samples of h(t) and maximize the new metric to estimate the fundamental periods of f(t) and g(t), we can reconstruct them one by one as in Fig.1. Theoretically, given enough number of samples we can reconstruct all uncorrelated periodic components in a nonperiodic signal individually.

To separate crosstalk jitter, the first step is to reconstruct the periodic signal using IUS. Crosstalk jitter can be visualized as

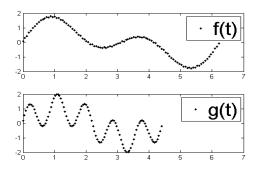


Fig. 1 Two periodic components reconstruction in example

voltage perturbation on the signal waveform induced due to power coupling into the channel from other links (aggressors). It causes timing deviation at the transition edge, and additive noise components at the flat region of the signal. Therefore, after the fundamental period estimation, the reconstructed signal resembles the waveform shown in step 1 in Fig.1. After averaging, the reconstructed signal, it resembles the red line shown in step 1, Fig. 2. The red line represents the major component of the victim signal (the first periodic component). If we subtract the victim signal, we will see the voltage perturbation as shown in step 2, Fig. 2. The reason that we see the aperiodic envelope shape is because the period of the victim and the period of the aggressors are different. The voltage perturbation in the crosstalk jitter is most prominent at the "bit-transition edge" of the aggressor signals. Here, we have made an assumption that the bit periods from aggressors and the bit period of the victim are same. The envelope shape has multiple voltage level (3 in this case) within in a time interval. This is because the estimated period is for the victim not the aggressors. Hence, we repeat the period estimation again to estimate the period for the crosstalk jitter (the second periodic component). The crosstalk jitter consists of high frequency components from the aggressors. The waveforms consist of pull-up and pull-down pulses as shown in step 3, Fig. 2. By using the estimated period and averaging the reconstructed jitter waveforms (step 4, Fig. 2), the impact of the aggressor on the reconstructed victim waveform can be compensated for to better visualize the residual jitter in the victim waveforms shown in step 5, Fig. 2. Once the victim signal and the crosstalk jitter are separated, further analysis can be done to characterize the random jitter. . We can also

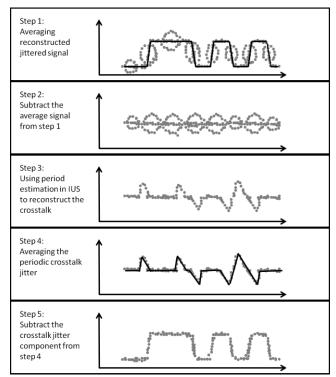


Fig. 2 Crosstalk Jitter Separation

estimate and reconstruct the period crosstalk jitter first and then estimate the victim signal since the cost function works well even at low signal to noise ratio.

III. JITTER SEPARATION & HARDWARE MEASUREMENT

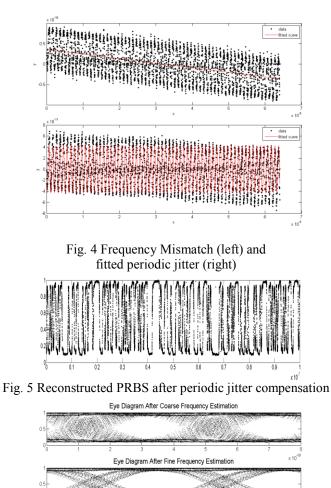
In this section, we provide the hardware measurement in support of the proposed techniques in section II for separating periodic jitter and crosstalk jitter. The first step is the same: use IUS to reconstruct the signal.

A. Periodic Jitter Separation

We use three different hardware setups to generate the jittered test signals and do jitter separation and compensation.



Fig. 3 Eye Diagram of the input jittered PRBS signal



The first one is using jittered pseudo random bit sequence (PRBS). The second and the third experiments we inject noise to power supply and to input reference clock of commercial off-the-shelf PLL evaluation board, and separate out the jitter. As for the signal acquisition hardware, we use ADC evaluation board from National Semiconductor, ADC12D1800RB, together with 15GHz bandwidth track-and-hold circuit HMC5640 evaluation board from Hittite Microwave. The sampling clock is 511MHz for all the setup.

1) Digital Bit Sequence Periodic Jitter Separation

In this hardware setup, Agilent 81133A is used to generate 2.5Gbps data pattern, and 1MHz sine wave is applied to the output delay control as the periodic jitter. The eye diagram of the jittered input signal is as shown in Fig. 3. By using the method described above, the period estimation error can be calculated by the slope of the regression line shown in Fig. 4 (left). The periodic jitter is extracted using sinusoidal fitting. We then compensate the deterministic component for each sample. After compensating for the jitter, the whole-period PRBS is shown as in Fig. 5. The eye diagram at different steps is shown in Fig. 6. The real jittered eye diagram is the eye diagram before compensation. By compensating out the periodic jitter, we're able to measure the histogram of the random jitter and the peak-to-peak value of the periodic jitter separately. In addition, we can estimate the potential gain in timing margin if additional circuitry is added to filter out the deterministic periodic jitter.

2) PLL Periodic Jitter Tolerance and Characterization Analysis

We apply the proposed methodology to test off-the-shelf PLL circuitry and explore the effect of two noise sources on the

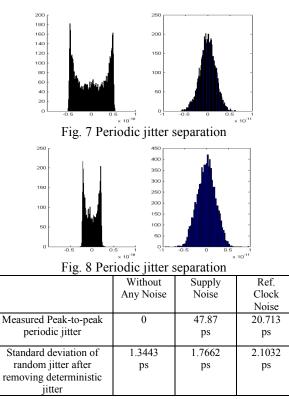


Fig. 6 Eye Diagram at different steps

Eye Diagram After Periodic Jitter Compensation

Table I

PLL output noise. We use the PLL evaluation board, LMX2531LQ1146E, from National Semiconductor, to generate reference test signal. Without injecting any noise, the edge histogram is a Gausian distribution. We then inject noise from two sources, power supply noise and reference clock noise. Using the proposed method, we separate the periodic jitter and the random jitter. The experiments are described in the following sub-sections.

a) PLL Power Supply Noise

We injected noise from two external sources into PLL circuit under test. One is the power supply noise. In this experiment, we applied 20 KHz 100mv peak-to-peak sinusoid to the power supply of PLL evaluation board. The extracted jitter is 20.441 KHz. The signal transition edge histogram before and after jitter compensation can be seen in Fig. 7.

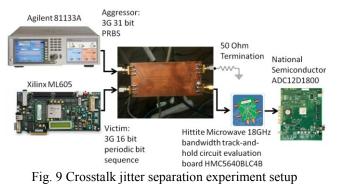
b) PLL Reference Clock Noise

In the second case external noise is injected at the reference clock port. In this experiment, we modulate the phase of the PLL evaluation board input reference clock with 0.1 rad and 50 KHz sinusoid. The measured periodic jitter is 51.103 KHz. The signal transition edge histogram before and after jitter compensation can be seen in Fig. 8. In both the experiments a 2% deviation in the estimated period of the jitter is seen due to a calibration error in the low frequency signal source used.

On the PLL evaluation board used in this experiment, the denoising circuitry is already implemented on the board. This cleans up most of the injected jitter. The separated periodic jitter and the standard deviation of the random jitter are shown as Table I. As can be seen, the amount of random jitter after injecting the noise is close to the reference signal. However, due to the non-linear effect of noise injection, the extracted random jitter is greater than the clean test signal.

B. Crosstalk Jitter Separation

To inject the crosstalk jitter, we designed a board with two



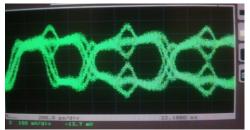
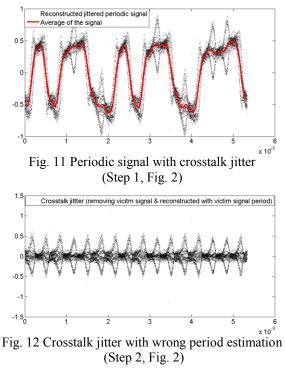


Fig. 10 Periodic signal with crosstalk jitter

traces to couple the high frequency components of signals from one link to another as shown in Fig. 9. We program the GTX pins of Xilinx ML605 FPGA evaluation board to generate a 3Gbps 16-bit periodic sequence as victim signal. We use Agilent 81133A to generate 3Gbps 31 bit PRBS signal as aggressor signal. The jittered signal can be observed from the other end of the crosstalk jitter injection board by an oscilloscope as shown in Fig. 10. We use National Semiconductor ADC12D1800 evaluation board to sample the signal at 511 Msps and use incoherent undersampling algorithm in section II to reconstruct the signal with crosstalk jitter and average the signal (step 1, Fig. 2) as shown in Fig. 11. As in step 2, Fig. 2, the signal after subtracting the average is shown in Fig. 12. The current period is still the period of the victim, not the period of the aggressor. We then apply period estimation again to estimate the period of the crosstalk jitter and get the correct waveform of the crosstalk jitter as shown in Fig. 13. The victim signal after separating the crosstalk jitter is shown in Fig. 14. Note that using the proposed method, turning off the victim or the aggressors is not required. However, in order to see how effective the separation method is, we first



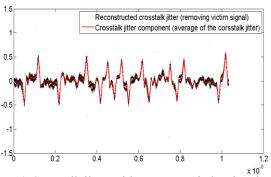


Fig. 13 Crosstalk jitter with correct period estimation (Step 3 & 4, Fig. 2)

turn off the aggressor and see the waveform of the victim signal before injecting any jitter using the oscilloscope. We also turn off the victim signal and measure the spectrum of the crosstalk jitter. Then we apply FFT to the average jitter waveform in Fig. 15 to see the jitter spectrum. Because we only have one period of the average spectrum, the resolution is limited. We fold the periodic signal in Fig. 14 by half since the oscilloscope can only be triggered by 1.5 GHz clock. The eye diagram before and after jitter separation can be seen in Fig. 16.

IV. CONCLUSION

In this paper, we propose an accurate fundamental period estimation method for incoherent undersampling. More importantly, we propose an effective IUS-based periodic and uncorrelated crosstalk jitter separation method. For periodic crosstalk jitter, the period of the aggressors and the period of the victim are different. The method relies on the ability to

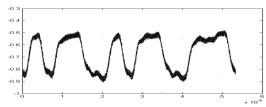


Fig. 14 Vitim signal after separate the crosstalk jitter (Step. 5, Fig. 2)

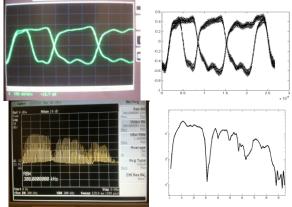


Fig. 15 Left: Periodic signal without crosstalk jitter and jitter spectrum (Turning off the aggressor)

Right: Vitim signal and crosstalk jitter spectrum by proposed method (Aggressor link is still active)

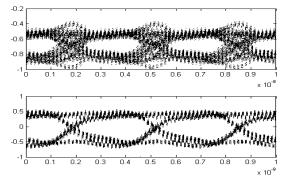


Fig. 16 Eye diagram before and after separate the crosstalk jitter

separate out periodic components from a multi tone noise signals for low signal to noise ratio.

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