Towards Improving Simulation of Analog Circuits using Model Order Reduction

Henda Aridhi, Mohamed H. Zaki and Sofiène Tahar
Dept. of Electrical and Computer Engineering, Concordia University, Montréal, Québec, Canada
Email: {h_aridh, mzaki, tahar@ece.concordia.ca}

***Abstract*—Large analog circuit models are very expensive to evaluate and verify. New techniques are needed to shorten time-to-market and to reduce the cost of producing a correct analog integrated circuit. Model order reduction is an approach used to reduce the computational complexity of the mathematical model of a dynamical system, while capturing its main features. This technique can be used to reduce an analog circuit model while retaining its realistic behavior. In this paper, we present an approach to model order reduction of nonlinear analog circuits. We model the circuit using fuzzy differential equations and use qualitative simulation and K-means clustering to discretion efficiently its state space. Moreover, we use a conformance checking approach to refine model order reduction steps and guarantee simulation acceleration and accuracy. In order to illustrate the effectiveness of our method, we applied it to a transmission line with nonlinear diodes and a large nonlinear ring oscillator circuit. Experimental results show that our reduced models are more than one order of magnitude faster and accurate when compared to existing methods.**

I. INTRODUCTION

The simulation of today's analog circuits is essential to address the problem of model selection, circuit sizing, parameter setting and to decrease the cost of producing a correct IC design. Model simplification and abstraction techniques are employed to enhance the capacities of circuit simulators at the cost of accuracy. Model Order Reduction (MOR) is a promising technique that transforms a large size dynamical model to a smaller one while preserving its main behavior. The reduced model can increase the speed of functional and statistical simulation, control and verification [1]. Most of the research on MOR of VLSI circuits has elaborated a set of methods suitable for linear problems [2]. Unfortunately, techniques that can help with the reduction of nonlinear analog circuits still require development [3] [4]. Qualitative simulation is a method used to characterize dynamical systems behavior using global optimization techniques when parameters and/or initial conditions are considered as uncertain values or fuzzy numbers [5] [6].

In this paper, we propose a new approach to MOR of nonlinear analog circuits using Krylov space projection [7]. We model the circuit using Fuzzy Differential Equations (FDE) and use qualitative simulation to characterize and determine over approximated envelopes of its state behavior. Then, we employ the K-means clustering algorithm to subdivide the circuit state space into discrete regions that contain its main responses. Moreover, we establish a set of conformance checking criteria and refine the reduced model to guarantee simulation acceleration and accuracy. We illustrate our proposed methodology on the model of a transmission line with nonlinear diodes considered in [4] [8] and a large ring oscillator. In Section II, we give an overview of MOR techniques developed for analog circuits. Then, in Section III, we briefly explain MOR through projection and Section IV details our proposed MOR scheme. In Section V, we examine our experimental results and in Section VI, we present our conclusions.

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II. RELATED WORK

In the electronic industry, a set of MOR algorithms were proposed for linear circuit models (RLC and RC networks). The most prevalent among them are based on the Krylov space projection [7] [2]. Transferring these methods to the case of nonlinear circuits is not straightforward and only few MOR methods were recently proposed.

The Proper Orthogonal Decomposition (POD) [9] was used in [10] to deal with the reduction of integrated circuits. This method is also a projection technique that finds a subspace that approximates a set of data in an optimal least squares sense. Also, a MOR based on quadratic Taylor approximations and Krylov space projections was proposed in [11] for the reduction of weakly nonlinear systems such as transmission lines.

The Trajectory PieceWise Linear (TPWL) method proposed in [3] for the reduction of nonlinear circuits and micro machined devices consist of building different linear models for a finite set of expansion points on the trajectory of the circuit determined using a training input and weighting those models on the fly to generate a single model for the system. The general purpose MOR method proposed in [4] is similar to the TPWL method but it uses piecewise polynomial representations rather than linear models. The accuracy and the simulation acceleration gain of the two previous methods depend on the training inputs, the number of expansion points and the weight functions. In fact, a large set of expansion points may slowdown the reduced model because of the extra time needed to select the points that will be used to approximate the actual state and compute their weights. In addition, the reduced model may fail to reach states that are distant from the set of expansion points since they were determined using a specific training input.

III. PRELIMINARIES

A. MOR via Projection

A large set of nonlinear circuits may be described using the set of Ordinary Differential Equations (ODE) in Equation 1, through Modified Nodal Analysis [12].

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y} &= \mathbf{C}^t\mathbf{x} \end{aligned} \tag{1}$$

where $\mathbf{f} : \mathbf{R}^n \to \mathbf{R}^n$ is a nonlinear vector valued function, $\mathbf{x} \in \mathbf{R}^n$ is a vector of states, $\mathbf{y} \in \mathbf{R}^m$ is a vector of outputs, $\mathbf{u} \in \mathbf{R}^p$ is a vector of inputs, and $\mathbf{B}$ is an $n \times p$ input selection matrix, and $\mathbf{C}$ is an $n \times m$ output selection matrix.

The reduction of the full order model in Equation 1 via projection consists of finding an $n \times q$ unitary projection matrix $\mathbf{V}$, such that $(\mathbf{V}\mathbf{V}^t = \mathbf{I}_n)$ and the vector $\hat{\mathbf{x}} = \mathbf{V}\mathbf{z}$ is a good approximate of the original state vector $\mathbf{x}$, where $\mathbf{z}$ is the reduced state vector of variables. Thus, the behavior of the full order model can be obtained through backward projection, using the matrix $\mathbf{V}$, of the behavior of the reduced model given in Equation 2.

$$\begin{aligned} \mathbf{V}\dot{\mathbf{z}} &= \mathbf{V}^t\mathbf{f}(\mathbf{V}\mathbf{z}) + \mathbf{V}^t\mathbf{B}\mathbf{u}(t) \\ \hat{\mathbf{y}} &= \mathbf{C}^t\mathbf{V}\mathbf{z} \end{aligned} \tag{2}$$

For linear models, the Arnoldi's or Lanczos algorithms [1] are advanced methods used to compute Krylov space projection basis [7]. They guarantee moments matching of the reduced model and the full order model transfer functions, at least up to the order q. If the nonlinear behavior is viewed as a sequence of linear models through Taylor Expansions, the Krylov space projection method may be applicable. However, this means that the projection matrix should be updated whenever the Jacobian matrix of $\mathbf{f}$ is updated. Unfortunately, this will increase substantially the time required to evaluate the reduced model.

B. Fuzzy Differential Equations

We suppose, without loss of generality, that the dynamical model of nonlinear circuits is modeled, as given in Equation 3.

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{P}) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y} &= \mathbf{C}^t\mathbf{x} \\ \mathbf{x}(0) &= \mathbf{x}_0 \end{aligned} \tag{3}$$

where $\mathbf{f} : \mathbf{R}^n \times \mathbf{R}^{n_p} \to \mathbf{R}^n$ is a nonlinear vector valued function, $\mathbf{P} \in \mathbf{R}^{n_p}$ a vector of n_p circuit parameters, $\mathbf{x} \in \mathbf{R}^n$ is a vector of states containing unknown node voltages and currents, $\mathbf{y} \in \mathbf{R}^m$ is a vector of outputs, $\mathbf{u} \in \mathbf{R}^p$ is a vector of inputs, $\mathbf{B}$ is an $n \times p$ input selection matrix, $\mathbf{C}$ is an $n \times m$ output selection matrix, and $\mathbf{x}(0)$ is the state initial conditions.

It is well known that analog circuits have an infinite state space because of the continuity of analog quantities, their nonlinear behavior and their dependence on the inputs, parameters P as well as the initial conditions $\mathbf{x}(0)$. In fact, a slight deviation of these quantities might affect surprisingly the possible trajectories of the circuit. For these reasons, we propose to incorporate uncertainty on the dynamical model of analog circuits, which leads to Fuzzy Differential Equations (FDE). FDEs are a formal mean to introduce uncertainty in deterministic dynamical models [5] [6]. The analog circuit model given in Equation 3 is transformed to the model of Equation 4, where the deterministic parameters P, B and initial conditions $\mathbf{x}(0)$ are replaced with the fuzzy numbers $\mu_{\mathbf{P}}$, $\mu_{\mathbf{B}}$ and $\mu_{\mathbf{x}(0)}$, respectively.

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{P}) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y} &= \mathbf{C}^t\mathbf{x} \\ \mathbf{x}(0) &= \mu_{\mathbf{x}(0)}, \mathbf{P} = \mu_{\mathbf{P}}, \mathbf{B} = \mu_{\mathbf{B}} \end{aligned} \tag{4}$$

The initial conditions $\mu_{\mathbf{x}(0)}$, the parameters $\mu_{\mathbf{P}}$ and the input selection $\mu_{\mathbf{B}}$ matrix are fuzzy numbers defined using a membership functions, as given in Figure 1. These fuzzy numbers form an initial fuzzy region in the state space that evolves in time according to the dynamics of the analog circuit, defined by the FDE in Equation 4.

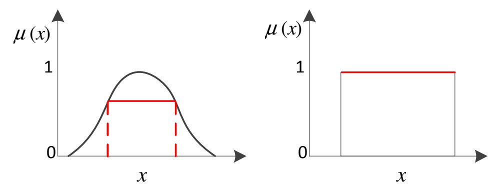

Fig. 1. Fuzzy number membership function examples

IV. PROPOSED METHODOLOGY

1) Overview: An overview of our proposed methodology to reduce the state space size of analog circuits, is shown in Figure 2.

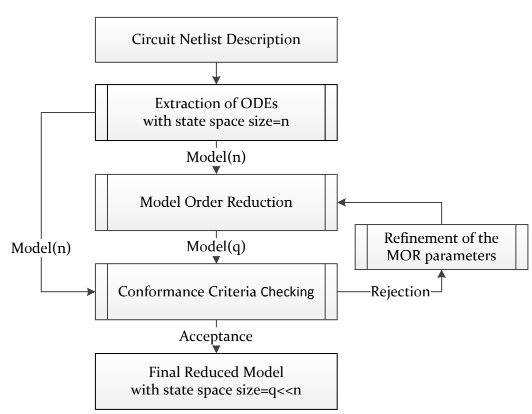

Fig. 2. A method for the MOR of large analog circuits

Given a nonlinear analog circuit description (e.g., a spice netlist), we extract a set of ODEs describing the analog circuit $Model(n)$, using Modified Nodal Analysis Formulation. After that, the full order $Model(n)$ is reduced using the technique detailed in Subsection IV-2 to obtain the reduced model $Model(q)$. This technique employs qualitative simulation [6], k-means clustering and Krylov space projections. After completion of the MOR step, the full order model $Model(n)$ and the reduced model $Model(q)$ are input to a Conformance Criteria Checking step that guarantees a minimum speedup of the simulation as well as a conformance of their transient behavior. The conformance of the transient behavior is defined as a minimum relative error between the outputs $\mathbf{y}$ and $\hat{\mathbf{y}}$ as well as the state vectors $\mathbf{x}$ and the backward projection of reduced state vector $\hat{\mathbf{x}} = \mathbf{V}\mathbf{z}$ of $Model(n)$ and $Model(q)$, respectively. If these requirements in terms of accuracy and speedup are not satisfied, a refinement of the MOR parameters step is started. During this step the failure of the model is investigated and the MOR parameters are iteratively adjusted until the requirements are satisfied which leads to an acceptance of the reduced model $Model(q)$.

2) MOR Scheme: Figure 3 details our proposed MOR methodology which consist of four steps:

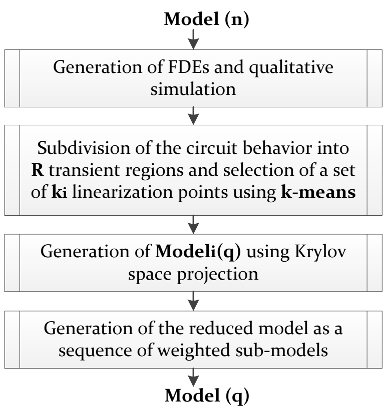

Fig. 3. Proposed MOR scheme

Step 1: Generation of FDEs and qualitative simulation: this step is necessary to explore all the analog circuit possible behaviors that cannot be determined through traditional simulation of $Model(n)$. To do so, the deterministic full order model $Model(n)$ is transformed to an FDE description, as given in Equation 4, where the initial conditions, the parameters and input sources are uncertain and modeled as fuzzy numbers. Then, the qualitative simulation is used to determine overapproximated envelopes of the transient behavior of the analog circuit described in Equation 4.

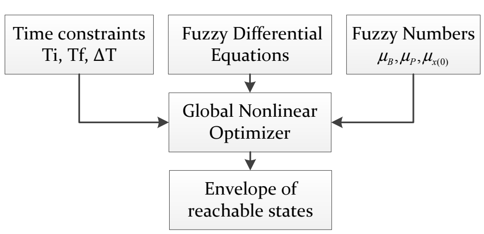

Fig. 4. Qualitative simulation

Figure 4 summarizes the qualitative simulation principle. In fact, for a set of initial fuzzy numbers that represent the system uncertainty and a fuzzy dynamical system description, the qualitative simulation outputs bounded envelopes of the system behavior for each instant of the transient simulation time. The qualitative simulation is based on a global nonlinear optimizer that determines overapproximated envelopes of the circuit state at each time instant. These overapproximated envelopes contain but are not equal to the set of all possible states of the dynamical system [6]. Thus, for the case of analog circuits a selection procedure of the useful data from of the qualitative simulation output is necessary.

Step 2: Subdivision of the circuit behavior into R transient regions and selection of a set of k_i linearization points using k-means: in order to perform Krylov space projections, a set of linearization points is needed to build the reduced model. We propose to select a proper set of points from the qualitative simulation result using k-means clustering [13], as follows:

1. We subdivide the qualitative simulation result into distinct R regions, on the time axis. The number of R regions is different for each circuit and is tuned during the MOR parameters refinement step to meet accuracy requirements. For example, an oscillator should have at least 2 regions since it has a start-up phase and a permanent oscillation phase. Other circuits might require more or less regions of operations.

2. We use k-means clustering to subdivide each of the R overapproximated reachable state space regions into k_i clusters, which result in $k = \sum_{i=1}^{R} k_i$ clusters. Then, a single state vector, that is the centroid of each region, is used as a linearization point to model locally the behavior of the circuit. The number k_i of clusters in each of the R regions is set to an initial guess at the first MOR tentative and is updated after that, during the MOR parameters refinement step, to guarantee accuracy and speedup criteria.

Step 3: Generation of $Model_i(q)$ using Krylov space projection: for each region i ($i = 1 \dots R$), we determine a set k_i of linearization points and a linear model $Model_j(n)$, as given in Equation 5, where $\mathbf{A_j}$ is the Jacobian of $\mathbf{f}$ at the linearization point $\mathbf{x_j}$ ($j = 1 \dots k_i$).

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x_j}, \mathbf{P}) + \mathbf{A_j}(\mathbf{x} - \mathbf{x_j}) + \mathbf{B}\mathbf{u}(t) \tag{5}$$

After that, we compute a Krylov type projection basis $\mathbf{V_j}$ using the Arnoldi process, and then, we determine a unified projection basis $\mathbf{V_i}$ as the Singular Value Decomposition of the union of these basis $\mathbf{V_i} = SVD(\cup_{j=1}^{k_i} \mathbf{V_j})$, where SVD is the singular value decomposition operator [1]. At the end, we have R projection basis $\mathbf{V_i}$, $i = 1 \dots R$, that will be used at different times of the transient behavior according to the actual state of the circuit.

$$\dot{\mathbf{z}} = \mathbf{V_i}^t \mathbf{f}(\mathbf{x_j}, \mathbf{P}) + \mathbf{V_i}^t \mathbf{A_j} \mathbf{V_i}(\mathbf{z} - \mathbf{z_j}) + \mathbf{V_i}^t \mathbf{B}\mathbf{u}(t) \tag{6}$$

The reduced sub models $Model_i(q)$ are given in Equation 6, where $\mathbf{z}$ is the reduced state variable, i is the region index and j is the linearization point index.

Step 4: Generation of the reduced model $Model(q)$ as a sequence of weighted sub-models: the reduced sub-models model $Model_i(q)$ are weighted to form the reduced model $Model(q)$. The weights are a mean to smooth transitions between state space regions and allow contributions of closest models. However, the weights computation should be simple, otherwise, the simulation time will increase extensively without any gain in terms of accuracy. In our MOR scheme, the circuit current state $\mathbf{z}$ is determined using the set of closest three linearization points to the current state. The weight functions at a point $\mathbf{z}$ in the reduced state space are computed as $w_s = \frac{\|\mathbf{z}-\mathbf{z_s}\|_2}{\sum_{s=1}^{3}\|\mathbf{z}-\mathbf{z_s}\|_2}$, $s = 1, 2, 3$. The final reduced model $Model(q)$ is given in Equation 7.

$$\begin{aligned} \dot{\mathbf{z}} &= \sum_{s=1}^{3} w_s(\hat{\mathbf{f}}(\mathbf{x_s}, \mathbf{P}) + \hat{\mathbf{A}}_\mathbf{s}(\mathbf{z}-\mathbf{z_s})) + \hat{\mathbf{B}}\mathbf{u}(t)) \qquad (7) \\ \hat{\mathbf{y}} &= \hat{\mathbf{C}}^t\mathbf{z} \end{aligned}$$

where i is the number of the region where the state $\mathbf{z}$ is located, $\hat{\mathbf{A}}_\mathbf{s} = \mathbf{V_i}^t\mathbf{A_s}\mathbf{V_i}$, $\hat{\mathbf{B}} = \mathbf{V_i}^t\mathbf{B}$, $\hat{\mathbf{C}}^t = \mathbf{V_i}^t\mathbf{C}$, $\hat{\mathbf{f}}(\mathbf{x_s}, \mathbf{P}) = \mathbf{V_i}^t\mathbf{f}(\mathbf{x_s}, \mathbf{P})$ and $\hat{\mathbf{y}}$ is the output of the reduced model that approximates the full order model output $\mathbf{y}$.

3) Conformance Criteria Checking: The objective of the conformance criteria checking is to verify that the reduced model $Model(q)$ mimics the behavior of the full order model $Model(n)$ in a faster way. We verify, for all transient regions, that relative error between the outputs $\mathbf{y}$ and $\hat{\mathbf{y}}$ and the relative error between the state vectors $\mathbf{x}$ and the backward projection of reduced state vector $\hat{\mathbf{x}} = \mathbf{V}^t\mathbf{z}$ are minimum and that the acceleration of the simulation is above the required minimum speedup, as given in Equation 8.

$$\begin{aligned} \frac{\|\hat{\mathbf{x}}-\mathbf{x}\|_2}{\|\mathbf{x}\|_2} &\leq \epsilon_1 \qquad (8) \\ \frac{\|\hat{\mathbf{y}}-\mathbf{y}\|_2}{\|\mathbf{y}\|_2} &\leq \epsilon_2 \\ \frac{tsim(Model(n))}{tsim(Model(q))} &\geq min\_speedup \end{aligned}$$

where ϵ_1 and ϵ_2 are the required relative errors and the level of accuracy of the application that will make use of the reduced model.

4) MOR Parameters Refinement: The MOR parameters refinement step consists in tuning several parameters of the MOR scheme which affect the accuracy and speedup of the reduced model, based on the conformance criteria checking result. The region where the model fails is determined and several steps are performed. In case the speedup criteria is not met, we increase the number of transient regions R while decreasing the number of clusters in each region. Otherwise, we decrease the order q of the reduced model. If the accuracy of the transient behavior is not satisfied, we check first if the error is localized. If it is the case, we add extra linearization points in that area otherwise we increase the number of regions.

V. APPLICATIONS

In this section, we present the results of the application of our MOR technique on the example of a transmission line with nonlinear diodes considered in [3] and a large nonlinear ring oscillator. All simulations and models descriptions were performed in the MATLAB environment and all simulation times of $Model(n)$ and $Model(q)$ are in seconds. All simulations were run on a 64-bit Windows 7 workstation with a 2.8 GHz processor and 24 GB of memory.

A. Transmission Line with Nonlinear Diodes

Figure 5 shows the transmission line model that is a chain of connected resistor, capacitor and diode cells. The input current source is $i(t)$ and all capacitors and resistor values are set to $1\ F$ and $1\ \Omega$, respectively. The behavior of the diodes is nonlinear and is given by $I_d(v) = \exp(40v) + v - 1$.

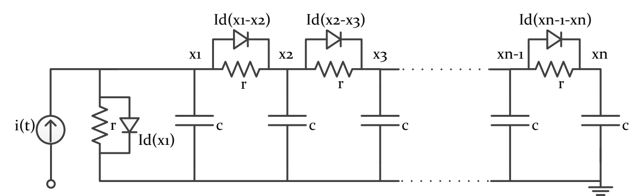

Fig. 5. Transmision line with nonlinear diode chain

The full order model of the transmission line is given in Equation 9, where $x_1, \ldots, x_n$ are the circuit node voltages.

$$\begin{aligned} \dot{x_1} &= -I_d(x_1) - I_d(x_1 - x_2) + b\ i(t) \qquad (9) \\ \dot{x_i} &= I_d(x_{i-1} - x_i) - I_d(x_i - x_{i+1}) \\ \dot{x_n} &= I_d(x_{n-1} - x_n) \\ y &= x_1 \\ \mathbf{x}(0) &= \mu_{\mathbf{x}(0)} \\ b &= \mu_b \end{aligned}$$

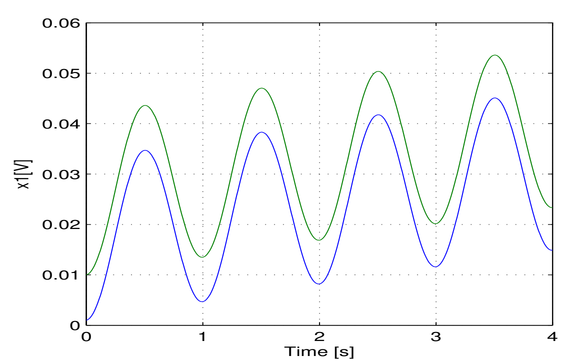

Fig. 6. Qualitative simulation of the nonlinear transmission line

Figure 6 illustrates the qualitative simulation result for the first node voltage of the transmission line model of Figure 5, when the input source is $i(t) = sin(2\pi t)$ and the initial conditions are uncertain and are represented by the fuzzy number $\mu_{\mathbf{x}(0)} = (\mu_{x_1(0)}, \ldots, \mu_{x_n(0)})$ and for $i = 1 \ldots n,\ \mu_{x_i(0)} = 1$

if $x_i(0) \in [0, 10^{-2}]V$ and $\mu_{x_i(0)} = 0$, otherwise. This means that all initial voltages are possibly in the interval $[0, 10^{-2}]V$ and any possible behavior of the first node voltage is always between these two extremum trajectories.

TABLE I
SIMULATION TIMES FOR THE NONLINEAR TRANSMISSION LINE

Input, Problem size	Speedup TPWL [3]	Speedup Current Method
$i(t) = H(t-3)$ $n = 1500, q = 30$	$\frac{9573.30}{80.80} \simeq 118$	$\frac{810.39}{0.64} \simeq 1248$
$i(t) = exp(-t)$ $n = 1500, q = 30$	$\frac{11713.10}{110.90} \simeq 105$	$\frac{1061.32}{0.82} \simeq 1284$
$i(t) = sin(\frac{2\pi t}{10})$ $n = 100, q = 10$	$\frac{25.40}{2.70} \simeq 9$	$\frac{1.84}{0.31} \simeq 6$

Table I compares the simulation times of the same reduction problems for the transmission line considered in [3]. Although these results were conducted using different processors, the speedup criteria can measure the improvement of our MOR method in terms of simulation acceleration. In fact, our reduced models are ten times faster than the TPWL and one thousand times faster than the full nonlinear model for the two first large scale problem sizes.

TABLE II
ACCURACY FOR THE TRANSMISSION LINE CIRCUIT USING $k = 20$ LINEARIZATION POINTS

Input, Problem size	$\frac{\|\hat{\mathbf{x}}-\mathbf{x}\|_2}{\|\mathbf{x}\|_2}$	$\frac{\|\hat{\mathbf{y}}-\mathbf{y}\|_2}{\|\mathbf{y}\|_2}$
$i(t) = H(t-3)$ $n = 1500, q = 30$	$0.45\ 10^{-3}$	$0.12\ 10^{-2}$
$i(t) = exp(-t)$ $n = 1500, q = 30$	$0.12\ 10^{-2}$	$0.18\ 10^{-2}$
$i(t) = sin(\frac{2\pi t}{10})$ $n = 100, q = 10$	$0.42\ 10^{-2}$	$0.37\ 10^{-2}$

Table II shows that the reduced model mimics the behavior of the full order model for the three considered problem sizes and the different current sources. The accuracy criteria is satisfied for the different experiments and the relative errors of the state variables and the output are always less than 10^{-2}, that is the maximum acceptable error during conformance checking.

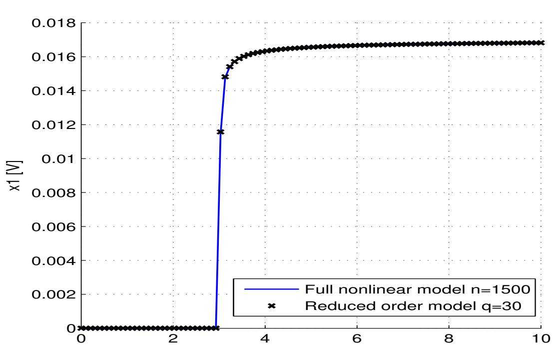

Fig. 7. Transmission line transient behavior, $i(t) = H(t-3)$

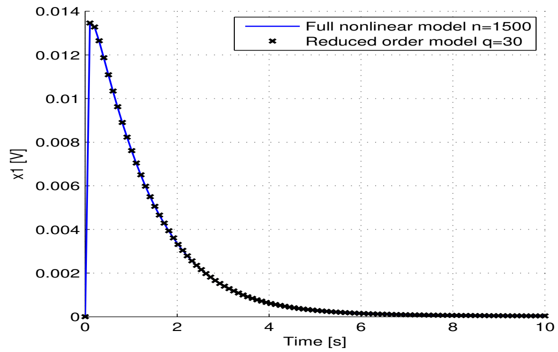

Fig. 8. Transmission line transient behavior, $i(t) = exp(-t)$

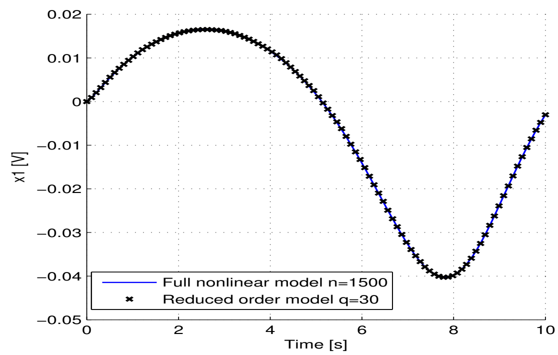

Fig. 9. Transmission line transient behavior, $i(t) = sin(\frac{2\pi t}{10})$

Figures 7, 8 and 9 show the transient behavior of the full order model and the reduced order model problems of Table II, for three current source types: a step source $i(t) = H(t-3) = 1\ if\ t\ \geq 3,\ 0\ otherwise$, an exponential source $i(t) = exp(-t)$ and a sinusoidal source $i(t) = sin(\frac{2\pi t}{10})$, respectively.

B. Ring Oscillator

Figure 10 represents a ring oscillator composed of a large odd number n of inverters connected in a circular chain. Each inverter is single ended and is composed of a cascaded n-channel and p-channel transistors and a capacitance C connected to their drains. The node voltages x_i of each of the n inverter oscillates between the ground $gnd = 0V$ and the power $vdd = 1.8V$. The circuit model is given in

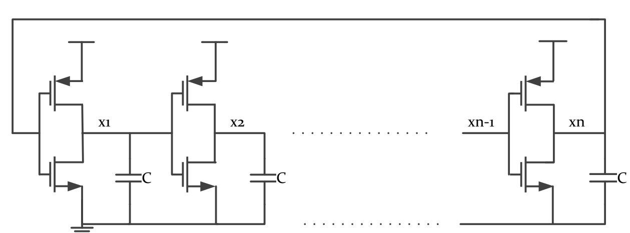

Fig. 10. Ring oscillator circuit

Equation 10, where x_i, $i = 1 \dots n$, are the node voltages, $C = 0.164fF$ and the functions I_n and I_p model the nonlinear current generated by the n-channel and p-channel transistors, respectively, based on their gate, drain and source voltages. The initial conditions $\mathbf{x}(0)$ are represented by the fuzzy number $\mu_{\mathbf{x}(0)}$.

$$\begin{aligned}
\dot{x_1} &= -\frac{1}{C}(I_n(x_n, x_1, gnd) + I_p(x_n, x_1, vdd)) \quad (10)\\
\dot{x_i} &= -\frac{1}{C}(I_n(x_{i-1}, x_i, gnd) + I_p(x_{i-1}, x_i, vdd))\\
y &= x_n\\
\mathbf{x}(0) &= \mu_{\mathbf{x}(0)}
\end{aligned}$$

When using one transient region for the ring oscillator model ($n = 131$, $q = 51$), it is hard to reproduce the oscillation behavior of the full order model with the required speedup constraint. This could be explained by the highly nonlinear initial startup transient region that needs to be accurately approximated by the reduced model. This highlights two key points of our method, namely the need for a further subdivision of the transient behavior into sub-regions, and the qualitative simulation that provides better coverage of the initial region.

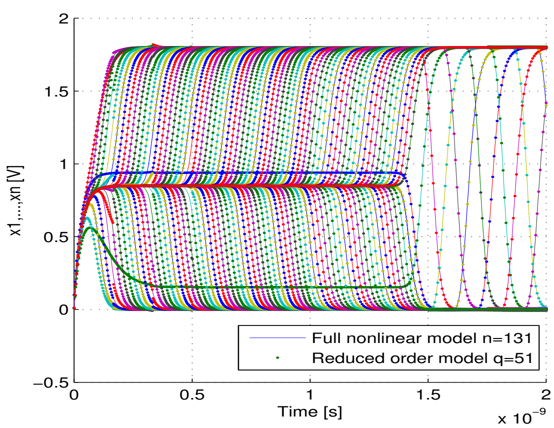

Fig. 11. Ring oscillator internal state transient responses

Our method used ten transient regions to provide a reduced model that mimics accurately the full order model behavior as shown in Figure 11, where the state vector of the original full order model $\mathbf{x}$ is represented by the solid line and the backward projection of the reduced order model $\hat{\mathbf{x}} = \mathbf{V}\mathbf{z}$ is represented by dotted line.

Table III presents the refinement of the MOR for the ring oscillator model. The requirements are a minimum speedup of 100, $\frac{\|\hat{\mathbf{x}}-\mathbf{x}\|_2}{\|\mathbf{x}\|_2} \leq 10^{-2}$ and $\frac{\|\hat{\mathbf{y}}-\mathbf{y}\|_2}{\|\mathbf{y}\|_2} \leq 10^{-2}$. The above requirements were satisfied after refinement of the number of clusters in each sub-region.

TABLE III
REFINEMENT OF $Model(q)$ FOR THE RING OSCILLATOR MODEL

Number of clusters k	Speedup	$\frac{\|\hat{\mathbf{x}}-\mathbf{x}\|_2}{\|\mathbf{x}\|_2}$	$\frac{\|\hat{\mathbf{y}}-\mathbf{y}\|_2}{\|\mathbf{y}\|_2}$	Status
10	$\frac{142.33}{0.68} \simeq 207$	$0.99\ 10^{-2}$	$2.33\ 10^{-2}$	rejected
14	$\frac{142.33}{0.71} \simeq 199$	$0.55\ 10^{-2}$	$1.30\ 10^{-2}$	rejected
18	$\frac{142.33}{0.96} \simeq 147$	$0.43\ 10^{-2}$	$1.76\ 10^{-2}$	rejected
19	$\frac{142.33}{1.02} \simeq 139$	$0.40\ 10^{-2}$	$0.63\ 10^{-2}$	accepted

VI. Conclusion

In this paper, we have presented a model order reduction methodology based on Krylov space projections. Our approach is based on modeling nonlinear behavior and uncertainty of initial conditions and parameters of analog circuits using fuzzy differential equations, qualitative simulation and k-means clustering of their transient behavior. The main advantage of the methodology is that it yields a minimum number of linearization points for each transient region, thereby providing better simulation acceleration and making the reduced model more accurate and robust for a fuzzy number of inputs, parameters and initial conditions. Also, the accuracy of the methodology is enhanced because of the MOR parameters refinement step. However, a systematic method for picking the initial guess of the number of clusters k and the criteria for conformance checking can both help improve the proposed MOR scheme each of which we are further developing at this time. The proposed methodology is limited to the transient time behavior and extensions to the DC characteristics and the frequency behavior are possible. Moreover, we plan to show the effectiveness of our method on larger nonlinear models such as RF communication circuits or high speed digital communication interfaces which exhibit a highly nonlinear analog behavior at speeds of up to Gbps [14].

References

[1] A. C. Antoulas, *Approximation of large-scale dynamical systems*. Society for Industrial and Applied Mathematic, 2005.

[2] S. Tab and L. He, *Advanced model order reduction techniques in VLSI design*. Cambridge University Press, 2007.

[3] M. Rewienski and J. White, “A trajectory piecewise-linear approach to model order reduction and fast simulation of nonlinear circuits and micromachined devices,” *IEEE Transactions on Computer Aided Design of Integrated Circuits and Systems*, vol. 22, no. 2, pp. 155–170, 2003.

[4] N. Dong and J. Roychowdhury, “General-purpose nonlinear model-order reduction using piecewise-polynomial representations,” *IEEE Transactions on Computer Aided Design of Integrated Circuits and Systems*, vol. 27, no. 2, pp. 649–654, 2008.

[5] L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.

[6] M. Nikravesh, L. A. Zadeh, and V. Korotkikh, “Fuzzy partial differential equations and relational equations,” *Reservoir Characterization and Modeling Series: Studies in Fuzziness and Soft Computing*, 2004.

[7] R. W. Freund, “Krylov-subspace methods for reduced-order modeling in circuit simulation,” *Computational and Applied Mathematics, Special Issue on Numerical Analysis*, vol. 123, pp. 395–421, 2000.

[8] L. Feng, “Review of model order reduction methods for numerical simulation of nonlinear circuits,” *Applied Mathematics and Computation*, vol. 167, no. 1, pp. 576–591, 2005.

[9] L. Sirovich, “Part i: Coherent structures part ii: Symmetries and transformations part iii: Dynamics and scaling,” *Quarterly of Applied Mathematics*, vol. 45, no. 3, pp. 561–590, 1987.

[10] A. Verhoeven, M. Striebel, and E. J. W. ter Maten, *Model order reduction for nonlinear IC models with POD*. Springer, 2010.

[11] Y. Chen, “Model order reduction for nonlinear systems,” M.S. Thesis, Massachusetts Institute of Technology, United States, 1999.

[12] W. K. Chen, *Analog and VLSI circuits*. CRC Press, 2009.

[13] G. Gan, C. Ma, and J. Wu, *Data clustering: theory, algorithms, and applications*. Society for Industrial and Applied Mathematic, 2007.

[14] “Rapidio specifications,” November 2011. [Online]. Available: http://www.rapidio.org/specs/current/