Statistical Static Timing Analysis using a Skew-Normal Canonical Delay Model

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***Abstract*—In its simplest form, a parameterized block based statistical static timing analysis (SSTA) is performed by assuming that both gate delays and the arrival times at various nodes are Gaussian random variables. These assumptions are not true in many cases. Quadratic models are used for more accurate analysis, but at the cost of increased computational complexity. In this paper, we propose a model based on skew-normal random variables. It can take into account the skewness in the gate delay distribution as well as the nonlinearity of the MAX operation. We derive analytical expressions for the moments of the MAX operator based on the conditional expectations. The computational complexity of using this model is marginally higher than the linear model based on Clark's approximations. The results obtained using this model match well with Monte-Carlo simulations.**

I. Introduction

Integrated circuits today have significant process parameter variations that affect the timing and hence operating frequency of the chip. Traditionally static timing analysis (STA) was performed over multiple process corners. However, this corner based approach assumes circuit to be operating at worst case values of process parameters, which have very low probability of occurrence in real conditions. This makes the corner based STA overly pessimistic. Moreover, as the number of sources of variation increases, the number of timing runs required for validation increases exponentially. To overcome the problems with STA, a statistical static timing analysis (SSTA) is used. Either a path based or block based SSTA can be performed. A path based SSTA finds the maximum of all the path delays in the circuit, which becomes computationally inefficient as the circuit size grows. A block based SSTA involves a PERT-like traversal of the circuit graph and is computationally more efficient [1], [2].

In its simplest form, a parameterized block based analysis is performed by assuming that gate delays and the arrival times at various nodes are Gaussian random variables. This essentially means that all process parameter variations can be represented as Gaussian random variables and the gate delay and the arrival times are linear functions of these variations. To handle globally correlated sources of variation, spatial correlations are usually modelled using quad-trees as in [1], [3]. In addition to these global sources of variation, spatially uncorrelated variations are taken into account by adding a single independent Gaussian random variable to the canonical delay model. This often results in a significant error in the arrival time due to reconvergent paths in the circuit [4]. The extended canonical form proposed in [4] attempts to correct this error, but at the cost of some computational overhead. The gate delay variation is more accurately modelled as a quadratic function of the process parameter variations. The delay distribution therefore has a significant skewness that cannot be ignored. In addition, the MAX operation that is used to propagate the arrival times is inherently non-linear. The quadratic models attempt to take both these non-linearities into account by matching the skewness in addition to the mean, standard deviation and correlations. While the overall errors do reduce, it is at the cost of significant additional computational complexity [5], [6], [7]. The models in [6], [7], [8] also handle non-Gaussian sources of variation. The other method used to reduce the error in the standard deviation of the circuit delay (and hence the yield) is to assume that the arrival times have a joint skew normal distribution [9]. Using a procedure similar to the one used by Clark, they find the tightness probability and the moments of this distribution. However, the method used to compute the moments is complicated and the details of how it was used to match moments when the standard canonical form is used is not clear.

In this paper, we propose a skew-normal canonical form that can be used to represent both the delay and the arrival times. This canonical form is a merger of the standard canonical model for the delay and the standard skew-normal representation proposed in [10]. We assume that the process parameter variations are Gaussian. However, SPICE simulations indicate that the resulting gate delay distributions exhibit a skewness. We show that these delay distributions can be fitted well to a skew-normal distribution. We derive the moments of the MAX of skew-normal random variables by integrating the conditional distribution. This turns out to be a much simpler method than the method used in [9]. The moments and the tightness probability are written in terms of the CDF of the standard normal distribution and the Owen's T-function [11]. Standard computer routines are available for both these routines. Computation of these moments has marginally higher complexity than use of Clark's formulas. With respect to a quadratic model it has the advantage of a significantly lower computational complexity, while matching the first three moments.

The paper is organized as follows. Section II contains a review of the models used for the arrival times and the gate delays. Section III introduces skew normal representations and

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contains some properties that are used for SSTA. In section IV, we describe the skew-canonical model and derive the tightness probability and the moments of the MAX of two skew normal variables. We also describe the moment matching procedure to obtain the coefficients of the canonical form. Section V contains the results and section VI, the conclusions.

II. Canonical models - Review

There are several models for the delay and the arrival time that have been used in the literature. In this section we briefly review these models.

A. Linear models

The two operations that are required for SSTA are the SUM and the MAX operations. If the sources of variation are Gaussian random variables and the model is a linear model, the SUM operation is a straightforward operation. The MAX operation is nonlinear, but it is linearized using Clark's formulas to match the first and second order moments [12].

1) Canonical model (CM): In this model, the arrival time at node i (A_i) is represented in terms of the principal components of the process parameters (ξ_j) and an independent standard normal random variable as follows [2]

$$A_i = a_{i0} + \sum_{j=1}^{n} a_{ij}\xi_j + p_i\eta_i \tag{1}$$

a_{ij} represents the sensitivity to the j^{th} principal component at node i. Clark's formulas are used to determine the sensitivities and the mean. p_i is adjusted so that the standard deviation of the arrival time matches the standard deviation computed using Clark's formula. It is therefore an artificial term that is added to match moments. However, η_i can also represent an actual independent source of variation for each gate. In this case, p_i is also determined using the tightness probability. The coefficients a_{ij} and p_i are then scaled to match standard deviation obtained using Clark's formula [13].

2) Extended Canonical model (ECM): In the canonical model, each arrival time has only one independent source of variation. This gives rise to large errors when the circuit has many reconvergent paths [4]. To circumvent this, in the extended canonical model, each independent component is carried forward. Therefore, arrival time is now represented as

$$A_i = a_{i0} + \sum_{j=1}^{n} a_{ij}\xi_j + \sum_{j=1}^{N} p_{ij}\eta_j \tag{2}$$

Therefore, for an N gate circuit, there are N independent sources of variation. Each of the sensitivities is determined using the tightness probability [4], [13]. The sensitivities are then scaled to match the standard deviation using Clark's formula. While the extended canonical model gives more accurate results, it has a significant computational overhead.

B. Quadratic models

The two main problems with the linear models is that they do not take into account the non-Gaussian nature of gate delay distributions and the inherent non-linearity of the MAX operator. As a result, the arrival time exhibits a significant skewness. One way to account for the skewness is to use quadratic models for the arrival time. This has the further advantage that each gate delay can also be modelled similarly. The arrival time at node i is now written as [5], [6], [7], [14]

$$A_i = a_{i0} + \mathbf{A}\xi + \xi^T\mathbf{B}\xi + p_i\eta_i \tag{3}$$

The SUM operation continues to be straightforward since each individual component can be added. The MAX operation is more involved since we now need to compute the moments of the MAX of two quadratic functions. The actual moment matching involves numerical integrations and convolutions (using FFT) [5] or approximation using a Fourier series along with a table lookup [6] or fitting of a quadratic model along with moment matching [7]. There is significant increase in computational complexity and in most cases, the cross terms are ignored to speedup computations. However, the quadratic delay models in [6], [7] are very general models that can handle non-Gaussian process parameter variations.

C. Other models

The other ways to take skewness into account are localised Monte-Carlo sampling [4] and using the moments of a skew-normal distribution to propagate skewness [9].

III. Skew normal random variables

A standard skew normal random variable has a PDF given by [10]

$$f(z;\lambda) = 2\phi(z)\Phi(\lambda z) \tag{4}$$

where $\phi(z)$ and $\Phi(z)$ are the PDF and the CDF of the standard normal random variable. The parameter λ determines the skewness of the distribution. The moments can be adjusted using a location and a scale parameter. The CDF is given by

$$F(z;\lambda) = \Phi(z) - 2T(z;\lambda), \qquad \lambda > 0 \tag{5}$$

where $T(z;\lambda)$ is the Owen's T-function. Standard computer routines are available to compute this function. The properties of the Owen's T-function can be used to get the CDF for $\lambda < 0$ (negative skewness).

There are several representations of the skew normal random variable that have this PDF [10], [15]. The representation that we are interested in is

$$Z = \alpha + \beta X \tag{6}$$

where

$$X = \frac{\lambda}{\sqrt{1+\lambda^2}}|U| + \frac{1}{\sqrt{1+\lambda^2}}V \tag{7}$$

where U and V are independent standard normal random variables. The moments as well as several interesting properties of the skew normal random variable can be found in [10], [15]. The properties that are of interest to us are

1. The sum of a skew-normal and a normal random variable is also a skew normal random variable.
2. If $X_1 = a_1V_1 + b_1|U|$ and $X_2 = a_2V_2 + b_2|U|$, then $X_3 = X_1 + X_2 = a_3V_3 + (b_1+b_2)|U|$, where $a_3 = \sqrt{a_1^2 + a_2^2}$. V_1, V_2, V_3 and U are independent standard normal random variables. X_3 is also a skew-normal random variable.

3. The conditional distribution $f(x|u)$ is a normal distribution.
4. $X_4 = X_1 - X_2$ is therefore also a skew-normal random variable and $P(X_1 > X_2)$ can be found from the CDF of the skew-normal distribution [15], [16].
5. The maximum skewness of the skew-normal random variable is 0.995272.

These properties also hold true when the skew-normal random variables have an arbitrary location and scale factor.

IV. Skew Canonical delay model

We propose a modification of the canonical form to a skew-canonical form as follows:

$$A_i = a_{i0} + \sum_{j=1}^{n} a_{ij}\xi_j + p_i\eta_i + q_i|z| \tag{8}$$

Therefore, the standard canonical form is augmented by the addition of a globally correlated random variable $|z|$, where z is standard normal random variable, independent of ξ_j and η_i. A_i is therefore a skew normal random variable. This form is much simpler than the quadratic or semi-quadratic forms that are used and matches the same number of moments, namely the mean, standard deviation and the skewness. As will be seen later, it is possible to represent gate delay distributions in terms of skew-normal random variables. In fact, given any quadratic delay model, it is possible to get a skew normal delay model by matching the appropriate moments. It can also be viewed as a particular case of the generalized canonical model described in [8].

The two operations that are required for SSTA are the SUM and MAX operation. If A and B are two delay variables represented in the skew-canonical form and $S = A + B$, the coefficients of S can be computed as,

$$\begin{aligned}
S &= s_0 + \sum_{i=1}^{n} s_i\xi_i + p_s\eta_s + q_s|z| \\
s_0 &= a_0 + b_0 \\
s_i &= a_i + b_i, \quad \text{for} \quad i = 1, 2, \cdots n \\
p_s &= \sqrt{p_a^2 + p_b^2} \\
q_s &= q_a + q_b
\end{aligned}$$

We wish to represent the result of the MAX operation in the skew-canonical form. To achieve this, we either need to find the moments of the joint PDF of two skew normal random variables and follow the method in [9], [12] or integrate the conditional distribution as is done in [8]. We use the latter method since the conditional distribution, given z, is a normal distribution, which makes it possible to use Clark's formulas to get the conditional moments. Since $z \sim N(0, 1)$, the actual moments can be found by integrating the conditional moments along with the PDF of the standard normal. Analytical forms of all moments can be obtained in terms of the Owen's T function and the CDF of the standard normal. We illustrate this for the first moment. If $C = MAX(A, B)$, the conditional expectation given the value of z is obtained from Clark's formula as

$$\begin{aligned}
E\{C|Z\} = \mu_{C|Z} &= \mu_{A|Z}T_{A_{cond}} + \mu_{B|Z}(1 - T_{A_{cond}}) \\
&\quad + \sigma_{(A-B)|Z}\phi(\theta)
\end{aligned} \tag{9}$$

where $\theta = \frac{\mu_{(A-B)|Z}}{\sigma_{(A-B)|Z}}$ and

$$\mu_{(A-B)|Z} = (a_0 - b_0) + (q_a - q_b)|z| \tag{10}$$

$$\sigma_{(A-B)|Z} = \sqrt{\sum_{i=1}^{n}(a_i - b_i)^2 + p_a^2 + p_b^2} \tag{11}$$

$$T_{A_{cond}} = \Phi\left(\frac{\mu_{(A-B)|Z}}{\sigma_{(A-B)|Z}}\right) \tag{12}$$

To evaluate the first moment, we need to evaluate integrals of the form

$$\begin{aligned}
&\int_0^\infty \phi(z)\ \Phi(a + bz)\ \mathrm{d}z, \qquad \int_0^\infty z\ \phi(z)\ \Phi(a + bz)\ \mathrm{d}z \\
&\int_0^\infty \phi(z)\ \phi(a + bz)\ \mathrm{d}z
\end{aligned} \tag{13}$$

The first integral can be evaluated using the analytical expression for the unconditional tightness probability given by property (4) of the skew-normal random variables. Therefore,

$$\begin{aligned}
T_A = \int_{-\infty}^{\infty} T_{A_{cond}}\ \phi(z)\ \mathrm{d}z &= 2\int_0^\infty \Phi(a + bz)\ \phi(z)\ \mathrm{d}z \\
&= \Phi(\tau) + 2\ T(\tau, b)
\end{aligned} \tag{14}$$

where,

$$a = \frac{a_0 - b_0}{\sigma_{(A-B)|Z}}, \qquad b = \frac{q_a - q_b}{\sigma_{(A-B)|Z}},$$
$$\tau = \frac{a}{\sqrt{1 + b^2}}$$

and $T(h, a)$ is the Owen's T function [11], given by

$$T(h, a) = \frac{1}{2\pi}\int_0^a \frac{e^{\frac{-h^2}{2}(1+x^2)}}{1 + x^2}\ \mathrm{d}x$$

The third integral can be evaluated in terms of $\Phi(\cdot)$. Since $\frac{d\phi}{dz} = -z\phi(z)$, the second integral can be obtained by integrating by parts. Using this, the first moment can be written as

$$\begin{aligned}
M_1 &= (a_0 - b_0)\Phi(\tau) + 2(a_0 - b_0)\ T(\tau, b) \\
&\quad + \sqrt{\frac{2}{\pi}}(q_a - q_b)\Phi(a) + \frac{2b}{t}(q_a - q_b)\phi(\tau)\Phi(-b\tau) \\
&\quad + \frac{2\theta}{t}\phi(\tau)\Phi(-b\tau) + b_0 + 2q_b
\end{aligned}$$

where $t = \sqrt{1 + b^2}$. It can be easily verified that it reduces to Clark's formula when $q_a = q_b = 0$.

All the higher order moments can be obtained similarly and using the fact that $z^2\phi(z)$ and $z^3\phi(z)$ can be written in terms of $\phi(z)$ and its derivatives.

If the mean, standard deviation and the skewness of C are denoted by μ_c, σ_c and γ_c, the coefficients of the canonical

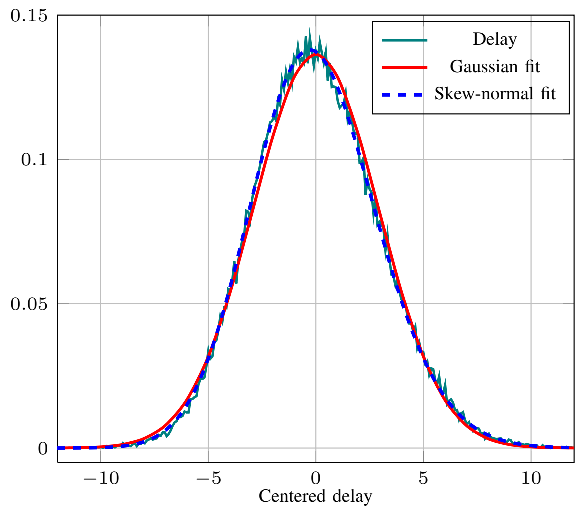

Fig. 1. Centered delay distribution of a NAND gate - Monte-Carlo simulations, its Gaussian and a skew-normal fit.

form can be found as follows

$$q_c = \sqrt[3]{\frac{\gamma_c \sigma_c^3}{\sqrt{\frac{2}{\pi}}\left(\frac{4}{\pi}-1\right)}} \tag{15}$$

$$c_0 = \mu_c - q_c\sqrt{\frac{2}{\pi}} \tag{16}$$

$$c_i = a_i\ T_A + b_i\ (1 - T_A) \tag{17}$$

$$p_c = \sqrt{(p_a T_A)^2 + (p_b(1-T_A))^2} \tag{18}$$

The coefficients c_i and p_c are scaled to match the standard deviation. The scaling factor s is found as follows

$$\sigma'_c = \sqrt{\sigma_c^2 - q_c^2\left(1 - \frac{2}{\pi}\right)} \tag{19}$$

$$S_0 = \sqrt{\sum_{i=1}^{n} c_i^2 + p_c^2} \tag{20}$$

$$s = \frac{\sigma'_c}{S_0} \tag{21}$$

V. Results

Assuming a 10% variation in V_T and W and a 5% variation in L, we performed Monte-Carlo simulations using SPICE and obtained the gate delay distribution for some of the gates in the standard cell library for both 90 and 180nm technology. The skewness in the gate delays of these gates were between 0.1 and 0.4. Based on the first three moments, we obtained a skew-canonical model for these gate delays. Fig.1 contains a comparison of the delay distribution of a NAND gate in 180nm technology, indicating it is a good fit. Also shown in the figure is a Gaussian fit to the distribution. Although there seems to be only a small difference, it results in a significant skewness in the final circuit delay as will be seen later.

In our first experiment, the gate delays were modelled using skew-canonical form. ISCAS85 benchmarks were synthesized using the standard cells for 180nm technology. In our experiment, L, W and V_{TH} variations were considered as normal random variables with σ/μ of 5%, 10% and 10% respectively. The skewness in the gate delay was randomly assigned a value between 0.2 and 0.3. For each of the parameters, a four layer quad-tree with equal weights in each layer was used to model correlations. 5% of the variation was assumed to be due to an independent variation. Table I shows the results of this experiment. The values obtained using SSTA are compared with Monte-Carlo simulations performed using 10^6 samples. The errors in μ and σ are similar to that obtained using the quadratic delay model [7], but the error in γ is significantly lower. Fig. 2 shows the plot of circuit delay PDF for c6288 benchmark obtained from this SSTA algorithm and from Monte-Carlo simulation. The delay PDF is seen to have a significant skew and matches well with Monte-Carlo plot. Although we have not used the extended canonical form, the match with Monte-Carlo simulations is very good. This is partly because the weight for the independent random variable is smaller than the weights for the correlated intra-die variations. A similar good fit is observed in [7] for slightly smaller total variation.

As discussed in the previous section, the sensitivities c_{ij} and p_c are scaled to match the standard deviation. Table II shows the statistics of the scale factor. The mean is seen to be very close to one and the probability that the scale factor is less than 0.98 is very small. This indicates that the error in the correlation with the principal component of the process parameter is also very small.

TABLE I. Percentage error in μ, σ, γ and 95% yield point of circuit delay using the skew canonical model

Circuit	$\%\mu_{err}$	$\%\sigma_{err}$	$\%\gamma_{err}$	$95\%YP_{err}$
c1355	-0.059	-0.403	-0.009	-0.282
c17	-0.165	0.383	-2.909	-0.258
c1908	0.050	0.310	-0.153	-0.027
c2670	-0.345	0.140	-1.832	-0.398
c3540	-0.136	0.665	-2.934	-0.144
c432	-0.756	0.167	-0.270	-0.646
c499	-0.049	-0.938	-0.639	-0.285
c5315	-0.545	0.454	-4.210	-0.559
c6288	-0.373	-0.176	-3.460	-0.511
c7552	-0.451	0.142	0.303	-0.427
c880	-0.234	0.051	-0.918	-0.317
Avg. err	0.361	0.430	2.165	0.390

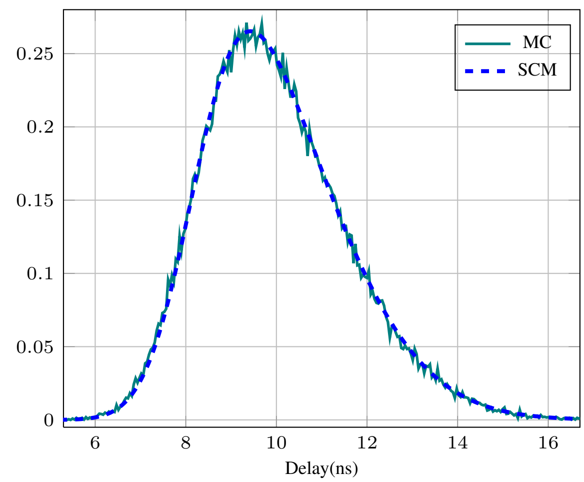

Fig. 2. Delay PDF of c6288 circuit. MC: Monte-Carlo simulations; SCM: Skew-Normal SSTA

TABLE II. STATISTICS OF THE SCALING FACTOR $s = \frac{\sigma_c'}{S_0}$

Circuit	Ratio of σ_c' to S_0				Probability of the ratio in each range				
	μ	σ	minimum	maximum	<0.98	[0.98,0.99]	(0.99,1.0)	[1.0,1.01]	>1.01
c1355	0.9988	0.0093	0.9394	1.0303	0.0230	0.0967	0.1566	0.6451	0.0783
c17	0.9992	0.0123	0.9832	1.0249	0.0	0.2857	0.0	0.5714	0.1428
c1908	1.0001	0.0063	0.9802	1.0427	0.0	0.0416	0.1958	0.7166	0.0458
c2670	0.9991	0.0065	0.9644	1.0469	0.0191	0.0383	0.1823	0.7389	0.0211
c3540	0.9990	0.0067	0.9542	1.0582	0.0198	0.0338	0.2686	0.6507	0.0268
c432	1.0001	0.0031	0.9867	1.0306	0.0	0.0068	0.1586	0.8275	0.0068
c499	0.9996	0.0082	0.9774	1.0366	0.0092	0.1059	0.1152	0.6958	0.0737
c5315	0.9988	0.0072	0.9403	1.0331	0.0215	0.0467	0.2048	0.6999	0.0269
c6288	0.9992	0.0046	0.9506	1.0152	0.0129	0.0323	0.3166	0.6371	0.0009
c7552	0.9994	0.0042	0.9641	1.0198	0.0102	0.0232	0.1931	0.7660	0.0074
c880	0.9974	0.0072	0.9346	1.0168	0.0348	0.0696	0.1811	0.7108	0.0034

TABLE III. PERCENTAGE ERROR IN μ AND σ USING THE CANONICAL AND SKEW-CANONICAL MODEL(SCM) AND SKEWNESS OF CIRCUIT DELAY.

Circuit	$\%\mu_{err}$		$\%\sigma_{err}$		Skewness
	CM	SCM	CM	SCM	
c1355	-0.893	-0.412	5.962	1.918	0.16
c499	-0.848	-0.377	4.643	1.051	0.14
c17	-0.400	-0.344	4.244	4.040	0.11
c1908	-0.954	-0.698	6.266	5.902	0.10
c3540	-0.522	-0.411	6.253	6.666	0.08
c5315	-0.984	-0.783	4.209	4.383	0.16
c2670	-0.848	-0.737	6.612	7.269	0.04
c432	-1.549	-1.430	9.981	11.376	0.05
c6288	-0.572	-0.506	3.116	5.013	0.05
c7552	-0.658	-0.645	5.297	5.564	0.01
c880	-0.440	-0.412	2.471	3.138	0.01
Avg. err	0.847	0.682	5.708	5.782	-

In the second experiment, we modelled gate delays as normal random variables. The skew-canonical form was used to find the MAX of the arrival times. The variation in the length and the threshold voltage is 6.67% of the mean. The threshold voltage variation was modelled entirely by an independent random variable and half the length variance was independent as in [9]. The other half was correlated and a four layer quad-tree was used to model the correlation. The canonical model typically results in large errors in the standard deviation when most of the correlation arises due the circuit structure (due to reconvergent paths) rather than the quad-tree. The actual numbers depended heavily on the correlations in the process parameters, but some trends could be observed.

Table III shows the percentage error obtained when the canonical and skew-canonical model is used. The skew-canonical form can compensate partly for the error in the standard deviation occurring due to reconvergent paths, if the skewness is significant. From the table, it can be seen that this is not the case for most circuits. Only c499 and c1355 benefit from using the skew-canonical form. In the most cases, the skewness is too small to make a significant difference and the errors in the two cases are within 1-2% of each other.

Note that these errors occur due to reconvergent paths and reduce considerably if the extended canonical form is used. In fact, the skew canonical model can very easily be modified to get an "extended skew-canonical model".

Table IV shows the CPU times required for MAX operations using the skew-canonical delay model and the linear canonical delay model. It shows that use of the skew-canonical model results in a marginal overhead in CPU times. Table V has the comparison of the CPU times for computing the circuit delay of the ISCAS85 benchmarks. The average overhead in using the skew-canonical delay model is only about 7%, indicating its computational efficiency.

TABLE IV. RUN TIME COMPARISON OF CLARK'S MAX(CM) AND SKEW-NORMAL MAX(SNM), WHERE N BEING THE NUMBER OF RANDOM VARIABLES.

N	$t_{CM}(ms)$	$t_{SNM}(ms)$	$\frac{t_{SNM}-t_{CM}}{t_{CM}} \times 100\%$
1000	9.149	9.542	4.29
10000	90.65	93.47	3.11
100000	907.12	929.97	2.51

TABLE V. RUN TIME COMPARISON OF SSTA USING CM AND SCM

Circuit	$t_{CM}(ms)$	$t_{SCM}(ms)$	$\frac{t_{SCM}-t_{CM}}{t_{CM}} \times 100\%$
c1355	2.045	2.132	4.25
c17	0.027	0.029	7.40
c1908	2.590	2.678	3.39
c2670	3.310	3.452	4.29
c3540	7.080	7.585	7.13
c432	1.364	1.461	7.11
c499	1.996	2.157	8.06
c5315	7.485	8.074	7.86
c6288	18.375	19.285	4.95
c7552	8.662	9.316	7.55
c880	1.999	2.191	9.60
Avg % increase in run-time			6.77

VI. CONCLUSION

In this paper, we have proposed a skew-canonical form for the gate delay and arrival times to take into account the skewness in the gate delay variation and the inherent non-linearity of the MAX operator. The advantage of using the skew-canonical form is that analytical expressions for the moments of the MAX operation can be obtained in terms of the CDF of the standard normal and the Owen's T-function. Standard computer routines are available for both these functions. The computational complexity of evaluating these moments is marginally more than using Clark's formulas. It is much simpler to use than the quadratic model and it matches the first three moments. Practical examples show that there is only a small error in the correlations with the independent component of the process parameters.

However, it has the same limitations and advantages as the canonical form. If the variations have a strong independent component, an extended (skew) canonical form may be essential to account for reconvergent paths in the circuit. Some more work is also required to see the effect of non-Gaussian process parameter variations.

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