Sensitivity-based weighting for passivity enforcement of linear macromodels in power integrity applications

A. Ubolli, S. Grivet-Talocia
Politecnico di Torino
C. Duca degli Abruzzi 24, 10129 Torino, Italy
e-mail {andrea.ubolli,grivet}@polito.it

M. Bandinu, A. Chinea
IdemWorks s.r.l.
C. Trento 13, 10129 Torino, Italy
e-mail {m.bandinu,a.chinea}@idemworks.com

***Abstract*—The electrical performance of Power Distribution Networks (PDNs) is usually assessed by computing frequency responses through quasi-static or full-wave electromagnetic solvers. Such responses, often available in the scattering form, are then fed to suitable macromodeling algorithms for the extraction of compact reduced-order behavioral models that can be seemlessly simulated in the time domain by standard circuit solvers. Such algorithms perform a rational fitting of the raw scattering responses, followed by a passivity check and enforcement step. The resulting macromodel is typically very accurate when compared to the raw scattering responses. It may however happen that the responses of the PDN macromodel exhibit significant deviation from the true system responses under realistic loading conditions, which include appropriate models for active device blocks, decoupling capacitors, voltage regulators, etc. We highlight the source of this accuracy loss, and we propose a sensitivity-based weighting strategy that is able to optimize and tune the macromodel accuracy based on its specific nominal termination network. The particular focus of this paper is the definition and the inclusion of optimal weigths in the passivity enforcement loop, which is recognized as the most challenging step. The result is a reliable macromodeling flow, which is able to produce passive, accurate and efficient reduced-order models of general PDN structures for power integrity analysis and verification.**

I. Introduction

Electrical verification for power and signal integrity is one of the most challenging tasks in the design flow of electronic systems. In fact, due to aggressive miniaturization and coexistence of tightly coupled signal and power distribution networks in close proximity, local and global electromagnetic interactions, crosstalk, and couplings cannot be neglected and must be carefully assessed. These parasitic effects, as well as frequency-dependent metal (including skin effect) and dielectric losses, must be correctly represented in the simulation models that designers run to qualify and approve the entire system [1]–[3].

All these phenomena are well represented in the frequency domain by means of frequency-domain full-wave solvers. Despite the huge amount of possibly multiscale geometrical details and complicated material properties, state of the art solvers are able to provide within reasonable time a frequency sweep of the response of a complex signal and/or Power Distribution Network (PDN). This data is usually available as tabulated scattering parameters, whose ports are defined at chip/package/board level, depending on the application. For power integrity verifications, the ports at the die side are meant to be terminated by the suitable models for the active device blocks, whereas the ports at the package or board level are usually terminated by models of decoupling capacitors and Voltage Regulator Modules (VRM) [4], [5]. After such system is assembled, extensive transient simulations are run, in order to estimate the voltage drop that occurs at the device locations excited by device switching. This voltage drop is usually expressed as an impedance $Z_{\rm PDN}$ of the loaded power distribution network, corresponding to a unit (normalized) switching current excitation.

The above described flow calls for reduced order compact PDN models, in order to reduce runtime. Many approaches have been presented for the generation of such compact macromodels. These approaches include classical model order reduction methods to reduce the size of an already existing simulation model by means of projection or truncation [6], [7], and black-box identification/approximation methods to estimate compact models from input-output frequency responses [8]– [21]. This work belongs to this second class.

Starting from frequency-domain scattering parameter data, state-space macromodels are easily obtained through rational curve fitting algorithms, aiming at the identification of dominant pole-residue pairs in a partial fraction representation of the model scattering matrix, by minimizing some approximation error norm [8]– [12]. The model is then checked for passivity, a fundamental condition ensuring not only physical consistency but also numerical robustness and guaranteeing stability in successive transient simulations. Any passivity violations are eliminated by means of model perturbation using one of the many available techniques [13]– [21]. Since rational transfer matrices correspond to lumped circuits or, equivalently, to systems of ordinary differential equations, the implementation of the macromodels in standard circuit solvers is straightforward.

It is well recognized by both academic and industrial

978-3-9815370-2-4/DATE14/©2014 EDAA

communities that the above flow leads to very accurate passive macromodels, whose frequency responses match closely the data samples used for the identification process. So, when starting from scattering data, the scattering responses of the macromodel will be accurate. Unfortunately, this is not sufficient for the macromodel to be reliably used in power integrity verification. In fact, it has been shown in [23] that, when connecting the proper terminations to the macromodel ports (VRM, decoupling capacitors, active device models) and computing the resulting PDN voltage drop excited by active device switching, the results may be inaccurate. The root cause for this problem turns out to be the sensitivity of the target impedance $Z_{\rm PDN}$ to perturbations in the scattering responses. The inevitable approximation error of the macromodel can be amplified by the feedback provided by the PDN termination network, although this error is well under control in the scattering domain, corresponding to 50 Ω terminations.

The developments in [23] provide a good solution to avoid this issue in the rational fitting stage of the macromodel generation. The sensitivity is computed numerically from the original scattering data based on the nominal termination network, and used as a frequency-dependent weight in the definition of the target norm that the rational fitting engine aims at minimizing during model identification. This simple trick is able to compensate the frequency-dependent feedback and provides excellent accuracy both in the scattering responses and in the the target impedance $Z_{\rm PDN}$ computed from the macromodel. Unfortunately, the resulting model can present passivity violations and is thus subject to potential instability in transient system-level simulations. Available approaches for passivity enforcement are likely to destroy model accuracy, since no information on the model sensitivity is included in the perturbation scheme. This is actually the case, as demonstrated in Sec. IV through a PDN model of a real design.

This paper fills this gap, by presenting a new approach to include sensitivity-based frequency-dependent weights in the cost function used for model perturbation in a passivity enforcement loop. This allows to compensate the frequency-dependent feedback from the terminations also during passivity enforcement, thus preserving model accuracy. We provide in Sec. II some background information and we state the problem at hand. The proposed sensitivity-weighted passivity enforcement scheme is presented in Sec. III. Numerical results are presented and discussed in Sec. IV.

II. Background and problem statement

Let us consider a P-port PDN structure known via its scattering matrix samples $\hat{\mathbf{S}}_k$ at frequencies ω_k for $k = 1, \ldots, K$, normalized to a port resistance R_0. We also consider a generic nominal termination scheme defined by a generalized Norton equivalent

$$-\boldsymbol{I}(s) = \mathbf{Y}_L(s)\boldsymbol{V}(s) - \boldsymbol{J}(s), \tag{1}$$

where $\boldsymbol{V}(s), \boldsymbol{I}(s)$ are vectors collecting the Laplace-domain port voltages and currents, $\mathbf{Y}_L(s)$ is the short-circuit load admittance, and $\boldsymbol{J}(s)$ collects all independent current excitations. We will assume a single unit current excitation at port j, resulting in a single nonvanishing component in the source vector $J_j(s) = 1$. We are interested in the voltage drop at port i, which can be obtained as element (i, j) of matrix

$$\hat{\mathbf{Z}}_k = \left\{ R_0^{-1}[\mathbf{I} - \hat{\mathbf{S}}_k][\mathbf{I} + \hat{\mathbf{S}}_k]^{-1} + \mathbf{Y}_L(\mathrm{j}\omega_k) \right\}^{-1}. \tag{2}$$

We will define $\hat{Z}_{\mathrm{PDN},k} = (\hat{\mathbf{Z}}_k)_{i,j}$ as the reference PDN impedance at frequency ω_k. The samples of this reference impedance are considered to be exact, except for the approximations of the field solver used to compute the initial scattering samples.

We next process the samples $\hat{\mathbf{S}}_k$ to compute a standard scattering-based macromodel in pole-residue form

$$\mathbf{S}(s) = \sum_{n=1}^{N} \frac{\mathbf{R}_n}{s - p_n} + \mathbf{R}_0 \tag{3}$$

through Vector Fitting (VF) [8]– [12]. A standard VF application leads to a model whose responses minimize the error metric

$$\mathcal{E}^2 = \sum_{k=1}^{K} \mathcal{E}_k^2 = \sum_{k=1}^{K} \|\mathbf{S}(\mathrm{j}\omega_k) - \hat{\mathbf{S}}_k\|^2. \tag{4}$$

It has been shown in [23] that the macromodel-based PDN impedance $Z_{\rm PDN}(\mathrm{j}\omega_k)$, obtained by replacing the original samples $\hat{\mathbf{S}}_k$ in (2) with $\mathbf{S}(\mathrm{j}\omega_k)$, may differ significantly from the reference $\hat{Z}_{\mathrm{PDN},k}$. This consideration led to the definition of a first-order sensitivity Ξ_k, defined as

$$E\{|Z_{\rm PDN}(\mathrm{j}\omega_k) - \hat{Z}_{\mathrm{PDN},k}|\} \approx \Xi_k\, \sigma\,, \tag{5}$$

where σ is the standard deviation of P^2 uncorrelated and zero-mean gaussian variables used to perturb independently all elements of the original scattering samples and $E\{\}$ extracts the expected value. This frequency-dependent sensitivity $\Xi(s)$ models the amplification of the macromodel approximation errors due to the nonlinear transformation from $\mathbf{S}(s)$ to $Z_{\rm PDN}(s)$ due to the loading network.

It was shown in [23] that the generation of the macromodel using a modified (weighted) error metric

$$\mathcal{E}_w^2 = \sum_{k=1}^{K} \mathcal{E}_{w,k}^2 = \sum_{k=1}^{K} w_k^2 \|\mathbf{S}(\mathrm{j}\omega_k) - \hat{\mathbf{S}}_k\|^2 \tag{6}$$

in the VF algorithm is able to compensate for the frequency-dependent sensitivity, leading to accurate macromodel-based PDN impedance. In the following, we will assume that the weights are such that $w_k = \Xi_k$, i.e., they coincide with the first-order sensitivity. A weight refinement procedure that further optimizes the weights is documented in [23].

III. Sensitivity-weighted passivity enforcement

In general, there is no guarantee that the model generated by VF and minizing (6) is passive. Model passivity can be readily checked by converting the pole-residue form into a regular (minimal) state-space system

$$\mathbf{S}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \leftrightarrow \left[\begin{array}{c|c} \mathbf{A} & \mathbf{B} \\ \hline \mathbf{C} & \mathbf{D} \end{array}\right], \tag{7}$$

which always exists due to the adopted scattering representation, by forming the Hamiltonian matrix $\mathcal{M}$ associated to the state-space realization, and by computing its eigenvalues. If any purely imaginary eigenvalues of odd multiplicity are present, then the macromodel is not passive [14]. Equivalently, at least one singular value $\sigma_i(\mathrm{j}\omega_\nu)$ of the scattering matrix $\mathbf{S}(s)$ at some frequency $\mathrm{j}\omega_\nu$ is strictly larger than one. This singular value represents the worst-case energy gain due to the macromodel under excitation at fixed frequency ω_ν, which in turn may be the root cause for numerical instabilities in transient simulations based on the macromodel.

The non-passive macromodel must then be subjected to a suitable perturbation to eliminate the passivity violations. We follow here the standard approach of perturbing the state matrix $\mathbf{C} \leftarrow \mathbf{C} + \delta\mathbf{C}$, and we formulate an iterative perturbation loop where a suitable norm of the macromodel perturbation $\delta\mathbf{S}(s)$ induced by $\delta\mathbf{C}$ is minimized, based on local passivity constraints

$$\sigma_i(\mathrm{j}\omega_\nu) + \delta\sigma_i(\mathrm{j}\omega_\nu) \leq 1\,, \tag{8}$$

where $\delta\sigma_i(\mathrm{j}\omega_\nu)$ is the perturbation induced on the i-th macromodel singular value at frequency ω_ν. A linearization of the constraints (8) leads to the following optimization loop

$$\delta\mathbf{C}_\mu = \arg\min_{\delta\mathbf{C}} \|\delta\mathbf{S}\|^2 \quad \text{s.t.} \quad \mathbf{F}\mathrm{vec}(\boldsymbol{\delta}\mathbf{C}) \prec \boldsymbol{g}\,, \tag{9}$$

where $\mathbf{F}, \boldsymbol{g}$ collect the linarization coefficients of constraints (8), $\mathrm{vec}(\boldsymbol{\delta}\mathbf{C})$ denotes the vectorized form of $\delta\mathbf{C}$, obtained by stacking all columns into a single vector, and $\prec$ operates elementwise. The problem (9) is repeated for $\mu = 1, 2, \ldots$ by accumulating all perturbations $\delta\mathbf{C}_\mu$ until the macromodel results passive. The above formulation is standard, for more details we refer the Reader to [13]– [20].

The key for accuracy preservation during the passivity enforcement loop is the choice of the norm $\|\delta\mathbf{S}\|$ to be minimized in (9). It is well known [14] that a good choice is the $\mathcal{L}_2$ norm of the impulse response perturbation, which corresponds in the current perturbation setting to

$$\|\delta\mathbf{S}\|_2^2 = \mathrm{tr}(\delta\mathbf{C}\,\mathbf{P}\,\delta\mathbf{C}^T)\,, \tag{10}$$

where tr is the matrix trace and $\mathbf{P} = \mathbf{P}^T > 0$ is the controllability Gramian associated to the state-space macromodel, computed as

$$\mathbf{A}\mathbf{P} + \mathbf{P}\mathbf{A}^T = -\mathbf{B}\mathbf{B}^T\,. \tag{11}$$

Unfortunately, the $\mathcal{L}_2$ norm (10) is not appropriate for our application, since it aims at minimizing the macromodel perturbation in the scattering representation. Our objective is here to minimize the perturbation $\delta Z_{\mathrm{PDN}}(s)$ induced on the PDN impedance $Z_{\mathrm{PDN}}(s)$. It is clear however that $\delta Z_{\mathrm{PDN}}(s)$ is related to $\delta\mathbf{S}(s)$ through the sensitivity function $\Xi(s)$, at least for small perturbations such that the first-order approximation holds. Therefore, we seek a procedure to incorporate the sensitivity $\Xi(s)$ as a frequency-dependent weight in the definition of the cost function in (9).

A direct weighting in the proposed framework is not straightforward. In fact, the optimization problem (9) and especially the norm characterization (10) are purely algebraic and based on the state-space matrices of the macromodel. Instead, we know the sensitivity function $\Xi(s)$ only at the discrete frequencies ω_k at which the original data samples are known. We therefore have two possibilities.

1) We can replace the algebraic norm characterization (10), which is equivalent [22] to

$$\|\delta\mathbf{S}\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{tr}(\delta\mathbf{S}(\mathrm{j}\omega)\delta\mathbf{S}(\mathrm{j}\omega)^H)d\omega \tag{12}$$

by a discrete weighted summation over the available frequency samples

$$\|\delta\mathbf{S}\|_2^2 \approx \frac{1}{2\pi}\sum_{k=1}^{K} \vartheta_k \mathrm{tr}(\delta\mathbf{S}(\mathrm{j}\omega_k)\delta\mathbf{S}(\mathrm{j}\omega_k)^H)\,. \tag{13}$$

This choice is simple to implement and to modify by including any arbitrary frequency-dependent weighting factor in the norm definition. Unfortunately, as discussed in [17], the inclusion of such sample-based norm as a cost function in the passivity enforcement loop (9) may require excessive computational resources for its solution.

2) A second possibility is to seek for an algebraic characterization of the weighted norm

$$\|\delta\mathbf{S}\|_{\Xi}^2 = \|\widetilde{\Xi}\,\delta\mathbf{S}\|_2^2\,, \tag{14}$$

where $\widetilde{\Xi}(s)$ is the transfer function of a state-space system such that

$$|\widetilde{\Xi}(\mathrm{j}\omega_k)|^2 \approx |\Xi_k|^2\,. \tag{15}$$

This choice enables the inclusion of the sensitivity weights with no additional cost for the solution of (9), as discussed below.

The first step is the extraction of a state-space system

$$\widetilde{\Xi}(s) = \widetilde{\mathbf{c}}(s\mathbf{I} - \widetilde{\mathbf{A}})^{-1}\widetilde{\mathbf{b}} + \widetilde{d} \leftrightarrow \left[\begin{array}{c|c} \widetilde{\mathbf{A}} & \widetilde{\mathbf{b}} \\ \hline \widetilde{\mathbf{c}} & \widetilde{d} \end{array}\right]\,, \tag{16}$$

such that the squared magnitude of its frequency response $\widetilde{\Xi}(s)$ approximates the known sensitivity frequency samples Ξ_k through (15). This step does not pose particular difficulties. The identification of this “sensitivity macromodel” can be performed by considering the pole-residue form

$$\widetilde{\Xi}(s)\widetilde{\Xi}(-s) = \sum_{m=1}^{M} \frac{r_m}{q_m^2 - s^2} + \widetilde{d} = \widetilde{d}\,\frac{\prod_{m=1}^{M}(z_m^2 - s^2)}{\prod_{m=1}^{M}(q_m^2 - s^2)} \tag{17}$$

and computing its coefficients through the Magnitude Vector Fitting algorithm [24], [25]. The (minimum-phase) sensitivity macromodel $\widetilde{\Xi}(s)$ is then constructed by extracting the subset of zeros and poles with negative real part from (17). The derivation of the corresponding minimal state-space realization (16) is standard.

Once (16) is available, we repeat for each matrix element $S_{ij}(s)$ of the PDN scattering macromodel the following steps:

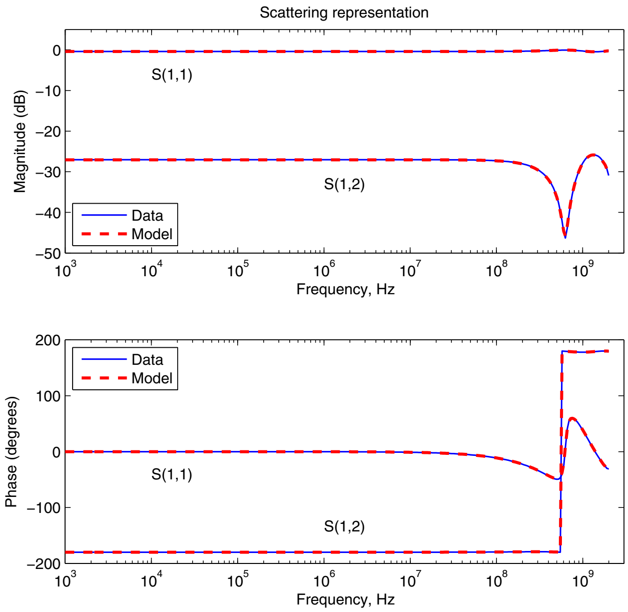

Fig. 1. Comparison between original data and model responses in the native scattering representation. The model is obtained with standard techniques for both fitting and passivity enforcement, in particular without specialized weighting.

1) we construct a state-space realization for the single element $S_{ij}(s)$, denoting the corresponding matrices as $\mathbf{A}_{ij}, \mathbf{b}_{ij}, \mathbf{c}_{ij}, d_{ij}$;
2) we form a state-space realization of the product system $S_{ij}(s)\widetilde{\Xi}(s)$ as

$$S_{ij}(s)\widetilde{\Xi}(s) \leftrightarrow \left[\begin{array}{cc|c} \mathbf{A}_{ij} & \mathbf{b}_{ij}\widetilde{\mathbf{c}} & \mathbf{b}_{ij}\widetilde{d} \\ \mathbf{0} & \widetilde{\mathbf{A}} & \widetilde{\mathbf{b}} \\ \hline \mathbf{c}_{ij} & d_{ij}\widetilde{\mathbf{c}} & d_{ij}\widetilde{d} \end{array}\right] \tag{18}$$

3) we compute the controllability gramian associated to the realization (18), which is further partitioned as

$$\mathbf{P}_{ij}^{\Xi} = \begin{bmatrix} \mathbf{P}_{ij}^{\Xi,11} & \mathbf{P}_{ij}^{\Xi,12} \\ \mathbf{P}_{ij}^{\Xi,21} & \mathbf{P}_{ij}^{\Xi,22} \end{bmatrix} \tag{19}$$

4) we define

$$\|\delta S_{ij}\|_{\Xi}^2 = \mathrm{tr}(\delta\mathbf{c}_{ij}\,\mathbf{P}_{ij}^{\Xi,11}\,\delta\mathbf{c}_{ij}^T) \tag{20}$$

Finally, all individual contributions (20) are assembled as

$$\|\delta\mathbf{S}\|_{\Xi}^2 = \sum_{i,j=1}^{P} \|\delta S_{ij}\|_{\Xi}^2\,, \tag{21}$$

which in turn is used as a cost function in the passivity enforcement loop (9). The above steps extend [18], [19] for the inclusion of sensitivity-based weighting. We show in next section how this approach proves very effective in the accuracy preservation of the PDN macromodel response, including the target impedance.

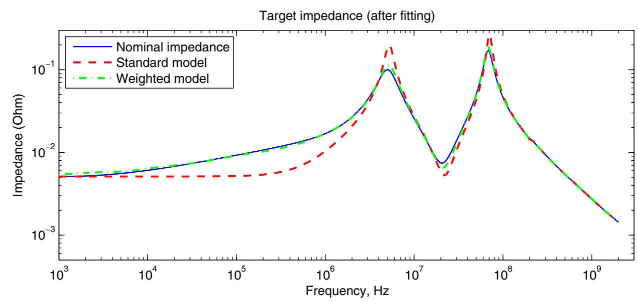

Fig. 2. Target impedance of PDN structure: comparison between nominal impedance (solid line) and impedance computed from PDN macromodels with and without inclusion of sensitivity-based weighting.

IV. Numerical results

We consider a PDN example from a real design. The structure under investigation is a single power domain at small form factor, few layers package (courtesy of Boaz Hirschl, Intel). A subset of $P = 45$ ports are considered. Of these ports, a total of $P_a = 24$ ports are connected to the power supply of active device blocks on the die, $P_c = 12$ ports are connected to decoupling capacitors on the board, and $P_v = 1$ port is connected to the VRM. All the other P_o ports are left open. The nominal termination scheme is thus defined as:

- a short-circuit is connected to the VRM port;
- appropriate decoupling capacitor models from the vendor are connected to the P_c board ports, including the associated parasitic series resistance (ESR) and inductance (ESL);
- series RC equivalent circuits are connected to each of the P_a ports to model active die devices; in addition, a total current of 1 A excites the PDN at these ports through identical equivalent current sources with value $1/P_a$.
- the resulting target impedance $Z_{\mathrm{ref}}(s)$ is obtained as the voltage resulting at one of the die ports.

Based on these definitions, $Z_{\mathrm{ref}}(s)$ represents the PDN voltage response on a specific die location excited by synchronous switching of multiple active device blocks uniformly spread on the die.

The system is known via its scattering responses (normalized to $R_0 = 50\,\Omega$) obtained by a field solver and tabulated from 1 kHz to 2 GHz with logarithmic sampling and including the DC point. Two representative scattering responses are depicted in Fig. 1 (solid blue lines). From these plots, we see that all scattering responses are very smooth over the frequency band of interest and should pose no problems for the extraction of a rational macromodel using standard techniques. In fact, the responses of a rational macromodel ($n = 12$ poles, common to all scattering matrix elements) match very closely the raw data, see Fig. 1, dashed lines.

We now compute the target impedance $Z_{\mathrm{ref}}(s)$ of the PDN connected to the nominal termination network. Figure 2 depicts the target impedance computed from the original scattering responses (solid blue line) and from the standard macromodel (red dashed line). We see that the accuracy

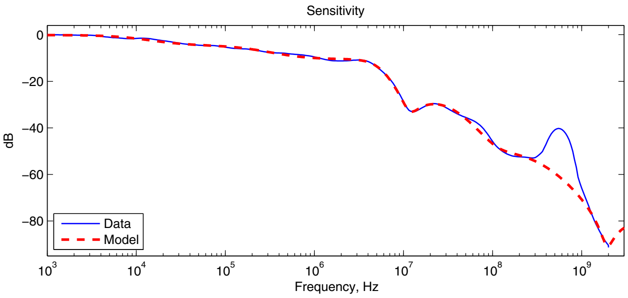

Fig. 3. First-order sensitivity of PDN target impedance to perturbations in the scattering responses. The solid line is the sensitivity obtained by direct perturbation analysis; the dashed line is the response of the corresponding rational macromodel obtained via Magnitude Vector Fitting.

of the standard macromodel is severely deteriorated when observed under nominal loading conditions. This is due to the sensitivity of the network transformation from the scattering representation to the target impedance, which amplifies the inevitable approximation errors present in the macromodel. This sensitivity is represented as a solid blue line in Fig. 3. The explicit inclusion of this sensitivity function as a weight in the linear least squares systems of the Vector Fitting process leads to a much better macromodel, whose target impedance is depicted with a green dash-dot line in Fig. 2.

Up to this point, the passivity of the rational sensitivity-based macromodel has not been considered. A frequency sweep of the scattering singular values of the weighted macromodel reveals multiple passivity violations, depicted in Fig. 4. Application of a standard (no weighting) passivity enforcement scheme based on iterative perturbations leads to a passive macromodel, which however suffers a major accuracy loss. This issue is clearly visible in Fig. 5, where it is observed that the target impedance derived from the macromodel deviates significantly at low frequencies. Although passive, this macromodel is thus useless for practical design and verification.

We then apply the proposed frequency weighting process in the passivity enforcement. The first step is the generation of a rational model for the sensitivity subsystem using Magnitude Vector Fitting. The result is depicted in Fig. 3, where a good match is observed between sensitivity data and model. Note that, in order to keep the order of the weighting subsystem low (we used $n_w = 8$), we did not care of matching the spike around 0.5–1 GHz, since both scattering responses and target impedance are very accurate in this frequency band. The weighting subsystem $W(s)$ was then used in the proposed formulation to obtain a frequency-weighted controllability Gramian (19), which was then used to define the norm used as a cost function in the passivity enforcement optimization loop.

A passive macromodel was obtained in 9 iterations. The singular values of the passive macromodel are depicted in Fig. 4, showing that all passivity violations have been removed (all singular values are not larger than one at all frequencies). The scattering responses of the macromodel are also accurate, as depicted in Fig. 6. No difference between the passive

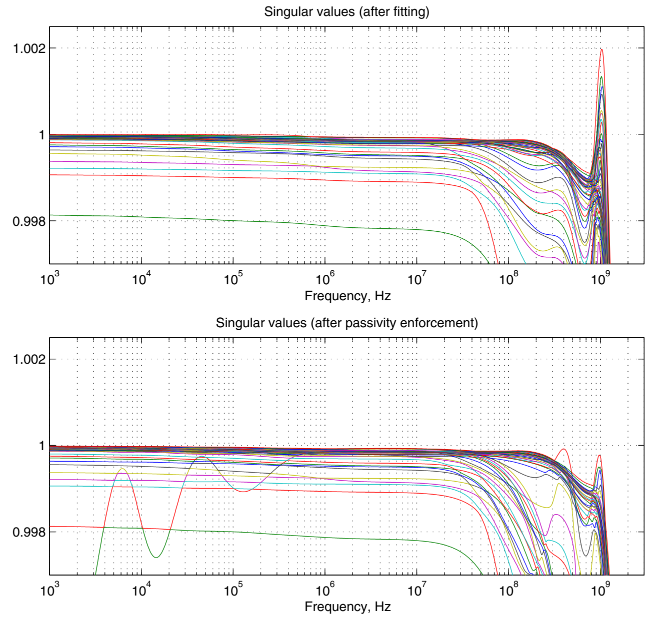

Fig. 4. Singular values of the PDN model before (top) and after (bottom) passivity enforcement.

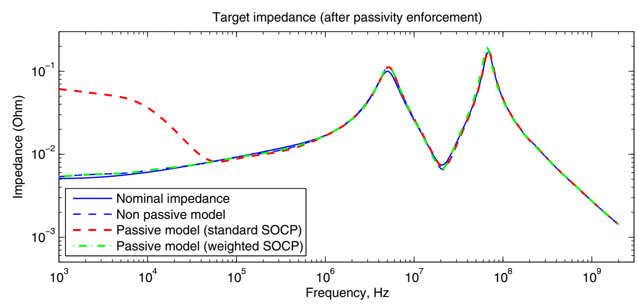

Fig. 5. Target impedance of PDN structure: comparison between nominal impedance (solid line) and impedance computed from PDN macromodels after passivity enforcement, with and without inclusion of sensitivity-based weighting.

macromodels obtained without weighting and with sensitivity-based weighting can be noted in the scattering representation by comparing Fig. 1 and Fig. 6.

The main result is depicted in Fig. 5, which reports with a green dash-dot line the target impedance of the sensitivity-based passive macromodel. It is then confirmed that the latter macromodel is accurate at all frequencies both in the scattering and especially under nominal termination conditions, thus proving the effectiveness of the sensitivity-based weighting approach, both in fitting and in passivity enforcement. This macromodel is then safely usable for transient power integrity verifications using circuit solvers.

V. Conclusions

We have presented a technique for the generation of passive macromodels of linear interconnect structures, with specific reference to power integrity applications. The main advantage of proposed method is a major reduction in the sensitivity

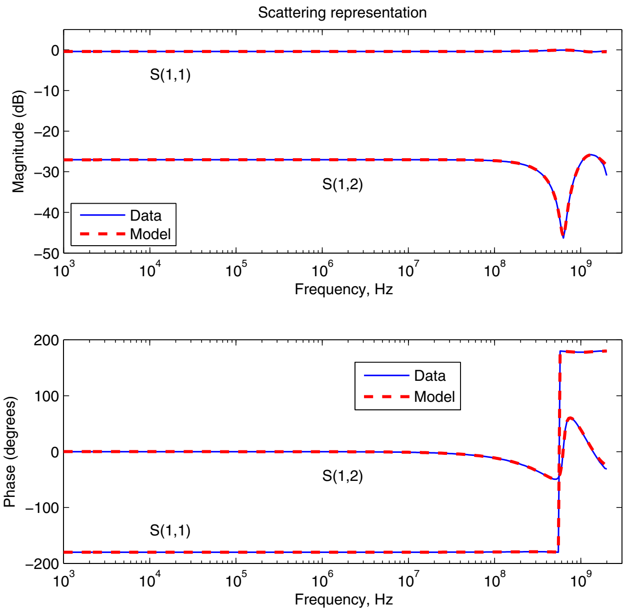

Fig. 6. Comparison between original data and model responses in the native scattering representation. The model is obtained with sensitivity-based weighting during passivity enforcement.

of the model responses to changes in the terminations used to load its interface ports. We have shown that an explicit inclusion of sensitivities as weighting factors in all steps of model extraction proves very effective for tuning model accuracy based on nominal operating conditions. The improvements over existing and more standard techniques have been documented on a real industrial testcase. The same improvement was observed for several other PDN models, not shown here.

A few remarks are in order. First, the computational cost for implementation of the sensitivity-based weights is negligible with respect to all other steps of model extraction, so that the accuracy improvement only requires a marginal overhead. Second, although we presented a model extraction flow starting from scattering parameters, the same sensitivity-based weighting process can be applied to native data in admittance or impedance form, as well as in scattering representations normalized to different port resistances. Suitably-defined weights will enable accuracy improvement independent on the starting data representation, due to the possibly strong frequency dependence of the feedback induced by the terminations. A detailed comparison of alternative model extraction strategies and possibly the selection of a recommended model extraction flow will be documented in a forthcoming report.

References

[1] M. Swaminathan, A.E. Engin, *Power Integrity Modeling and Design for Semiconductors and Systems*, Prentice Hall, 2007.

[2] Swaminathan, M.; Joungho Kim; Novak, I.; Libous, J.P., "Power distribution networks for system-on-package: status and challenges," *IEEE Trans. on Advanced Packaging*, vol. 27, no. 2, pp. 286–300, May 2004.

[3] Swaminathan, M.; Daehyun Chung; Grivet-Talocia, S.; Bharath, K.; Laddha, V.; Jianyong Xie, "Designing and Modeling for Power Integrity," *IEEE Trans. on Electromagnetic Compatibility*, vol. 52, no. 2, pp. 288–310, May 2010.

[4] Haihua Su; Sapatnekar, S.S.; Nassif, S.R., "Optimal decoupling capacitor sizing and placement for standard-cell layout designs," *IEEE Trans. on Computer-Aided Design of Integrated Circuits and Systems*, vol. 22, no. 4, pp. 428–436, Apr 2003.

[5] Kose, S.; Friedman, E.G., "Distributed On-Chip Power Delivery," *IEEE Journal on Emerging and Selected Topics in Circuits and Systems*, vol. 2, no. 4, pp. 704–713, Dec. 2012.

[6] A. C. Antoulas, *Approximation of large-scale dynamical systems*, SIAM, 2005.

[7] Schilders, W. H., Van Der Vorst, H. A., Rommes, J. (Eds.), *Model order reduction: theory, research aspects and applications*. Springer, 2008.

[8] B. Gustavsen, A. Semlyen, "Rational approximation of frequency responses by vector fitting", *IEEE Trans. Power Delivery*, Vol. 14, N. 3, pp. 1052–1061, July 1999.

[9] D. Deschrijver, B. Haegeman, T. Dhaene, "Orthonormal Vector Fitting: A Robust Macromodeling Tool for Rational Approximation of Frequency Domain Responses", *IEEE Transactions on Advanced Packaging*, Vol. 30, No. 2, pp. 216–225, May 2007.

[10] D. Deschrijver, M. Mrozowski, T. Dhaene, D. De Zutter, "Macromodeling of Multiport Systems Using a Fast Implementation of the Vector Fitting Method," *IEEE Microwave and Wireless Components Letters*, Vol. 18, N. 6, June 2008, pp.383–385.

[11] A. Chinea, S. Grivet-Talocia, "On the Parallelization of Vector Fitting Algorithms," *IEEE Trans. on Components, Packaging, and Manufacturing Technology*, Vol. 1, n. 11, Nov. 2011, pp. 1761–1773.

[12] S. Grivet-Talocia, S.B. Olivadese, P. Triverio, "A compression strategy for rational macromodeling of large interconnect structures," EPEPS 2011, San Jose, CA (USA), October 23-26, 2011, pp. 53–56.

[13] C. P. Coelho, J. Phillips, L. M. Silveira, "A Convex Programming Approach for Generating Guaranteed Passive Approximations to Tabulated Frequency-Data", *IEEE Trans. CAD*, Vol. 23, No. 2, pp. 293–301, Feb. 2004.

[14] S. Grivet-Talocia, "Passivity enforcement via perturbation of Hamiltonian matrices", *IEEE Trans. CAS-I*, Vol. 51, No. 9, pp. 1755-1769, Sept. 2004.

[15] D. Saraswat, R. Achar and M. Nakhla, "Global Passivity Enforcement Algorithm for Macromodels of Interconnect Subnetworks Characterized by Tabulated Data", *IEEE Trans. VLSI Systems*, Vol. 13, No. 7, pp. 819–832, July 2005.

[16] S. Grivet-Talocia, A. Ubolli "On the Generation of Large Passive Macromodels for Complex Interconnect Structures", *IEEE Trans. Adv. Packaging*, vol. 29, No. 1, pp. 39–54, Feb. 2006.

[17] S. Grivet-Talocia and A. Ubolli, "A comparative study of passivity enforcement schemes for linear lumped macromodels," *IEEE Trans. Advanced Packaging*, vol. 31, pp. 673–683, Nov 2008.

[18] S. Grivet-Talocia, A. Ubolli, "Passivity Enforcement With Relative Error Control" , *IEEE Trans. Microwave Theory and Techniques*, Vol. 55, No. 11, pp. 2374–2383, Nov. 2007.

[19] A. Ubolli, S. Grivet-Talocia, "Weighting Strategies for Passivity Enforcement Schemes", *16th IEEE Topical Meeting on Electrical Performance of Electronic Packaging, Atlanta, GA, 29–31 October, 2007*

[20] B. Gustavsen, A. Semlyen, "Enforcing passivity for admittance matrices approximated by rational functions", *IEEE Trans. Power Systems*, Vol. 16, N. 1, pp. 97–104, Feb. 2001.

[21] D. Deschrijver, T. Dhaene, "Fast Passivity Enforcement of S-Parameter Macromodels by Pole Perturbation," *IEEE Trans. MTT*, Vol. 57, no. 3, pp. 620–626, 2009.

[22] K. Zhou, J. C. Doyle, K. Glover, *Robust and Optimal Control*, Prentice Hall, 1996.

[23] S. Grivet-Talocia, A. Ubolli, M. Bandinu, A. Chinea, "An iterative reweighting process for macromodel extraction of power distribution networks," *Electrical Performance of Electronic Packaging and Systems (EPEPS)*, 27–30 October, 2013, San Jose (CA), USA.

[24] De Tommasi, L.; Gustavsen, B.; Dhaene, T., "Accurate Macromodeling Based on Tabulated Magnitude Frequency Responses," *12th IEEE Workshop on Signal Propagation on Interconnects*, pp.1–4, 12-15 May 2008

[25] W. Hendrickx, D. Deschrijver, L. Knockaert, T. Dhaene, "Magnitude Vector Fitting to interval data," *Mathematics and Computers in Simulation*, Vol. , No. 3, Nov. 2009, pp. 572–580