An Efficient Wirelength Model for Analytical Placement

B.N.B. Ray*
Dept. of Computer Science
Utkal University
Bhubaneswar, Odisha, India - 751004
Email: bnbray@yahoo.com

Shankar Balachandran
Dept. of Computer Science and Engineering
Indian Institute of Technology Madras
Chennai, Tamil Nadu, India - 600036
Email: shankar@cse.iitm.ac.in

***Abstract*—Smooth approximations to half-perimeter wirelength are being investigated actively because of the recent increase in interest in analytical placement. It is necessary to not just provide smooth approximations but also to provide error analysis and convergence properties of these approximations. We present a new approximation scheme which uses a non-recursive approximation to the `max` function. We also show the convergence properties and the error bounds. The accuracy of our proposed scheme is better than those of the popular Logarithm-Sum-Exponential (LSE) wirelength model [7] and the recently proposed Weighted Average(WA) wirelength model[3]. We also experimentally validate the comparison by using global and detail placements produced by NTU Placer [1] on ISPD 2004 benchmark suite. The experimentations on benchmarks validate that the error bounds of our model are lower, with an average of 4% error in the total wirelength.**

I. Introduction

Placement problem decides the actual physical locations of blocks in a chip. Analytical placers use either a constrained or an unconstrained optimization framework to decide the locations. Widely used analytical placers are Aplace [4], mPL6 [2], FastPlace [6] and NTUPlacer [1]. All of them use minimization of half-perimeter wirelength (HPWL) as their objective due to its simplicity, ease of calculation and the strong correlation between HPWL and Steiner tree wirelength [4].

Since HPWL uses `max` and `min` functions, and these functions are nondifferentiable and non-smooth, analytical placers replace these functions by their smooth continuous approximations before the optimization problem is solved. There have been many approximations to `max` and `min` functions [4], [5], [3]. Hsu et al. [3] applied their wirelength model to TSV aware placement and showed that the error upper bound of their wirelength model was smaller than that of LSE wirelength model. As these smooth functions are only approximations to HPWL, better approximations to HPWL are still an open issue in analytical placement literature. To this end we are motivated to explore smooth approximations for HPWL model and apply them to analytical placement.

Our contributions in this paper are listed below.

1) We propose a new smooth approximation to the `max` function. Using the proposed approximation, we derive a new smooth wirelength function for half-perimeter wirelength model.
2) We study the convexity and the convergence properties, and derive an upper bound of error of the proposed `max` functions. Compared to LSE wirelength model [7] and two recent wirelength models viz. WA wirelength model [3] and ABSWL model [5], our wirelegth model has smaller upper bound error both in theory and practice.
3) We also discuss implementation issues of the proposed model and compare its accuracy with other wirelength models conducting experiments on standard set of IBM benchmarks. The experimental results validate our theoretical findings.

The remainder of this paper is organized as follows. Section II discusses existing wirelength models. In Section III, we present our new wirelength model and study its convergence properties and derive an upper bound of error. Runtime consideration and implementation issues are described in Section IV. Finally conclusions and future scope of the work are offered in Section V.

II. Background on HPWL

The circuits in placement are represented as a hypergraph $H(V, E)$, where V is the set of fixed or movable blocks or pads, and E is a set of nets. If the bottom left corner of a block in chip is represented by $(x_i, y_i)(1 \leq i \leq \mid V \mid)$, HPWL of a net e is defined as

$$HPWL_e = \max_{i \in e}\{x_i\} - \min_{i \in e}\{x_i\} + \max_{i \in e}\{y_i\} - \min_{i \in e}\{y_i\} \quad (1)$$

$HPWL$ of a placement is given by sum of the HPWL of all nets in the netlist:

$$HPWL = \sum_{e \in E} HPWL_e \quad (2)$$

A. Review of Existing HPWL Wirelength Models

The wirelength function given in (Eqn(1) and (2)) is hard to minimize due to the presence of `max` and `min` functions. As a result several smooth approximations to wirelength function have been proposed by replacing the `max` and `min` functions by their smooth counterparts. Some of them are as follows.

*Currently a Ph.D. student in the CSE Dept. of IIT Madras.

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1. Logarithm-Sum-Exponential Wirelength Model(LSE)[7]
For real parameter $\gamma \to 0$, smooth approximation to HPWL of a net e is given by

$$\begin{aligned} LSEWL_e &= \gamma \ln(\sum_i e^{x_i/\gamma}) + \gamma \ln(\sum_i e^{-x_i/\gamma}) + \\ &\quad \gamma \ln(\sum_i e^{y_i/\gamma}) + \gamma \ln(\sum_i e^{-y_i/\gamma}) \end{aligned} \quad (3)$$

This wirelength model is very popular and used by analytic placers discussed in [1], [4], [2].

2. Weighted Average Wirelength Model(WAWL)[3]
If x and y coordinates of blocks of a net e are denoted by x_i and y_i then the weighted average HPWL of a net is given by

$$\begin{aligned} WAWL_e &= X_{max}(x_e) - X_{min}(x_e) \\ &\quad + Y_{max}(y_e) - Y_{min}(y_e) \end{aligned} \quad (4)$$

where

$$X_{max}(x_e) = \frac{\sum_{v_i \in e} x_i \cdot e^{x_i/\gamma}}{\sum_{v_i \in e} e^{x_i/\gamma}}$$
$$X_{min}(x_e) = \frac{\sum_{v_i \in e} x_i \cdot e^{-x_i/\gamma}}{\sum_{v_i \in e} e^{-x_i/\gamma}}$$
$$Y_{max}(y_e) = \frac{\sum_{v_i \in e} y_i \cdot e^{y_i/\gamma}}{\sum_{v_i \in e} e^{y_i/\gamma}}$$
$$Y_{min}(y_e) = \frac{\sum_{v_i \in e} y_i \cdot e^{-y_i/\gamma}}{\sum_{v_i \in e} e^{-y_i/\gamma}}$$

and γ is chosen such that $\gamma \to 0$. If $ErrWA(x_e)$ is the estimation error of WA wirelength model for x co-ordinates of the net e, then Hsu et al. [3] proved the following theorems concerning the bounds of error.

Theorem 1. $0 \leq ErrWA(x_e) \leq \frac{\gamma \Delta x}{1+exp(\Delta x)/n}$, where $\Delta x = x_{max} - x_{min}$.

Theorem 2. The estimation error upper bound of the WA wirelength model is smaller than that of the LSE wirelength model i.e

$\frac{\gamma \Delta x}{1+exp(\Delta x)/n} \leq \gamma \ln n$, $\forall n \geq 2$

3. Absolute Wirelength Model (ABSWL)[5]
For real parameter $\beta \to \infty$, an approximation to the two variable max function $\max(x_1, x_2)$ is given by

$$\begin{aligned} & ABSBMAX(x_1, x_2) \\ = \; & \tfrac{1}{2}(x_1 + x_2 + |x_1 - x_2|) \\ \approx \; & \tfrac{1}{2}\left(x_1 + x_2 + \tfrac{1}{\beta}(\ln 2 + \ln(1 + \cosh(\beta(x_1 - x_2))))\right) \end{aligned}$$

Generalizing $ABSBMAX(x_1, x_2)$ to n recursively, a smooth formulation for HPWL was obtained in [5]. It was shown through simulation that the estimation upper bound error of ABSWL model is less than LSEWL model.

III. Proposed Wirelength Model

In this section, we formulate a novel wirelength model. Let $x_e = (x_1, x_2, ...x_n)$ and $y_e = (y_1, y_2, ...y_n)$ be x and y be coordinates of net e respectively. Without loss of generality, assume these coordinates are positive real constants. Then for real parameters $\gamma \to +0$, $p \to +\infty$, we define the (γ, p)-mean of x_e by

$$X^{(\gamma,p)}(x_e) = \frac{\sum_{i=1}^{n} x_i^p exp(x_i/\gamma)}{\sum_{i=1}^{n} x_i^{p-1} exp(x_i/\gamma)} \quad (5)$$

A. Convergence Properties

We have the following convergence properties of (γ, p)-mean.

Lemma 1. $X^{(p,\gamma)}(x_e)$ is strictly convex and is a continuously differentiable function of x_e.

Proof: $X^{(p,\gamma)}(x_e)$ is twice differentiable for $x_i \in x_e$ and the derivative is positive. Thus, the lemma follows.

Theorem 3. If x_{max} and x_{min} are maximum and minimum of $x_1, x_2, ...x_n$, then we have

$(i) \lim_{(\gamma,p) \to (+0,+\infty)} X^{(\gamma,p)}(x_e) = x_{max}$

$(ii) \lim_{(\gamma,p) \to (-0,-\infty)} X^{(\gamma,p)}(x_e) = x_{min}$.

Proof: Without loss of generality, let us assume $x_1 \geq x_2 \geq ... \geq x_n$. Setting $\gamma = \frac{1}{p}$, we have

$X^{(p,1/p)}(x_e) = x_1 \frac{1+\sum_{i=2}^{n}(x_i/x_1)^p exp((x_i-x_1)p)}{1+\sum_{i=2}^{n}(x_i/x_1)^{p-1} exp((x_i-x_1)p)}$

Letting $p \to +\infty$ Theorem 3(i) follows. Similarly Theorem 3(ii) can be proved.

It is interesting to note that for $p = 1$, (γ, p) mean of (x_e) reduces to WA maximum function $X_{max}(x_e)$ defined in Section II. Thus (γ, p)-mean is a generalized version of weighted average function defined in [3].

Using (γ, p) mean, smooth approximation to HPWL model is given by

$\sum_{e \in E} \left(X^{(\gamma,p)}(x_e) - X^{(-\gamma,-p)}(x_e) + X^{(\gamma,p)}(y_e) - X^{(-\gamma,-p)}(y_e)\right)$

B. Error Bounds

Let $ErrX^{(\gamma,p)}(x_e)$ be the estimation error of (γ, p)-mean wirelength model. Then we have the following upper bound of the error.

Theorem 4:

$$0 \leq ErrX^{(\gamma,p)}(x_e) \leq \frac{\gamma \Delta x}{1+((x_{max}/x_{min})^{p-1} \cdot exp(\Delta x))/n}$$

where $\Delta x = x_{max} - x_{min}$.
Proof: Let us assume $x_1 \geq x_2 \geq ... \geq x_n$ and denote $\Delta x_i = (x_1 - x_i)/\gamma$. Now the error expression for maximum function for net e by (γ, p) mean is given by

$$\begin{aligned} & ErrX^{*(\gamma,p)}(x_e) = x_1 - X^{(\gamma,p)}(x_e) \\ = \; & \frac{\sum_{i=2}^{n} x_i^{p-1}(x_1 - x_i) exp(x_i/\gamma)}{\sum_{i=1}^{n} x_i^{p-1} exp(x_i/\gamma)} \end{aligned} \quad (6)$$

In order to get an upper bound of error, differentiate equation (6) partially with respect to x_i for $(2 \le i \le n)$ and make them equal to 0's. That is for any i, $\partial ErrX^{*(\gamma,p)}(x_e)/\partial x_i = 0$, implies

$$\begin{aligned} & \frac{\sum_{i=1}^{n} x_i^{p-1} exp(x_i/\gamma)}{\sum_{i=2}^{n} x_i^{p-1}(x_1 - x_i) exp(x_i/\gamma)} \\ = \quad & \frac{(p-1) + (x_i/\gamma)}{-x_i + (x_1 - x_i)(p-1) + (x_1 - x_i)(x_i/\gamma)} \end{aligned} \tag{7}$$

Now solving the system of equations (7) for $2 \le i \le n$, one can conclude that error is maximum when $x_2 = x_3 = ... = x_n$. Using $x_1 = x_{max}$, $x_2 = x_3 = ... = x_n = x_{min}$ and multiplying $exp(-x_1/\gamma)$ both in numerator and denominator of equation (6) we have

$$\begin{aligned} ErrX^{*(\gamma,p)}(x_e) &= \frac{\gamma \Delta x (n-1) x_{min}^{p-1} exp(-\Delta x)}{x_{max}^{p-1} + (n-1) x_{min}^{p-1} exp(-\Delta x)} \\ = \quad & \frac{\gamma \Delta x}{1 + ((x_{max}/x_{min})^{p-1} exp(\Delta x))/(n-1)} \\ \le \quad & \frac{\gamma \Delta x}{1 + ((x_{max}/x_{min})^{p-1} exp(\Delta x))/n} \end{aligned} \tag{8}$$

where $\Delta x = x_{max} - x_{min}$.

Similarly we can have the same bounds of error for minimum function $ErrX^{*(-\gamma,-p)}(x_e)$. From the definitions of maximum and minimum functions(See Theorem 3), we have $ErrX^{*(\gamma,p)}(x_e) \le x_{max}$ and $x_{min} \le ErrX^{*(-\gamma,-p)}(x_e)$. This implies $ErrX^{(\gamma,p)}(x_e) \ge 0$.

Hence Theorem 4 is proved. We have adopted a proof technique similar to that of Theorem 1 shown in [3].

It is interesting to note that for $p = 1$, Theorem 4 reduces to Theorem 1. Further, from results of Theorem 2 and Theorem 4 we have the following theorem.

Theorem 5: The estimation error upper bound of (γ, p)-mean wirelength model is smaller than WA wirelength model which in turn smaller than LSE wirelength model, i.e

$$ErrX^{(\gamma,p)}(x_e) \le ErrWA(x_e) \le \gamma \ln n, \forall n \ge 2.$$

IV. Experimental Validation

In this section we shall discuss the choice of parameters γ and p, which will keep the implementation numerically stable.

A. Choice of γ and p

If datatype `double` is used to represent wirelength, the largest value the datatype can take is $1.797E * 10^{308} \approx e^{710}$. Since $x^p e^{x/\gamma}$ can not exceed this value, p should satisfy the relationship: $p \le \frac{710 - x/\gamma}{\ln x}$. In WA and ABSWL wirelength models, γ and β) cannot be chosen arbitrarly close to 0 and ∞ respectively because of numerical instability. In our wirelength model trade-off between p and γ can be leveraged by fixing one parameter (say $\gamma(p)$) and varying the other(say γ)). Therefore our model is less susceptible to numerical instability than the other two models.

Though in theory p is supposed to be large, in practice one need to scale down the chip dimension W and H sufficiently so that the implementation remains stable. To illustrate the effect of increase in the value of p, we choose *ibm01* from ISPD 2004 fixed die benchmark suite. Using 1550×1530 grids we place the circuit using NTUPlace[1]. Then we measure the half perimeter wirelegth using exact calculations. Withought scaling the chip dimensions, the largest value p can take is $\frac{710 - 1530/\gamma}{\ln 1530}$. We pick $\gamma = 14$ and choose increasing values of p and simultaneously scale down the chip dimensions. The effect of larger p on errors for this calculation is shown in Table I. From the table it is evident that the errors go down steadily as p increases.

TABLE I. Effect of p on Approximation

$\gamma = 14, p$	6	12	25	50	100
Error %	35.9	27.15	17.28	10.04	5.27

B. Runtime Consideration

As it was done in [5], we compare the runtimes for two-variable maximum function for LSEWL, ABSWL, WA and (γ, p)-mean wirelegth models. For this, we generated 60×10^6 pairs of random real numbers and passed them as arguments to these two-variable max functions. The averaged runtimes over several experiments are listed in Table II. From the table it is clear that LSE maximum function has least runtime and other functions runtimes are comparable.

TABLE II. Runtime of 2-Variable Approximations

Method	LSE	WAWL	ABSWL	(γ, p)
Runtime (s)[1]	24.5	25.2	24.6	25.3

C. HPWL Accuracy

For comparing approximations for various wirelength models we choose circuits from IBM ISPD 2004 benchmark suite. The number of cells in these benchmarks vary from 12K to 210K. We obtain both global and detailed placements for each circuit using a widely used placement tool NTUPlace [1]. We used the global placements for the first round of comparisons. We read the placement back along with the netlist and calculate the HPWL for each net. The summation of exact HPWL over all nets is listed in column 2 of Table III.

To compare the different approximation schemes, we picked $\gamma = 0.01$, $\beta = 120$ [5] (satisfies the condition $K > 0.177$) and $p = 12$. We then scaled down the chip dimension to 4×4, calculated the approximated HPWL and scaled it back to the original dimensions by multiplying the result with $\frac{W+H}{8}$. The results from LSEWL, WA, ABSWL, and (γ, p)-mean approximations are presented in columns 3, 4, 5 and 6 of Table III. It is evident from the table that our (γ, p)-mean wirelength model gives the closest approximation to HPWL compared to the other schemes with an average of less than 4% absolute error in the total wirelength.

For detailed placement HPWL comparisons, we chose the same values for the parameters γ, β, and p. The detailed placement was generated using NTUPlace. The results of our comparisons are also shown in Table III. From the table we

TABLE III. HPWL MEASURED ON PLACEMENTS USING DIFFERENT APPROXIMATION SCHEMES

Circuit	Global Placement									Detailed Placement								
	Total HPWL ($\times 10^7$)					%Absolute Error in Approximation				Total HPWL ($\times 10^7$)					%Absolute Error in Approximation			
	A	L	W	B	G	L	W	B	G	A	L	W	B	G	L	W	B	G
ibm01	.170	.176	4.562	.175	.166	3.69	2583.5	2.73	2.35	.183	.187	4.48	.186	.18	2.58	2348.08	2	1.639
ibm02	.371	.381	7.460	.378	.365	2.66	1910.7	1.97	1.61	.381	.389	7.63	.387	.377	2.01	1902.62	1.55	1.04
ibm03	.496	.511	11.232	.508	.488	3	2164.5	2.23	1.61	.513	.524	11.48	.521	.507	2.17	2137.81	1.65	1.169
ibm04	.592	.612	11.538	.607	.581	3.25	1848.98	2.41	1.8	.624	.639	15.52	.635	.616	2.39	2387.17	1.82	1.282
ibm05	1.03	1.05	13.412	1.05	1.024	1.9	1202.13	1.42	0.5	1.07	1.08	13.37	1.08	1.06	1.42	1149.53	1.08	0.934
ibm06	.525	.547	14.902	.541	.512	4.28	2738.47	3.18	2.4	.541	.560	15.33	.556	.533	3.48	2733.64	2.66	1.478
ibm07	.873	.918	26.098	.907	.850	5.19	2889.46	3.89	2.6	.902	.938	27.40	.929	.88	3.96	2937.69	2.97	2.439
ibm08	.963	1.02	29.693	1.00	.938	5.44	2983.38	4.1	2.5	.990	1.03	30.37	1.02	.96	4.27	2967.67	3.21	3.030
ibm09	.980	1.05	37.310	1.03	.946	6.67	3707.14	4.99	3.4	1.02	1.07	36.50	1.06	.99	5	3478.43	3.73	2.94
ibm10	1.84	1.94	60.331	1.91	1.785	5.3	3178.85	3.94	2.9	1.88	1.96	61.93	1.94	1.83	4.44	3194.14	3.28	2.659
ibm11	1.42	1.53	58.094	1.50	1.372	7.43	3991.12	5.61	3.3	1.50	1.58	59.68	1.56	1.45	5.36	3878.66	3.98	3.333
ibm12	2.40	2.50	62.637	2.47	2.348	3.93	2509.87	2.9	2.1	2.40	2.48	65.46	2.46	2.35	3.48	2627.5	2.56	2.083
ibm13	1.77	1.91	75.318	1.87	1.707	7.47	4155.25	5.59	3.5	1.81	1.92	72.69	1.89	1.74	6.19	3916.02	4.6	3.86
ibm14	3.36	3.68	157.77	3.60	3.214	9.34	4595.53	7.05	4.3	3.39	3.67	153.04	3.60	3.24	8.18	4414.45	6.1	4.42
ibm15	4.08	4.47	184.19	4.38	3.897	9.54	4414.46	7.19	4.4	4.15	4.49	180.27	4.41	3.97	8.24	4243.85	6.12	4.33
ibm16	4.35	4.87	214.00	4.75	4.124	12	4819.54	9.14	5.1	4.67	5.12	216.34	5.01	4.44	9.64	4532.54	7.22	4.92
ibm17	6.65	7.16	236.01	7.04	6.394	7.79	3449.02	5.86	3.8	6.81	7.27	245.73	7.15	6.55	6.78	3508.37	5.03	3.81
ibm18	4.53	5.13	239.43	4.98	4.276	13.15	5158.43	10.02	5.6	4.68	5.21	233.03	5.08	4.42	11.24	4879.27	8.42	5.55
Average Error (in %)						6.22	3240.41	4.68	3.04	Average Error (in %)					5.05	3179.862	3.78	2.831

A: Actual L:Logsum W:WA B:ABSWL G:$(\gamma, p) - mean$

may notice that our scheme has an average less than 3% absolute error in the total wirelength of detailed placements.

V. CONCLUSIONS AND FUTURE WORK

We proposed an efficient wirelength model for HPWL function which can be used in analytical placers. We also studied its convergence properties and derived the bounds of error. The error bounds of the proposed model is less than the error bounds of extremely popular Logarithm-Sum-Exponent and recently proposed weighted average wirelength models. In comparable runtimes, our scheme has better performance than Logarithm-Sum-Exponent, Weighted average and Absolute wirelength models. Generating global and detailed placement using NTUPlace, we conclude that the accuracy of the proposed model is better than other wirelength models with an average 4% absolute error in total wirelength. In future we propose to apply this model in an analytical placer and study its performance.

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