

Event Density Analysis for Event Triggered Control Systems

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Abstract—In event triggered control systems, events occur aperiodically. For the real-time analysis of such systems, an appropriate approximation of the events' stimulation is necessary. Upper bounds have already been found for event triggered systems. For now, lower bounds have been assumed zero within the real-time analysis of event triggered systems. This work derives an approximated lower bound representing the maximum inter-sampling time. The bounds depend on the control system and the event generating mechanism. The beneficial effect is shown by analyzing an event triggered control system in a real-time analysis framework.

I. INTRODUCTION

One part of the real-time analysis is to integrate insights of control theory to obtain tighter bounds, schedulability and the corresponding response time of tasks. In control engineering, normally sensors are used to measure the output performance of the device being controlled. These measurements are used to give feedback to the input actuators that can make corrections toward desired performance. One common way to integrate digital control systems in real-time analysis bases on a periodically sampled sensor. On advantage of a periodic stimulation is the simple implementation. After a counter exceeds a pre-defined threshold, the sensor value is gathered. The threshold can be obtained by satisfying the Nyquist theorem [1]. This theorem states, that the sampling rate must be at least two times faster than the highest frequency that may occur in the control loop. The mathematical model of a system is represented by differential equations. This allows to obtain directly the threshold. In general, the physical system is oversampled to reach a better response on disturbances and changes in the reference value. The integration of a time triggered event stream into the real-time analysis is very effective and therefore very common.

As in most cyber-physical systems, timing effects the functional behavior. On the one hand, if the control loop is triggered too infrequently, the system can become unstable. On the other hand, additional sensor readings will not increase the stability of the system significantly. Especially in network controlled systems a big effort is to reduce communication. In contrast to time triggered control systems, event triggered control systems can guarantee a stable system with the least

amount of sensor events. Event triggered control mechanisms reduce the resource usage and the utilization.

In an event triggered control systems a sensor generates an event when a system state exceeds a defined threshold. This technique has a more efficient usage of system resources than conventional periodic sampling [2]. Events in event triggered control systems occur due to environmental circumstances. In consequence events do not occur in a strictly periodic manner anymore but occur aperiodic. To obtain an upper and lower bound of occurring events for the real-time analysis, a specification of the stimulation is needed. In literature, mostly bounds for the minimum inter-sampling times are regarded.

In figure 1 the y-axis shows the absolute value of the state vector. This value shows the absolute deviation from the last sampled state value to the current state value. The current value of the state vector is indicated by the solid red line. When the deviation exceeds a defined threshold $|x(t)|$ at point x_i , an event is triggered. This event leads to a correction of the input actors, which correct the output performance of the device towards the desired performance. Hence, the deviation goes back to zero. In a stable system, the threshold $|x(t)|$ is monotonically decreasing.

The specification of the stimulation tries to find an approximation for a minimum inter-sampling time and for a maximum inter-sampling time. With knowledge of the system, a minimum inter-sampling time (green dotted line in figure 1) and a maximum inter-sampling time (blue dashed line in figure 1) can be obtained. With the specifications of the stimulations, an upper bound (Δt_{\max}) and a lower bound (Δt_{\min}) for the real value (Δt_i) are obtained which are be used in the real-time analysis of the system. This work will introduce the approximation for the maximum inter-sampling time (blue dashed line in figure 1) and therefore a lower bound (Δt_{\min}) for the real-time analysis. For now the the lower bound is assumed to be zero.

The paper is structured as follows. The following section gives a brief overview of the related work. On the one hand the work that has been done to connect control systems and the timing behavior of the underlying hardware architecture. On the other hand the research in the filed of event-triggered control systems. In the third section models are introduced which characterize the occurrence of events over time. The

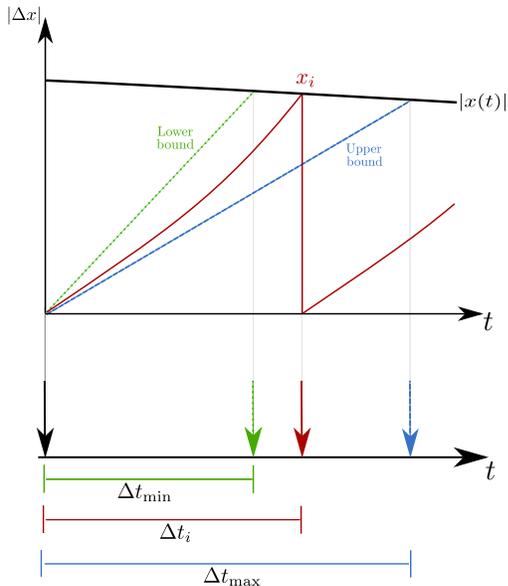


Fig. 1. Characteristics of an event triggered control system and its approximations.

control system model and its event generation mechanism lead to the problem formulation in this section. Section four derives a maximum inter-sampling time from the dynamics of the control system. The maximum inter-sampling time is equivalent to the minimum stimulation in the real-time analysis. Section five shows the benefits of our approach in a distributed system architecture. Our experimental results in section six demonstrate the positive effect in embedded system architectures, when a minimum stimulation for an event-triggered control system is available for the real-time analysis of such systems. This work closes with a conclusion.

II. RELATED WORK

The recent years have brought significant improvements in integrating control systems into the underlying hardware platform. This research focuses towards networked control systems and sensor actuator networks. Both find their application in a wide field of cyber physical systems. Most publications in this field deal with the influence of the delays. The delays are caused by the hardware and the scheduling of the control performance [3] [4]. Real-time analysis methods allow to obtain guaranteed bounds for the delays in a control loop. In [5], a functional model is connected to a real-time analysis model by using a bijective mapping. All so far stated methods presume periodic sampling, which is not resource friendly.

As mentioned before, event triggered control systems use system resources more efficiently than conventional periodic sampling [2]. There are different approaches to implement event triggered control systems [2] [6] [7]. This work focuses towards the input-to-state stability (ISS) triggering mechanism from [7]. The proof of ISS for the event triggered control

system is a useful and significant property, as it guarantees a minimum control performance. This proof is used in our work to show that the norm of the state space is decreasing over time. Our methods can also be adapted for similar triggering conditions as for example in [8].

The real-time analysis allows to validate schedulability of a system and to calculate the response times of its tasks and messages. For this analysis, we need an implementation of an event triggered control system on a distributed system architecture. For control systems with a periodic sampling, the representation of events is very basic because the tasks are stimulated strictly periodical. For event triggered control systems, events are generated aperiodically. This requires a more complex representation of the stimulation. Models that provide this additional degree of freedom are the event model from Gresser [9] and the arrival curves, as used by Wandeler [10].

III. MODEL AND PROBLEM FORMULATION

A time delay in a closed-loop control system has a direct impact on the control performance of networked control systems. In embedded systems these time delays are mainly induced by tasks which do not hold resources exclusively. Therefore, tasks or messages can be blocked or interrupted by other tasks or messages which lead then to time delays, which are denoted as response times. One possibility to determine these times is to perform a real-time analysis delivering absolute bounds for the worst-case and best-case response times. A methodology to obtain these response times is the modular performance analysis (MPA) as introduced in [10]. The idea behind the MPA is the convolution between curves, that describe the available capacity (β) for a task and the incoming stimulation (η). As an output, the mathematics behind the real-time calculus determines system properties as tasks response times, needed buffer size and remaining resource capacity of lower prior tasks.

The arrival curves are a specification of the tasks stimulation. For every time window, the arrival curves map to the corresponding number of events that can occur in that time interval. Thereby, the arrival curve is composed of two curves ($\eta^+(\Delta t), \eta^-(\Delta t)$), describing the maximal and minimal event density. Arrival Curves originate from the network calculus [11] and describe in our context stair function. The upper arrival curve ($\eta^+(\Delta t)$) is defined as the maximum number of events, that may occur in an arbitrary time interval with length $\Delta t + \epsilon$, with ϵ as marginal small value. Therefore, in the time interval of zero seconds, one event can occur. In contrast to the upper arrival curve, the lower arrival curve ($\eta^-(\Delta t)$) denotes the minimum number of events that may occur in the time interval Δt .

As additional information, the real time calculus requires a specification of the computation capacity of the tasks. When multiplying the arrival curves with the execution time of the task, we obtain the requested computation time. The available computation time can also be described in the time interval

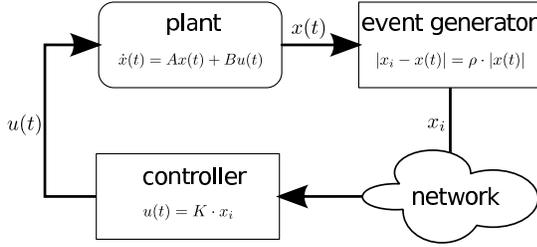


Fig. 2. Structure and interaction of signals in an event triggered control system.

domain. This is done by service curves $(\beta^+(\Delta t), \beta^-(\Delta t))$. Upper and lower service curves are defined analog to the arrival curves. The upper service curve defines the maximum amount of computation time, that can be provided a task in a given time interval. Respectively, the lower service curve defines the minimum calculation time available for a certain task.

For ordinary control systems, there exist stimulation models that can be applied to the arrival curves. These are stimulations with a strict periodic event generation. Considering event triggered control systems, events are generated depending on the dynamic behavior of the system. In the context of control systems, an event is represented by the sampled sensor value, which is afterwards propagated over a network. In [7] the existence of a lower bound for inter-sampling times (Δt_{min}) was proven. This lower bound can be direct transformed to a valid upper arrival curve by

$$\eta^+(\Delta t) = \left\lceil \frac{\Delta t}{\Delta t_{min}} \right\rceil \quad (1)$$

This upper arrival curve is not an exact solution but is a valid bound, due to the overestimation. What is still missing is a description of the lower arrival curve. One valid lower arrival curve would be zero, which could be assumed, when there is no further knowledge of the control system. Based on the minimum dynamics of the control system we derive a minimum stimulation, which leads to an improved lower arrival curve $(\eta^-(\Delta t))$.

In this paper we derive a valid lower arrival curve for an event triggered control system. Based on the system parameters of the control system and the event generating mechanism, we present a way to derive the lower arrival curve with small overestimation. For the sake of simplification, we assume a linear time-invariant systems without transport delays, which is stabilized by a state space controller. The dynamics of the system, that needs to be controlled can be modeled as differential equations. For linear systems, these differential equations can be combined to a matrix differential equation, which has in undisturbed case the form

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2)$$

with $x(t)$ as state vector, x_0 as initial state vector, $u(t)$ as system input A and B as system matrices with appropriate

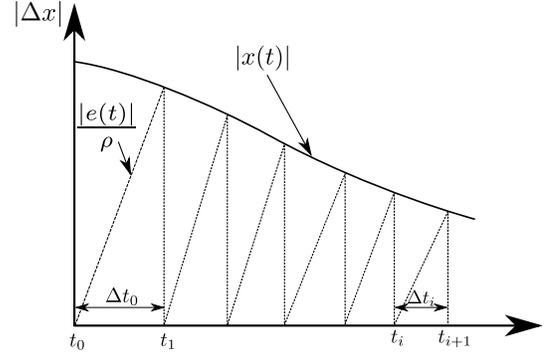


Fig. 3. Characteristics of the norms $|x(t)|$ and $|e(t)|$ for a undisturbed stable control system.

matrix dimensions. Thereby, the system matrix A is quadratic and invertible. The system is controlled by an event triggered control mechanism, as described by Tabuada et al. [7]. In the work of Tabuada, a new control value

$$u(t) = K \cdot x_i \quad (3)$$

is applied to the system at time t_i . As previously described, this happens, when the norm of an error $|e(t)|$ exceeds the norm of the state variable $|x(t)|$ multiplied by a factor ρ . The structure of such an event triggered control system is sketched in figure 2. An event is generated when the following equation holds

$$|e(t)| = \rho \cdot |x(t)| \quad (4)$$

The error is defined as $e(t) = x_i - x(t)$, where x_i is previous sampled state vector at time-step t_i . As a consequence the norm of the error $|e(t)|$ after the event generation is zero. The relation between the signals characteristic and the times an event is generated, is sketched in figure 3. By choosing an adequate factor ρ , this control method satisfies input-to-state stability, even for small enough transport delays, as shown in [7].

Based on the state space equations of the control system and the event generation rule, we approximate a valid lower bound of the arrival curve in the next chapter.

IV. APPROXIMATION OF STIMULATION

The goal of this paper is to find a valid lower bound of the arrival curve $\eta^-(\Delta t)$. This is done by determining the maximum time interval Δt_{max} between two consecutive events. First, we consider the state space representation of the controlled system in the time interval Δt_i . In this time interval the control value is held constant according to equation (3), leading to the state space representation of the close loop control system based on equation (2)

$$\dot{x}(t) = Ax(t) + BKx_i \quad \forall t_i \leq t < t_{i+1} \quad (5)$$

The solution of the ordinary matrix differential equation (5) in the time interval Δt_i can be calculated by solving

$$x(t) = e^{(t-t_i)A}x_i + \int_0^{(t-t_i)} e^{\tau A}BKx_i d\tau \quad \forall t_i \leq t < t_{i+1}. \quad (6)$$

To obtain the state x_{i+1} when the next event is generated we need to replace $(t - t_i)$ with Δt_i in the previous equation

$$x_{i+1} = e^{\Delta t_i A}x_i + \int_0^{\Delta t_i} e^{\tau A}BKx_i d\tau \quad (7)$$

leading to

$$\begin{aligned} x_{i+1} &= e^{\Delta t_i A}x_i + (e^{\Delta t_i A}A^{-1} - A^{-1})BKx_i \\ &= (e^{\Delta t_i A}(I + A^{-1}BK) - A^{-1}BK) \cdot x_i \\ &= (e^{\Delta t_i A}F + H) \cdot x_i \end{aligned} \quad (8)$$

with $F = I + A^{-1}BK$, $H = -A^{-1}BK$ and I as identity matrix. A^{-1} is the inverse matrix of A .

Equation (8) calculates the next sampled state x_{i+1} depending on the time interval Δt_i , the previous sampled state x_i and the system matrices.

As we want to calculate the maximum on this time interval, we need an extra constraint, given by the event triggering mechanism from equation (4). Substituting the solved ordinary differential equation from equation (8) in the event triggering mechanism gives us the following equation:

$$\begin{aligned} |x_{i+1}| &= \frac{1}{\rho}|x_i - x_{i+1}| \\ |(e^{\Delta t_i A}F + H) \cdot x_i| &= \frac{1}{\rho} |(I - e^{\Delta t_i A})F \cdot x_i| \end{aligned} \quad (9)$$

Unfortunately, this equation is hard to dissolve to Δt_i , so we need make an approximation. The approximation is done in a way, to reach an upper bound on Δt_i . Based on the characteristics of $|x(t)|$ and $|e(t)|$ for a stable system, we can make the assumption that the norm of the state vector $|x(t)|$ is decreasing and the norm of the error $|e(t)|$ increasing over the considered time interval. The inter-sampling time Δt_i can be seen as intersection between the function $|x(t)|$ and $\frac{|e(t)|}{\rho}$ as displayed in figure 3. Consequently, to get an upper bound on Δt_i , we have to find an upper bound for the left side of equation (9).

Theorem 4.1: The left part of equation (9) can be approximated as $|x_{i+1}| = |(e^{\Delta t_i A}F + H) \cdot x_i| \leq |x_i|$

Proof: As mentioned above, the event generation mechanism forces the system to fulfill ISS. Thus, the norm of the states $|x(t)|$ is a decreasing function [12] and therefore $|x_{i+1}| < |x_i|$. In other words, the right part of equation (9) is bounded by the norm of the previous sampled state vector $|x_i|$. ■

Based on the upper theorem and equation (9), we reformulate the problem by the following inequality

$$|x_i| \leq \frac{1}{\rho} |(I - e^{\Delta t_i A})F \cdot x_i| \quad (10)$$

Our goal is to find a Δt_i as small as possible which satisfies inequality (10). As our solution should be universally applicable, we separate the state space vector x_i from equation (10). This is done as we apply the approximation

$$|Mx| \geq \frac{|x|}{M^{-1}} \quad (11)$$

to the upper inequality leading to the following equation

$$\begin{aligned} 1 &\leq \frac{1}{\rho} \left| ((I - e^{\Delta t_i A})F)^{-1} \right|^{-1} \\ \rho \left| (e^{\Delta t_i A}F - F)^{-1} \right| &\leq 1. \end{aligned} \quad (12)$$

It is easy to see, that an evanescent time interval Δt_i would violate the inequality, whereas with a large time interval Δt_i the left part of equation (12) tends to zero.

The norm of equation (12) is the spectral norm of the included matrix, which is defined as their maximum singular value. Equation (12) can be written as

$$\rho \cdot \bar{\sigma} \left((e^{\Delta t_i A}F - F)^{-1} \right) \leq 1 \quad (13)$$

with $\bar{\sigma}(M)$ as maximum singular value from matrix M . To eliminate the inverse matrix operation from within this equation we make use of a singular value property. Due to the singular value definition, it can be shown that following property for singular values hold

$$\bar{\sigma}(M) = \frac{1}{\underline{\sigma}(M^{-1})} \quad (14)$$

with $\underline{\sigma}(M)$ as the minimum singular value of M . For equation (13) this results in

$$\begin{aligned} \rho \cdot \frac{1}{\underline{\sigma}(e^{\Delta t_i A}F - F)} &\leq 1 \\ \underline{\sigma}(e^{\Delta t_i A}F - F) &\geq \rho \end{aligned} \quad (15)$$

The left part of equation (15) maps the scalar Δt_i to the minimal singular value of the matrix function $M(\Delta t_i) = (e^{\Delta t_i A}F - F)$.

$$\Delta t_i \rightarrow \underline{\sigma}(M(\Delta t_i)) \quad (16)$$

Finally, our problem of finding a valid maximum inter-sampling time is equivalent to finding a time interval Δt_i . Hence all singular values of $M(\Delta t_i)$ have to be greater than ρ . Related to our problem, in mathematics there is the inverse singular value problem (ISVP). The ISVP tries to find a $c \in \mathbb{R}^n$ for which a matrix $L(c) \in \mathbb{R}^{n \times n}$ has a specific set of singular values $\sigma_1 \dots \sigma_n$. In [13] the authors describe a method to solve the ISVP. Our problem differs from ISVP in some aspects. First, our singular values should only exceed a threshold $\sigma_1 \dots \sigma_n \geq \rho$ and not equal predefined singular values. The second difference to the ISVP is, that our matrix function $M(\Delta t_i)$ only depends from a scalar $\Delta t_i \in \mathbb{R}$.

V. INFLUENCE ON DISTRIBUTED SYSTEM ARCHITECTURE

This section shows the effect of the improved minimum stimulation for event triggered control systems we derived in the last chapter. In a distributed system architecture, the timing of tasks and messages are coupled. This is because an extended or reduced execution time effects the start time of the next task in the task chain. A second reason is the shared resource usage of multiple tasks. The timing properties can be calculated by the above mentioned modular performance analysis from Wandeler [10]. This analysis framework extends the network calculus [11], which describes the flow of events through a network, whereas the real-time calculus focuses on the characteristics of embedded systems. The network calculus theory relies on the max-plus algebra. In the max-plus algebra, the convolution and deconvolution computations can be defined, which is used by Wandeler to calculate outgoing arrival and service curve based on their incoming correspondents.

$$\eta_{out}^+ = \min((\eta^+ \otimes \beta^+) \oslash \beta^-, \beta^+) \quad (17)$$

$$\eta_{out}^- = \min((\eta^- \oslash \beta^+) \otimes \beta^-, \beta^-) \quad (18)$$

$$\beta_{out}^+ = (\beta^+ - \eta^-) \overline{\otimes} 0 \quad (19)$$

$$\beta_{out}^- = (\beta^- - \eta^+) \overline{\otimes} 0 \quad (20)$$

The meaning of the upper equations and the definition of the convolution and deconvolution operations $\oslash, \otimes, \overline{\otimes}, \overline{\otimes}$ can be found in [10]. Arrival curve and service curve yield to the maximum response time over following equation

$$r^+ \leq \sup_{\lambda \geq 0} (\inf(\tau \geq 0 : \eta^+(\lambda) \leq \beta^-(\lambda + \tau))) \quad (21)$$

Baohua et al. proofed in [14] the isotonicity property of the convolution operator in the max-plus algebra. Isotonicity means, that if an operand of the convolution is increased, the result is increased. We use this property to determine the influence of an improved lower arrival curve.

Based on equation (18) the outgoing lower arrival curve (η_{out}^-) is increased, if the incoming lower arrival curve (η^-) of the task is increased. In other words, all tasks in the task chain of the event triggered control system have a higher minimum stimulation. Equation (19) says, the remaining upper service curve is decreased, if the incoming lower arrival curve is increased. This has an influence to all tasks on the same resource with smaller priority.

The upper equations (18) - (21) can be transferred in a simplified representation, showing only the direction of influence

$$\begin{aligned} \eta_{out}^+ \downarrow & \text{ if } \beta^+ \downarrow \text{ or } \eta^+ \downarrow \text{ or } \beta^- \uparrow \\ \eta_{out}^- \uparrow & \text{ if } \beta^+ \downarrow \text{ or } \eta^- \uparrow \text{ or } \beta^- \uparrow \\ \beta_{out}^+ \downarrow & \text{ if } \beta^+ \downarrow \text{ or } \eta^- \uparrow \\ \beta_{out}^- \uparrow & \text{ if } \beta^- \uparrow \text{ or } \eta^+ \downarrow \\ r^+ \downarrow & \text{ if } \beta^- \uparrow \text{ or } \eta^+ \downarrow \end{aligned}$$

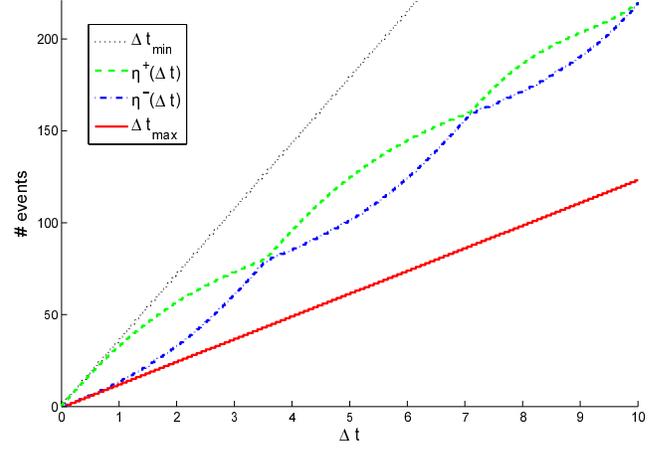


Fig. 4. Comparison of simulated upper and lower arrival curves (η^+, η^-) and corresponding calculated bounds.

VI. EXPERIMENTAL RESULTS

In this section we present the effect of the improved arrival curves, we extracted from the dynamics of the event triggered control system on the timing of the distributed system. The system, that we want to control has the following state-space equation

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t_i) \quad (22)$$

The system is stabilized by the feedback controller $u(t_i)$

$$u(t_i) = [1 \quad -4] x(t_i) \quad (23)$$

and the event generation mechanism $e(t)$

$$e(t) = 0.05 \cdot x(t_i) \quad (24)$$

With the usage of equation (15) we can calculate the maximum time between two events as $\Delta t_{max} = 80.9ms$. Similar to equation (1) we obtain the lower arrival curve as

$$\eta^-(\Delta t) = \left\lceil \frac{\Delta t}{\Delta t_{max}} \right\rceil - 1, \forall \Delta t > 0 \quad (25)$$

Figure 4 displays the arrival curves, that were obtained from a simulation of the event triggered control system for a duration of 10 seconds. The upper bound on the arrival curve was calculated from the lower bound on the inter-sampling time (28ms) as described in [7]. The lower curve was calculated using our method applying equations (25) and the solution of equation (15).

The effect of the improved arrival curve approximation is shown for a small distributed system architecture. The architecture is composed as displayed in figure 5. The task chain consisting of τ_1 over c_1 to τ_2 represents a distributed event triggered control system. The source ($S1$) stimulates a sensor-task with the above described arrival curve, build as composition of two periodic stimulations, such as the upper and lower curve in figure 4. Afterwards the sensor value

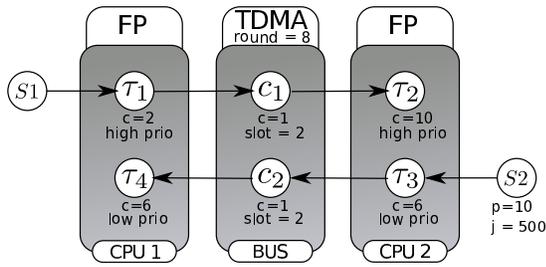


Fig. 5. System architecture

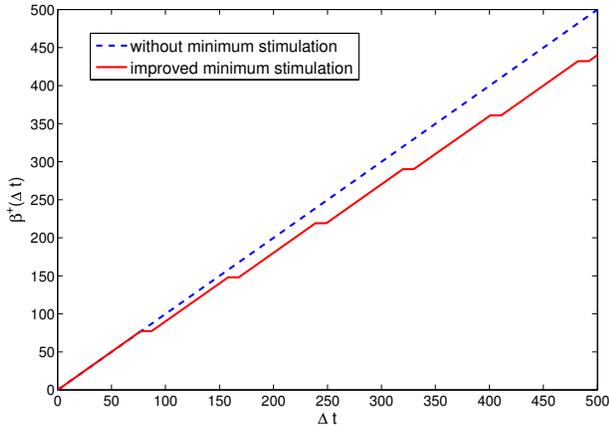


Fig. 6. Different upper service curves for task τ_3 for undefined and improved specification of the minimum stimulation of the event triggered control system.

is transmitted over a network, scheduled with time division multiple access (TDMA) policy. In task τ_2 the control value is calculated and allied to the plant. CPU1 and CPU2 are scheduled with fix priorities. The appropriate priorities and execution times are denoted under the appropriate task in figure 5.

As the arrival curve propagates through the distributed system architecture, we also see a higher lower bound on the stimulation on task τ_2 . As discussed above, this results in a reduction of the maximum remaining service curve, as displayed in figure 6. Due to the smaller priority of task τ_3 compared to τ_2 , the available service curve for τ_3 is the remaining service curve of τ_2 . Task τ_3 is stimulated by a source with a period of $10ms$ and a jitter of $500ms$. With the mathematics of the real time calculus, we can calculate a reduced outgoing upper arrival curve of task τ_3 . Finally we can see a huge effect on task τ_4 , where the maximum response time is reduced to $18ms$. The same system without knowledge of the lower arrival curve would cause a four times higher response time on task τ_4 of $72ms$.

As consequence, we can calculate less conservative, but still valid values for the system's time behavior by introducing a more realistic specification on the lower stimulation of an event triggered control system. With the new intelligence of the system, it may hold stronger deadlines or would allow a higher utilization.

VII. CONCLUSION AND FUTURE WORK

When applying event triggered control systems to real applications, it is necessary to perform a real-time analysis to obtain valid values for the timing behavior of the system. In distributed system architectures the timing behavior of one component affects the performance of the whole system. So it is necessary to define the inter-sampling time and therefore the stimulation of tasks as realistic as possible. We could show in our paper, that a disregarded minimum stimulation of tasks or a minimum stimulation of zero is an inadequate bound. Hence, we describe a method to calculate an improved minimum stimulation. This gives a system engineer the possibility to increase the utilization of the whole system.

As our lower bound on arrival curve is obtained from the maximum inter-sampling time it can be represented as a stepwise straight line. In our future work, we are interested in fining a lower bound on the arrival curve, which is closer to the real event density of the event generator. We are also interested in adopting our methods on systems with nonlinear system behavior and finding a relation between a disturbance specification of the control system and the resulting arrival curves.

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