Predictive Control of Networked Control Systems over Differentiated Services Lossy Networks

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***Abstract*—Networked control systems are feedback systems where plant and controller are connected through lossy wired/wireless networks. To mitigate communication delays and packet losses different control solutions have been proposed. In this work the model predictive control (MPC) has been improved by introducing transmission options offering different probabilities of packet drops (high priority service and low priority service). This Differentiated Services architecture introduces Quality-of-Service (QoS) guarantees and can be used to jointly design the control command and the transmission strategy. A novel MPC-QoS controller is proposed and its design is obtained by solving a mixed integer quadratic problem.**

I. Introduction

The study of Networked Control Systems (NCS) has produced over the past decades a large amount of research papers both from the theoretical and practical perspectives [6]. Networked control systems are spatially distributed systems where the communication between controller and plant occurs through lossy (wired/wireless) network. They are extremely useful for example to operate in dangerous environment (e.g. nuclear plant maintenance) or to improve accuracy and safety in teleoperated plant (e.g. robotic surgery).

The main issue in assuring plant controllability in NCS is related to the presence of a packet-based network which may introduce communication delay and packet dropouts. The research was mainly focused on adapting the linear quadratic (LQ) approach and the model predictive control (MPC) to this kind of systems. The crucial point is about the analytical model of the communication delays and packet dropouts. In [17], [18] packet dropout has been modeled as a Bernoulli random process. We improve this network model by considering a more complex communication architecture where different transmission options with different values of packet loss probability (still modeled as Bernoulli processes) are available. This behavior is produced by novel techniques to introduce Quality-of-Service (QoS) guarantees in IP networks such as the Differentiated Services (DiffServ) architecture ([12]) according to which packets are marked depending to their importance and sent through the network by using either a high-priority low-loss class, or a regular, unguaranteed class. DiffServ architecture has been proposed initially for multimedia communications ([5]) and recently extended on networked control systems (e.g. [11]).

The paper is organized as follows. Section II defines the mathematical model of the system with all the input/output signals and disturbances. The MPC design is briefly recalled to set the notation and the MPC problem is stated with reference to QoS-based transmission. The solution of the problem based on the mixed-integer quadratic programming (MIQP) is described in Section III. Simulation results are reported in Section IV, while conclusions are drawn in Section V.

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II. Background and Problem Statement

In this work we assume that the plant is a linear time-invariant discrete-time full-information system

$$P(z): \begin{cases} x_{k+1} &= Ax_k + Bu_k \\ y_k &= x_k \end{cases} \tag{1}$$

controlled by $C(z)$ located on the other side of a packet-based communication network as shown in Figure 1. The goal is to

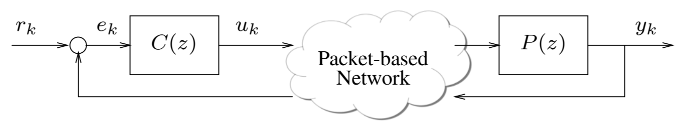


Fig. 1. Block diagram of a networked control system.

combine the classical MPC control design (section II-A) with the Differentiated Services network architecture (section II-B).

A. Review of MPC

For the linear plant (1), the quadratic cost function $J_{MPC}(\cdot)$ is defined as

$$J_{MPC}(k) = \sum_{i=0}^{N_p} \|\hat{x}_{k+i|k}\|^2_{Q_i} + \sum_{i=0}^{N_c} \|\hat{u}_{k+i|k}\|^2_{R_i} \tag{2}$$

where $Q_i > 0$, $R_i > 0$ are weighting matrices, N_p is the prediction horizon, N_c is the control horizon, and $\hat{x}_{k+i|k}$ and $\hat{u}_{k+i|k}$ are the i-ahead predictors of the state and of the command, respectively. It is common to assume $N_c < N_p$ and $\hat{u}_{k+i|k} = 0$ for $i \geq N_c$). The MPC control u_k at time k is computed based on the following algorithm ([9]):

1) get the new state measurement x_k,
2) solve the constrained optimization problem

$$\begin{aligned} \{\hat{u}^\star(k+\cdot|k)\} = \arg \min_{\{\hat{u}(k+\cdot|k)\}} \quad & J_{MPC}(k) \\ \text{subject to:} \quad & m_u \leq u_{k+\cdot|k} \leq M_u, \\ & m_x \leq x_{k+\cdot|k} \leq \bar{M}_x, \end{aligned}$$

where $\{m_u, M_u\}$ and $\{m_x, M_x\}$ are upper and lower bounds for the input and the state, respectively,

3) set $u_k = \hat{u}^\star_{k+0|k}$.

By re-writing the evolution of the system over the optimization horizon N_p as

$$\hat{X}(k) = \mathcal{A}x_k + \mathcal{B}\hat{U}(k)$$

where

$$\hat{X}(k) := \begin{bmatrix} \hat{x}_{k|k} \\ \hat{x}_{k+1|k} \\ \vdots \\ \hat{x}(k+N_p|k) \end{bmatrix}, \hat{U}(k) := \begin{bmatrix} \hat{u}_{k|k} \\ \hat{u}_{k+1|k} \\ \vdots \\ \hat{u}(k+N_c|k) \end{bmatrix},$$

and $\mathcal{A}$, $\mathcal{B}$ are appropriate block-matrices, the cost function becomes

$$\begin{aligned} J_{MPC}(k) \quad = \quad & \hat{U}^T(k)\underbrace{\left[\mathcal{B}^T\mathcal{Q}\mathcal{B}+\mathcal{R}\right]}_{\mathcal{H}}\hat{U}(k) + \\ & + \underbrace{2x_k^T\mathcal{A}^T\mathcal{Q}\mathcal{B}}_{\mathcal{G}}\hat{U}(k) + \underbrace{x_k^T\mathcal{A}^T\mathcal{Q}\mathcal{A}x_k}_{\mathcal{K}} \end{aligned}$$

where $\mathcal{Q} = \mathrm{diag}\{Q_i\}_{i=0}^{N_p}$, $\mathcal{R} = \mathrm{diag}\{R_i\}_{i=0}^{N_c}$ and the last term $\mathcal{K}$ is independent of $\hat{U}(k)$. The constraints on the command and on the state can be easily written as:

$$\underline{U} \leq \hat{U}(k) \leq \bar{U}, \qquad \underline{X} - \mathcal{A}x_k \leq \mathcal{B}\hat{U}(k) \leq \bar{X} - \mathcal{A}x_k \quad (3)$$

where $\underline{U}$, $\bar{U}$, $\underline{X}$ and $\bar{X}$ are matrices containing the upper and lower values. The optimal control $\hat{U}^\star(k)$ on the horizon $[0, N_c]$ is computed by solving the constrained Quadratic Programming (QP) problem

$$\begin{aligned} \hat{U}^\star(k) = \quad & \arg\min_{\hat{U}(k)} \quad \hat{U}^T(k)\mathcal{H}\hat{U}(k) + \mathcal{G}\hat{U}(k) \\ & \text{s.to} \qquad (3). \end{aligned}$$

The optimal control $\hat{u}^\star_k$ at time k is the first component in $\hat{U}^\star(k)$, i.e. $\hat{u}^\star_{k|k}$.

In this work we made the following stability assumption:

Assumption 1: The matrix A is asymptotically stable.

Under this assumption, it is possible to show that by setting $Q_1 = \ldots = Q_{N_p-1} = Q \geq 0$ and Q_{N_p} equal to the solution of the Lyapunov equation $A^TPA - A = Q$, the closed loop system with the above MPC controller is asymptotically stable (for details, see [3], [1], [10], [15]). Other approaches to guarantee stability can be found in [9].

Moreover we assume without loss of generality that

Assumption 2: The matrix B has full column rank.

The prediction of state and command values is the basis of the design of the optimal MPC controller; it could be improved by knowing the reliability level of the network upon which commands and measurements are sent. This original contribution of the work will be described in Section II-C.

B. Differentiated Services on the Network

With reference to Figure 1, commands u_k and measurements y_k may be affected by transmission delay and packet loss which compromise NCS performance. The Differentiated Services network architecture can be used to control such delays and losses without increasing the use of network resources. It consists in assigning a different forwarding priority to each packet. Without loss of generality, this work deals with two transmission priorities, i.e., H and L. Intermediate network systems favor the forwarding of H packets which therefore experience lower delay and lower loss probability than L packets. The control of the fraction of H packets on the overall traffic amount is crucial for the success of the mechanism; clearly, if all packets are sent with high priority, the un-differentiated case is re-established.

Another contribution of the work is the use of Differentiated Services to enhance control performance of NCS; in this scenario, each message (i.e., commands or measurements) is analyzed and then transmitted as either high- or low-priority packet. From the control perspective, packets sent with the H policy lead to better control performance than packets sent with the L policy, provided that the H fraction is kept low. Therefore, the optimal marking strategy can be formalized by assigning a cost to each policy and minimizing the total cost under performance constraints. Clearly, to maximize performance, the most important packets should be sent with the H policy as demonstrated in literature in the context of multimedia communication [5].

The identification of the most important packets can be performed during the MPC optimization which is affected by network reliability as said at the end of Section II-A. Therefore, the work aims at jointly designing the optimal MPC controller and the DiffServ marking strategy as described in Section II-C. Furthermore, the packet loss probability also depends on the forwarding priority.

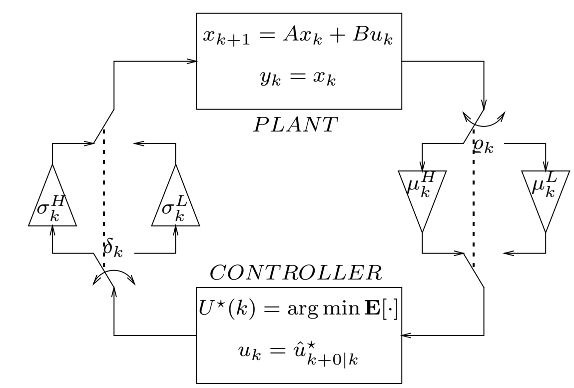


Fig. 2. Block diagram of a NCS with Differentiated Services network.

C. Problem statement

Figure 2 shows the block diagram of a NCS with a two-classes Differentiated Services network; the data paths from controller to plant and viceversa can be decomposed into two "virtual wires" representing the forwarding services, i.e., H and L, characterized by different loss probabilities. The effect of packet loss on the transmitted data can be modeled by multiplying it with a binary random variable (r.v.). Let $\sigma_k^H, \sigma_k^L, \mu_k^H, \mu_k^L$ be independent and identically distributed Bernoulli variables. For instance, the Bernoulli variables σ_k^H, σ_k^L take value 1 with probability

$$P\{\sigma_k^L = 1\} = \bar{\sigma}^L, \qquad P\{\sigma_k^H = 1\} = \bar{\sigma}^H,$$

where $\bar{\sigma}^H > \bar{\sigma}^L$ because the H service is more reliable than service L. The original model (1) becomes

$$\left\{ \begin{array}{rcl} x_{k+1} & = & Ax_k + \sigma_k^{\pi_f} Bu_k \\ y_k & = & \mu_k^{\pi_b} x_k \end{array} \right. \tag{4}$$

where $\pi_f, \pi_b \in \{H, L\}$, i.e. $\sigma_k^{\pi_f} \in \{\sigma_k^H, \sigma_k^L\}$ and $\mu_k^{\pi_b} \in \{\mu_k^H, \mu_k^L\}$. Let δ_k and ϱ_k be the binary variables representing the marker strategies for commands and measurements, respectively. For instance, δ_k represents the priority for the control command at time k, as follows:

$$\delta_k = \left\{ \begin{array}{ll} 1, & \text{high priority service,} \\ 0, & \text{low priority service.} \end{array} \right.$$

To simplify the analysis, the marking strategy is limited to the controller-to-plant path while the plant-to-controller path is assumed to be reliable, i.e., ϱ_k, μ_k^H and μ_k^L are not used in the model. Therefore, the model (4) can be re-written as

$$\left\{ \begin{array}{rcl} x_{k+1} & = & Ax_k + \left[(1-\delta_k)\sigma_k^L + \delta_k \sigma_k^H\right] Bu_k \\ y_k & = & x_k. \end{array} \right. \tag{5}$$

The equation links the marking strategy (represented by the binary variable δ_k) with the channel behavior (represented by the random variable σ_k) and the control command (represented by the value u_k) at time k. In fact, with $\delta_k = 1$ the channel behavior is described by the Bernoulli variable σ_k^H, otherwise σ_k^L is considered.

The performance index $J_{MPC}(k)$ defined in (2) has now to be modified for two reasons:

1) the performance index has to penalize the use of the high priority, otherwise the control architecture should always select the H service. The new index is

$$J_{MPC-QoS}(k) = J_{MPC}(k) + \sum_{i=0}^{N_c} \left[\|\delta_{k+i}\|^2_{W_i}\right]$$

where the weights W_i are positive;

2) the plant (5) is now a stochastic system due to the randomness introduced by the Bernoulli processes. This means that the performance index needs the expectation operator. However, since the state is known at time k, the conditional expectation given the state x_k is used as follows:

$$\ell_{MPC-QoS}(k) := \mathbf{E}\left[J_{MPC-QoS}(k)|\, x_k\right]. \tag{6}$$

Let MPC-QoS be the name of the following problem:

Problem 3: (Model Predictive Control problem over lossy networks) Given the system (5), find the optimal control $u_k^\star$ and the optimal transmission strategy $\delta_k^\star$ for the corresponding packet by solving the stochastic MPC-QoS problem

$$\left\{ \begin{array}{rcl} \{u_k^\star, \delta_k^\star\} = & \arg\min & \mathbf{E}[J_{MPC-QoS}(k)|x_k] \\ & \text{subject to} & m_u \leq \hat{u}_{k+i|k} \leq M_u \\ & & m_x \leq \hat{x}_{k+i|k} \leq M_x \\ & & \delta_k \in \{0,1\} \\ & & \sigma_k^L, \sigma_k^H \ \text{i.i.d. Bernoulli} \end{array} \right.$$

In the following section, the conditional expectation will be computed in order to re-write the above problem in a solvable mixed integer quadratic programming (MIQP) problem.

III. Solution of MPC-QoS

This section presents a solution of the stochastic MPC-QoS problem defined above by rewriting the minimization problem as a Linear Quadratic Programming problem. To reach this goal, the first step is to recognize that the system (5) belongs to the family of Mixed Logical Dynamical (MLD) systems introduced in [2] as

$$\left\{ \begin{array}{rcl} x_{k+1} & = & Ax_k + B_u u_k + B_\delta \delta_k + B_a a_k \\ y_k & = & Cx_k + D_u u_k + D_\delta \delta_k + D_a a_k \\ E & \leq & E_x x_k + E_u u_k + E_\delta \delta_k + E_a a_k \end{array} \right.$$

where x_k is the state, u_k is the input, y_k is the output, δ_k is a logical variable (0-1 variable) and a_k is an auxiliary variable. The components within the auxiliary variable usually take care of the products between decision variables and states or inputs.

By defining the auxiliary variable $a_k = \delta_k u_k$, the state equation takes the form

$$\begin{array}{rcl} x_{k+1} & = & Ax_k + \left[(1-\delta_k)\sigma_k^L + \delta_k\sigma_k^H\right] Bu_k \\ & = & Ax_k + \sigma_k^L Bu_k + \left[-\sigma_k^L + \sigma_k^H\right] Ba_k. \end{array}$$

The decision variable "disappears" and an auxiliary variable shows up. The two equations are equivalent if and only if the following inequalities hold

$$a_k \leq M_u \delta_k \tag{7}$$
$$a_k \geq m_u \delta_k \tag{8}$$
$$a_k \leq u_k - m_u(1-\delta_k) \tag{9}$$
$$a_k \geq u_k - M_u(1-\delta_k) \tag{10}$$

where $M_u = \max\{u(\cdot)\}$ and $m_u = \min\{u(\cdot)\}$ are vectors defining the range of the commands, [2].

With respect to the original MLD model, the present state equation is inherently stochastic: there are products of stochastic variables with input and auxiliary variables. The reader can refer to the papers [4], [14], [8] and the reference therein for a presentation of the stochastic MPC and for the analysis of the stability of MPC controllers applied to hybrid systems.

The second step has to bring the constrained minimization $\min \mathbf{E}[J_{MPC-QoS}(k)|x_k]$ into a LQ programming problem. By explicitly computing the conditional expectations, we will end up with a particular Mixed Integer LQ problem where the integer variables are now of the 0-1 type.

Using for $\hat{X}(k)$ the same matrix notation as before, and the following for the terms $\hat{u}_{k+i|k}\sigma_{k+i}^L$, $\hat{a}_{k+i|k}(-\sigma_{k+i}^L + \sigma_{k+i}^H)$ and $\delta_{k+i|k}$

$$\hat{U}_\sigma(k) = \begin{bmatrix} \hat{u}_{k|k}\sigma_k^L \\ \hat{u}_{k+1|k}\sigma_{k+1}^L \\ \vdots \\ \hat{u}_{k+N_c|k}\sigma_{k+N_c}^L \end{bmatrix}, \hat{\Delta}(k) = \begin{bmatrix} \delta_{k|k} \\ \delta_{k+1|k} \\ \vdots \\ \delta_{k+N_c|k} \end{bmatrix}$$

$$\hat{A}_\sigma(k) = \begin{bmatrix} \hat{a}_{k|k}(-\sigma_k^L + \sigma_k^H) \\ \hat{a}_{k+1|k}(-\sigma_{k+1}^L + \sigma_{k+1}^H) \\ \vdots \\ \hat{a}_{k+N_c|k}(-\sigma_{k+N_c}^L + \sigma_{k+N_c}^H) \end{bmatrix}$$

the matrix notation of the state equation is

$$\hat{X}(k) = \mathcal{A}x_k + \mathcal{B}\hat{U}_\sigma(k) + \mathcal{B}\hat{A}_\sigma(k), \tag{11}$$

whereas the index $J_{MPC-QoS}$ becomes

$$\begin{aligned} J_{MPC-QoS} &= \hat{X}(k)^T\mathcal{Q}\hat{X}(k) + \hat{U}^T(k)\mathcal{Q}\hat{U}(k) + \\ &\quad + \Delta^T(k)\mathcal{W}\Delta(k) \end{aligned} \tag{12}$$

with $\mathcal{W} = \mathrm{diag}\{W_i\}_{i=0}^{N_c}$. Since the index weights the binary variables through the matrix $\mathcal{W}$, there is no need to also weight the auxiliary vector $\hat{A}_\sigma(k)$. The constraints for the problem are (3) and $\Delta(k) \in \{0,1\}^{N_c}$. Moreover, since the inequalities (7)-(10) have to hold $\forall k$, they can be put in matrix form as

$$\mathcal{E}_u\hat{U}(k) + \mathcal{E}_a\hat{A}(k) + \mathcal{E}_\delta\Delta(k) \leq \mathcal{E} \tag{13}$$

where $\hat{A}(k)$ is equal to $\hat{A}_\sigma(k)$ without the contribution of the Bernoulli r.v., and for opportune matrices $\mathcal{E}_u, \mathcal{E}_a, \mathcal{E}_\delta, \mathcal{E}$.

We will now analyze one by one all the kinds of product that can occur within the performance index (12) when the state equation (11) is inserted. Let

$$N_{i,\ell} \triangleq (A^i)^T Q_p A^\ell B, \qquad M_{\ell,q} \triangleq B^T(A^\ell)^T Q_k A^q B$$

be the generic matrix products. Since all the terms $\hat{x}_{k+i|k}, \hat{u}_{k+i|k}, \hat{a}_{k+i|k}$ are measurable w.r.t. x_k by construction and by using well known results about the Bernoulli random variable and the conditional expectation, [13], the following expectations can be derived:

<u>state–input</u>

$$\mathbf{E}[x_k^T N_{0,i}\sigma_{k+i}^L \hat{u}_{k+i|k}|x_k] = x_k^T N_{0,i}\hat{u}_{k+i|k}\bar{\sigma}^L$$

<u>state–auxiliary variable</u>

$$\mathbf{E}[x_k^T N_{0,j}(-\sigma_{k+j}^L + \sigma_{k+j}^H)\hat{a}_{k+j|k}|x_k] = x_k^T N_{0,j}\hat{a}_{k+j|k}(-\bar{\sigma}^L + \bar{\sigma}^H)$$

<u>input–input</u>

$$\begin{aligned} &\mathbf{E}[\sigma_{k+i}^L \hat{u}_{k+i|k}^T M_{i,j}\sigma_{k+j}^L \hat{u}_{k+j|k}|x_k] = \\ &= \begin{cases} \hat{u}_{k+i|k}^T M_{i,j}\hat{u}_{k+j|k}\bar{\sigma}^L, & \text{if } i = j \\ \hat{u}_{k+i|k}^T M_{i,j}\hat{u}_{k+j|k}(\bar{\sigma}^L)^2, & \text{if } i \neq j \end{cases} \end{aligned}$$

<u>input–auxiliary variable</u>

$$\begin{aligned} &\mathbf{E}[\sigma_{k+i}^L \hat{u}_{k+i|k}^T M_{i,j}(-\sigma_{k+j}^L + \sigma_{k+j}^H)\hat{a}_{k+j|k}|x_k] = \\ &= \begin{cases} \hat{u}_{k+i|k}^T M_{i,j}\hat{a}_{k+j|k}(-\bar{\sigma}^L + \bar{\sigma}^L\bar{\sigma}^H), & \text{if } i = j \\ \hat{u}_{k+i|k}^T M_{i,j}\hat{a}_{k+j|k}(-(\bar{\sigma}^L)^2 + \bar{\sigma}^L\bar{\sigma}^H), & \text{if } i \neq j \end{cases} \end{aligned}$$

<u>auxiliary variable–auxiliary variable</u>

$$\begin{aligned} &\mathbf{E}[(-\sigma_{k+i}^L + \sigma_{k+i}^H)\hat{a}_{k+i|k}^T M_{i,j}(-\sigma_{k+j}^L + \sigma_{k+j}^H)\hat{a}_{k+j|k}|x_k] = \\ &= \hat{a}_{k+i|k}^T M_{i,j}\hat{a}_{k+j|k}\mathbf{E}[(-\sigma_{k+i}^L + \sigma_{k+i}^H)(-\sigma_{k+j}^L + \sigma_{k+j}^H)] \\ &= \begin{cases} \hat{a}_{k+i|k}^T M_{i,j}\hat{a}_{k+j|k}(\bar{\sigma}^L + \bar{\sigma}^H - 2\bar{\sigma}^L\bar{\sigma}^H), & \text{if } i = j \\ \hat{a}_{k+i|k}^T M_{i,j}\hat{a}_{k+j|k}(\bar{\sigma}^H - \bar{\sigma}^L)^2, & \text{if } i \neq j \end{cases} \end{aligned}$$

Based on the definitions

$$\begin{aligned} \mathcal{N} &:= \mathcal{A}^T\mathcal{Q}\begin{bmatrix} \mathcal{B} & \mathcal{B} \end{bmatrix} = \begin{bmatrix} \mathcal{N}^u & \mathcal{N}^a \end{bmatrix} \\ \mathcal{M} &:= \begin{bmatrix} \mathcal{B} & \mathcal{B} \end{bmatrix}^T\mathcal{Q}\begin{bmatrix} \mathcal{B} & \mathcal{B} \end{bmatrix} = \begin{bmatrix} \mathcal{M}^{uu} & \mathcal{M}^{ua} \\ \mathcal{M}^{au} & \mathcal{M}^{aa} \end{bmatrix} \end{aligned}$$

with $\mathcal{N}^u = \{\mathcal{N}_{i,j}^u\}_{i,j=0}^{N_c}$, $\mathcal{N}^a = \{\mathcal{N}_{i,j}^a\}_{i,j=0}^{N_c}$ and the same for the four blocks of $\mathcal{M}$, the index becomes

$$\begin{aligned} \mathbf{E}\left[J_{MPC-QoS}|\, x_k\right] &= x_k^T\mathcal{A}^T\mathcal{Q}\mathcal{A}x_k + \hat{U}^T(k)\mathcal{R}\hat{U}^T(k) + \\ &+ \hat{\Delta}^T(k)\mathcal{W}\hat{\Delta}^T(k) + 2x_k^T\mathcal{N}^\sigma\begin{bmatrix} \hat{U}(k) \\ \hat{A}(k) \end{bmatrix} + \\ &+ \begin{bmatrix} \hat{U}(k) \\ \hat{A}(k) \end{bmatrix}^T\mathcal{M}^\sigma\begin{bmatrix} \hat{U}(k) \\ \hat{A}(k) \end{bmatrix} \end{aligned}$$

where

$$\mathcal{N}^\sigma = \begin{bmatrix} \mathcal{N}^{\sigma u} & \mathcal{N}^{\sigma a} \end{bmatrix}, \qquad \mathcal{M}^\sigma = \begin{bmatrix} \mathcal{M}^{\sigma uu} & \mathcal{M}^{\sigma ua} \\ \mathcal{M}^{\sigma au} & \mathcal{M}^{\sigma aa} \end{bmatrix}$$

are defined according to the equalities derived above as

$$\begin{aligned} \mathcal{N}_{i,j}^{\sigma u} &= \mathcal{N}_{i,j}^u\bar{\sigma}^L \\ \mathcal{N}_{i,j}^{\sigma a} &= \mathcal{N}_{i,j}^a(-\bar{\sigma}^L + \bar{\sigma}^H) \\ \mathcal{M}_{i,j}^{\sigma uu} &= \begin{cases} \mathcal{M}_{i,j}^{uu}\bar{\sigma}^L, & \text{if } i = j \\ \mathcal{M}_{i,j}^{uu}(\bar{\sigma}^L)^2, & \text{if } i \neq j \end{cases} \\ \mathcal{M}_{i,j}^{\sigma ua} &= \begin{cases} \mathcal{M}_{i,j}^{ua}(-\bar{\sigma}^L + \bar{\sigma}^L\bar{\sigma}^H), & \text{if } i = j \\ \mathcal{M}_{i,j}^{ua}(-(\bar{\sigma}^L)^2 + \bar{\sigma}^L\bar{\sigma}^H), & \text{if } i \neq j \end{cases} \\ \mathcal{M}_{i,j}^{\sigma aa} &= \begin{cases} \mathcal{M}_{i,j}^{aa}(\bar{\sigma}^L + \bar{\sigma}^H - 2\bar{\sigma}^L\bar{\sigma}^H), & \text{if } i = j \\ \mathcal{M}_{i,j}^{aa}(\bar{\sigma}^H - \bar{\sigma}^L)^2, & \text{if } i \neq j \end{cases} \end{aligned}$$

The performance index is now deterministic and can be put in the usual form

$$\min_{V(k)} V^T(k)\mathcal{H}V(k) + \mathcal{P}V(k) + \mathcal{K}(k)$$

where the unknown vector is $V(k) := \begin{bmatrix} \hat{U}(k) & \hat{A}(k) & \Delta(k) \end{bmatrix}^T$. The matrices $\mathcal{H}$ and $\mathcal{P}$ take the form

$$\begin{aligned} \mathcal{H} &= \begin{bmatrix} \mathcal{M}^{\sigma uu} + \mathcal{R} & \mathcal{M}^{\sigma ua} & 0 \\ \mathcal{M}^{\sigma au} & \mathcal{M}^{\sigma aa} & 0 \\ 0 & 0 & \mathcal{W} \end{bmatrix}, \\ \mathcal{P} &= 2x_k^T\begin{bmatrix} \mathcal{N}^{\sigma u} & \mathcal{N}^{\sigma a} & 0 \end{bmatrix} \end{aligned}$$

whereas $\mathcal{K}(k)$ collects all the contributions independent of $V(k)$. Since the constraints (3), (13) can be written as $\mathcal{C}V(k) \leq \mathcal{D}$, the mixed integer quadratic programming (MIQP) related to the MPC-QoS problem is

$$\begin{aligned} V^\star(k) &= \arg\min_{V(k)} & &V^T(k)\mathcal{H}V(k) + \mathcal{P}V(k) \\ &\text{s. to} & &\mathcal{C}V(k) \leq \mathcal{D} \\ & & &\Delta_f(k) \in \{0,1\}^{N_u} \end{aligned} \tag{14}$$

Collecting these results we can say that the following theorem holds:

Theorem 4: The stochastic MPC-QoS Problem 3 is equivalent to the deterministic MIQP problem in (14).

In [2] this kind of controller are called mixed integer predictive controller. The matrix $\mathcal{H}$ is positive definite because $\mathcal{R}$ and $\mathcal{Q}$ are positive definite and thanks to the following result (together with the matrix inversion lemma):

Proposition 5: Let σ^L and σ^H be real numbers such that $0 < \sigma^L < \sigma^H < 1$. If the weighting matrix Q is positive definite and the Assumption 2 holds, then the matrices $\mathcal{M}^{\sigma uu}$ and $\mathcal{M}^{\sigma dd}$ are positive definite.

Proof. Since $Q > 0$ and Assumption 2 holds, the matrices $\mathcal{M}^{uu}$ and $\mathcal{M}^{dd}$ are both equal to $\mathcal{B}^T\mathcal{Q}\mathcal{B}$ and positive definite by construction. The matrix $\mathcal{M}^{\sigma dd}$ can be written as

$$\mathcal{M}^{\sigma dd} = (\bar{\sigma}^H - \bar{\sigma}^L)^2 \left(\mathcal{M}^{dd} + (k-1)\mathcal{D}\right)$$

where $\mathcal{D}$ has on the diagonal the elements (i,i) of $\mathcal{M}^{dd}$ and zero otherwise and k is given by

$$k = \frac{\bar{\sigma}^L + \bar{\sigma}^H - 2\bar{\sigma}^L\bar{\sigma}^H}{(\bar{\sigma}^H - \bar{\sigma}^L)^2}$$

Since $0 < \sigma^L < \sigma^H < 1$, the constant k is a real number larger than one. To prove that the eigenvalues of $\mathcal{M}^{dd} + (k-1)\mathcal{D}$ are positive, we will show that λ_1^σ is larger than λ_1, where λ_1 and λ_1^σ are the smaller eigenvalues of $\mathcal{M}^{dd}$ and $\mathcal{M}^{dd} + (k-1)\mathcal{D}$, respectively. The Rayleigh-Rits theorem ([7]) tells us that the smaller eigenvalue of a Hermitian matrix $A \in \mathbb{C}^{n\times n}$ is $\lambda_{min} = \min_{\mathbb{C}^n \ni x \neq 0} \frac{x^*Ax}{x^*x}$. In the present case the following relationships holds

$$\lambda_1^\sigma = \min_{x\neq 0} \frac{x^T(\mathcal{M}^{dd} + (k-1)\mathcal{D})x}{x^Tx} \geq$$
$$\geq \min_{x\neq 0} \frac{x^T\mathcal{M}^{dd}x}{x^Tx} + \underbrace{(k-1)\min_{x\neq 0}\frac{x^T\mathcal{D}x}{x^Tx}}_{\geq 0} \geq \lambda_1.$$

proving the theorem. The same line of reasoning can be applied to $\mathcal{M}^{\sigma uu}$. □

When the optimal solution $V^\star(k) = \begin{bmatrix} \hat{U}^\star(k) & \hat{A}^\star(k) & \Delta^\star(k) \end{bmatrix}^T$ of the problem (14) is available, the optimal control and the optimal transmission strategy at time k are

$$\begin{aligned} u^\star(k) &= \begin{bmatrix} I & 0 & \cdots & 0 \end{bmatrix} \hat{U}^\star(k) \\ \delta^\star(k) &= \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \Delta^\star(k). \end{aligned}$$

In this work we do not analyze the stability issue related to the closed loop control neither the feasibility of the MIQP problem. However, the present formulation can be written in a form where stability results exist (e.g. [9], [10], [1], [15]). We simply claim that for stable plant, there exist initial conditions x_0 and weighting matrices such that stability and feasibility can be guaranteed.

IV. Simulation Results

The MPC-QoS controller is used to steer to zero the system with matrices and initial condition equal to

$$A = \begin{bmatrix} 0.99 & 0 \\ 0 & 0.9 \end{bmatrix}; \quad B = \begin{bmatrix} 0.540 \\ 0.6 \end{bmatrix}; \quad x_0 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

The sample rate is 50 ms and the weighting matrices in the index (6) are time-invariant and equal to

$$Q = \begin{bmatrix} 40 & 0 \\ 0 & 20 \end{bmatrix}; \quad R = 2; \quad W = 10.$$

The control and prediction horizons are both equal to $N = 10$. In this example the input command is constrained to belong to the interval $[-5, 5]$.

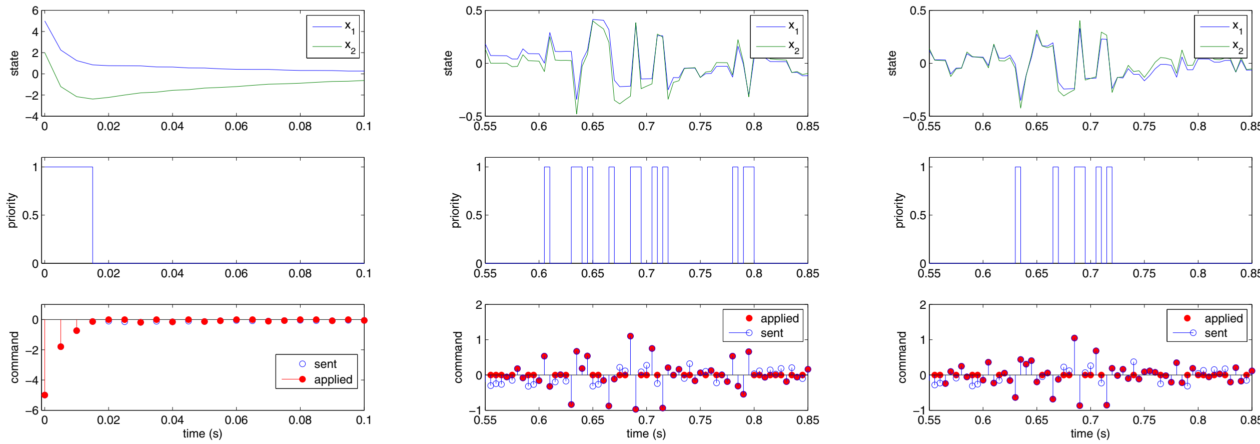
The proposed control strategy is tested on three cases:

(a) the MPC-QoS steers to zero the state starting from the non-zero initial condition; the high priority and the low priority services have a loss rate of 10% and 50%, respectively; i.e. $\bar{\sigma}^H = 0.9$ and $\bar{\sigma}^L = 0.5$;
(b) the measurements of the state are affected by Gaussian white noise with zero mean and variance 0.05; the Bernoulli r.v. have $\bar{\sigma}^H = 0.9$ and $\bar{\sigma}^L = 0.5$, respectively;
(c) same as (b) with $\bar{\sigma}^H = 0.9$ and $\bar{\sigma}^L = 0.65$.

The goal of these simulation experiments is to show that the control strategy adopts the high priority service when the commands to be sent to the plant are "important" according to the performance index. The importance is strictly related to the weighting matrices Q and R and to the cost of the H service (matrix W). Moreover, the policy choice is also related to the $\bar{\sigma}^H$ and $\bar{\sigma}^L$. For simplicity's sake, only the forwarding priority from controller to plant is taken into account here.

Figure 3(a) shows the time series for the two components of the state variable x_k, the optimal marking strategy $\delta_k^\star$ and the sent and applied commands. The command computed by the MPC-QoS controller $u_{k|k}^\star$ is sent to the network through the service H if $\delta_k^\star$ is set to 1, and through the service L if $\delta_k^\star$ is set to 0. The plot of $\delta_k^\star$ shows that at the beginning the priority is set to H because the state error is large. After the third sample the priority is set to zero and even if the packet loss probability is higher, this is the optimal solution with respect to the selected weighting matrices. The bottom plot in the figure shows with empty blue circles the sent commands and with full red circles the applied commands ($\sigma_k^H u_{k|k}^\star$ or $\sigma_k^L u_{k|k}^\star$). If there are no losses, these values are exactly the same, otherwise the applied command is zero. It is worth noting that after the third sample the value of the sent commands is close to zero (as in case of packet loss) and therefore the H policy is not used. The comparison between setting the command to zero or equal to the previous command when the packet is lost is described in detail in [16] where it is proven that the optimal solution is related to the plant dynamics.

Figures 3(b) and 3(c) reports the same plots when a noise is added to the state measurements and when σ_k^L changes from 0.5 to 0.65. In these cases, the priority is set H not only at the beginning (not shown in these figures) but also later when the noise brings the state far away from zero. Since the weight W is kept fixed in these two cases, the control architecture uses more the H policy in the first case because the packet loss probability is higher. In other words, MPC-QoS control strategy uses the H policy either when commands are important to compensate large errors or when the L service is particularly unreliable as in case of sudden network problems.



(a) $x_0 \neq 0$ (b) $\bar{\sigma}^H = 0.9$, $\bar{\sigma}^L = 0.5$ (c) $\bar{\sigma}^H = 0.9$ and $\bar{\sigma}^L = 0.65$

Fig. 3. Closed loop regulation: (a) $x_0 \neq 0$; (b),(c) noise added to the state measurements.

V. Conclusion

In this paper a novel model predictive controller for networked control systems has been proposed. The improvement with respect to the available solutions in literature is the introduction of the Differentiated Services architecture in the communication channel. Two forwarding policies are considered, i.e., a high-priority high-cost service, and a low-priority and low-cost service. The control architecture jointly designs the optimal command and the optimal transmission strategy. Performance improvement is obtained by using the communication channel in a smarter way, i.e., the high cost and more reliable service is used only when important packets have to be sent.

This paper focused mainly on the presentation of the idea behind the coupling of the DiffServ architecture with the MPC. However, several points deserving a deeper study have not been introduced to simplify the explanation. Future work will integrate the following aspects:

- introduction of packet losses also on the plant-to-controller channel and taking into account their contribution on the performance index $J_{MPC-QoS}$; this implies that some computational power should be available at the plant side (e.g. smart sensor, decentralized MPC) or that the transmission strategy of the measurement is also computed by the controller and sent to the other side through the network,
- design of an estimator at the controller side whenever the state of the plant is not available,
- extension of the proposed approach to more complex packet loss models such as the Markov model.

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